

Pascal's Law/ Force of a hydraulic lift

= Use reasoning to solve equations and justify the solution method

Program Task: NMTCC AR-14: Industrial Pneumatics

PA Core Standard: CC.2.2.HS.D.9

POS 2100: Demonstrate knowledge of fluid power systems

Description: Use reasoning to solve equations and justify the solution method

Program Associated Vocabulary:

PRESSURE, FORCE, AREA OF A PISTON, LOAD, BORE (DIAMETER)

Math Associated Vocabulary:

INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT, RADIUS

Program Formulas and Procedures:

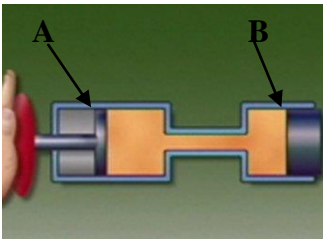
FORMULA: Force output of cylinder during extension— typical application of the $F = P \times A$ formula, where F=Force, P=Pressure and A=Area.

Formulas and Procedures:

Pascal's Law: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. Although the pressure is the same within the enclosure, it is extended over a much larger area, multiplying the force that moves a piston. The force of a small cylinder must be exerted over a much large distance—a small force exerted over a large distance is traded for a large force over a small distance.

Example 1:

If the hand pushing the plunger = 50psi. of pressure and the Diameter of Piston A=3" and the Diameter of Piston B=6" what Force (F_2) output?



$$F_2 = P \times A_2$$

$$F_2 = 50 \times (\pi 1.5^2)$$

$$F_2 = 50 \times (7.069)$$

$$F_2 = 353.4lbs$$

Example 2:

In this example, if an auto repair shop has a compressor that produces 100psi. of compressed air and acts on piston A (2" d) what is the Force (F_2) produced by piston B (22" d). The car weighs 4500 lbs. Does the system pictured produce enough Force (F_2) to raise the vehicle?

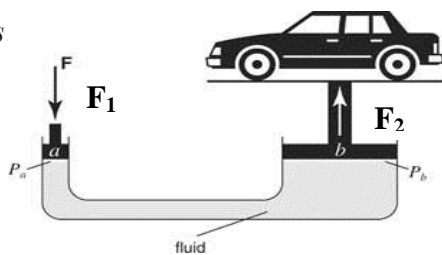
$$100 \text{ psi} = \frac{F_2}{A_2}, = \frac{F_2}{\pi 11^2} = \frac{F_2}{380}$$

Multiple both sides by 380

$$100 \times 380 = \frac{F_2}{380} \times \frac{380}{1}$$

$$F_2 = 38000lbs$$

Enough to raise the car!



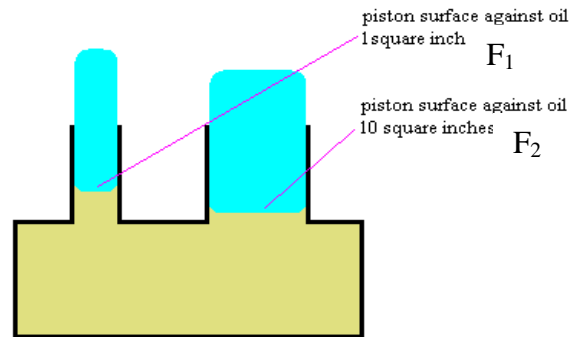
Example: Hydraulic lift principle. The surface area of the right piston is 10 times greater than the left one. The resulting force is 10 times larger. In this example, if 1lb of Force (F_1) is applied to the 1in.sq. piston, then 10lbs of Force (F_2) is applied to the 10in.sq. piston. The pressures remain the same.

$$F = P \times \text{Area}, \quad P = \frac{F}{A}$$

Check your work...

$$P = \frac{\text{Force}(F_1)}{\text{Area}}, \quad P = \frac{1}{1}, \quad P = 1$$

$$P = \frac{\text{Force}(F_2)}{\text{Area}}, \quad p = \frac{10}{10}, \quad P = 1$$



Area

Formula for Pascal's Law: $\frac{F_2}{P}$

$$F_2 = P \times A$$

Teacher's Script - Comparing and Contrasting

Understanding direct vs. inverse proportions can be very useful when students are making quick decisions as to how a change in a system may affect the result. For example, knowing that fuel mileage and miles driven are directly proportional means that an increase in miles driven with a single tank of fuel means the fuel mileage must also increase.

Within technical applications, proportional math comes from problems based on a given formula where a value is held steady and two other values are allowed to adjust.

It's important that students understand the distinction between direct and inverse. **Direct** indicates that the 2 values allowed to change in the proportion will go up together or go down together. An inverse proportion indicates the 2 values will change in opposite directions (one higher, one lower).

When working with a proportional problem with 2 ratios you can eliminate common factors on the top or bottom of both ratios (i.e. fractions on either side).

Common Mistakes Made By Students

When students compare Direct and Inverse Proportional relationships, they may become confused and have difficulty differentiating one from the other. One way to keep them straight is to share this problem: $F_1 = 50 \text{ lbs.}$ $\text{Radius}_1 = 1$ $\text{Radius}_2 = 4$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} = \frac{50}{\pi 1^2} = \frac{F_2}{\pi 4^2} = \frac{50}{3.14} = \frac{F_2}{50.27}$$

1. Set up one pair of values on the same line, e.g.

$$\text{Cross Multiple } \frac{50 \times 50.27}{3.14} = 800.47 \text{ lbs}$$

2. Cross multiply (50 times 50.27) and divide by A_1 , but first determine if you have to invert one ratio.
3. If you have to invert one ratio, then it is an inverse proportion.
4. If need be, set up the problem and do it both ways to see which answer makes sense. It must be an inverse proportion.

Lab Teacher's Extended Discussion

Technical tasks are usually not presented using this model. Therefore, it is important that technical instructors demonstrate to students how these math concepts link to and are relevant in their technical training. Technical instructors should also present the math in a way that CTE students are exposed to math concepts in their academic school settings. Using a program's tasks, technical teachers no doubt have many examples of this math concept to share with students.

Mathematics is such an integral part of our work that many times we use it on a subconscious level.

To become a well-rounded teacher do the math; make it your business to reach the comfort level necessary for teaching the math concepts and formulas that make engineering the profitable and satisfying career that we all know it can be.

“Remember, we need math constructs to understand any aspect of the world and those numbers and figures and equations are both beautiful and give life meaning and bring knowing to the unknown “(unknown author).

Problems	Occupational (Contextual) Math Concepts	Solutions
1. You have a cylinder with a bore of 5.0 in and a pressure of 100 psi. What would the force output of the cylinder be?		
2. The gauge reading on the outlet of a hydraulic fitting is 750 psi. If the force (F_1) on the fluid is 250 lbs., what is the diameter and area of the piston? Use the formula: $A = \frac{F_2}{P} \quad r = \sqrt{\frac{Area}{\pi}}$		
3. A hydraulic press has an input cylinder 1 inch in diameter and an output cylinder 6 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 10 pounds is applied to the input piston.		

Problems	Related, Generic Math Concepts	Solutions
4. A car master cylinder with a bore of 1.25 in. and a pressure of 500 psi. What would the force output be at the wheel cylinders?		
5. The output force (F_2) of fire truck's pumper is 1750 psi. If the pressure (F_1) on the water is 200psi, what is the diameter and area of the piston?		
6. A hydraulic jack has an input cylinder 2 inches in diameter with an output cylinder 4 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 50 pounds is applied to the input piston.		

Problems	PA Core Math Look	Solutions
7. Diameter=2.5"; $F_1=350$ psi. Find F_2 .		
8. $F_1=25,000$ lbs.; $P =350$ psi; find the diameter and area of the piston.		
9. Input cylinder=7" diameter; output cylinder=15" diameter. Find F_2 when 800 lbs., is applied to the input cylinder.		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. You have a cylinder with a bore of 5.0 in and a pressure of 100 psi. What would the force output of the cylinder be?	$F = P \times A$ $F = 100 \text{ psi} \times (\pi 2.5^2)$ $F = 100 \times (19.63)$ $F = 1,963 \text{ lbs.}$	
2. The gauge reading on the outlet of a hydraulic fitting (P) is 750 psi. If the force (F_1) on the fluid is 250 lbs., what is the diameter and area of the piston? Use the formula: $A = \frac{F_2}{P} \quad r = \sqrt{\frac{\text{Area}}{\pi}}$	Area = Force/Pressure $\text{Area} = 750 \text{ lbs.}/250 \text{ lbs./in}^2 \quad \text{Area of piston} = 3 \text{ in}^2$ Radius = $\sqrt{3/\pi} = .98 \text{ inches}$ Diameter = $2 \times \text{radius} = 2 \times .98 = 1.96 \text{ inches (diameter)}$	
3. A hydraulic press has an input cylinder 1 inch in diameter and an output cylinder 6 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 10 pounds is applied to the input piston.	$F_1/A_1 = F_2/A_2 \quad 10/(\pi .5^2) = F_2/(\pi 3^2)$ $10/.79 = F_2/28.27$ $F_2 = 357.85 \text{ lbs.}$	
Problems	Related, Generic Math Concepts	Solutions
4. A car master cylinder with a bore of 1.25 in. and a pressure of 500 psi. What would the force output be at the wheel cylinders?	$F = P \times A$ $F = 500 \times (\pi .625^2) \text{in}^2$ $F = 613.59 \text{ lbs.}$	
5. The output force (F_1) of fire truck's pumper is 1750 lbs. If the pressure (P) on the water is 200psi, what is the diameter and area of the piston?	$A = \frac{1750}{200} \quad A = 8.75 \text{in}^2$ $r = \sqrt{\frac{8.75}{\pi}} \quad r = 1.67 \quad D = r2 \quad D = 3.34$	
6. A hydraulic jack has an input cylinder 2 inches in diameter with an output cylinder 4 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 50 pounds is applied to the input piston.	$F_1/A_1 = F_2/A_2 \quad 50/(\pi 1^2) = F_2/(\pi 2^2) \text{in}^2$ $50/3.14 = F_2/12.57$ $F_2 = 200.16 \text{ lbs.}$	
Problems	PA Core Math Look	Solutions
7. Diameter=2.5"; $F_1=350$ psi. Find F_2 .	$F = P \times A$ $F = 350 \text{ psi} \times (\pi 1.25^2)$ $F = 350 \times (4.9)$ $F = 1,715 \text{ lbs.}$	
8. $F_1=25,000$ lbs.; $P = 350$ psi; find the diameter and area of the piston.	$A = \frac{25,000}{350} \quad A = 71.4 \text{in}^2$ $r = \sqrt{\frac{71.4}{\pi}} \quad r = 4.77 \quad D = r2 \quad D = 9.54$	
9. Input cylinder=7" diameter; output cylinder=15" diameter. Find F_2 when 800 lbs. is applied to the input cylinder.	$F_1/A_1 = F_2/A_2 \quad 800 \text{ lbs.}/(\pi 3.5^2) \text{in}^2 = F_2/(\pi 7.5^2) \text{in}^2$ $800/38.48 = F_2/176.71$ $F_2 = 3,673.8 \text{ lbs.}$	