

**Perform angular measurement = Apply geometric concepts to model and solve real world problems**

**Program Task:** Demonstrate use and care of measuring tools.

**Program Associated Vocabulary:**  
CENTERLINE ANGLE, INCLUDED ANGLE, DEGREES, COMPLEMENT

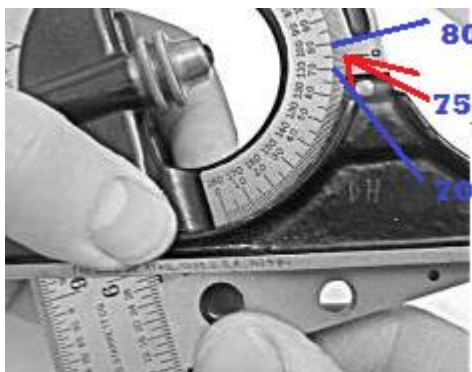
**Program Formulas and Procedures:**  
Measuring angles is very common in the machining field. A plain steel protractor or the combination set protractor can be used for semi-precision angular measurement.

**Example 1:** What is the included angle of the countersink?



Since the countersink angle and the angle on the left side of the protractor are vertical angles, they are equal and the resulting measurement is an 82° included angle. The centerline angle is the measurement from the axis of the countersink to one side and is half of the included angle, in this case 41°.

**Example 2:** What is the angle of the work piece opposite the shortest leg?



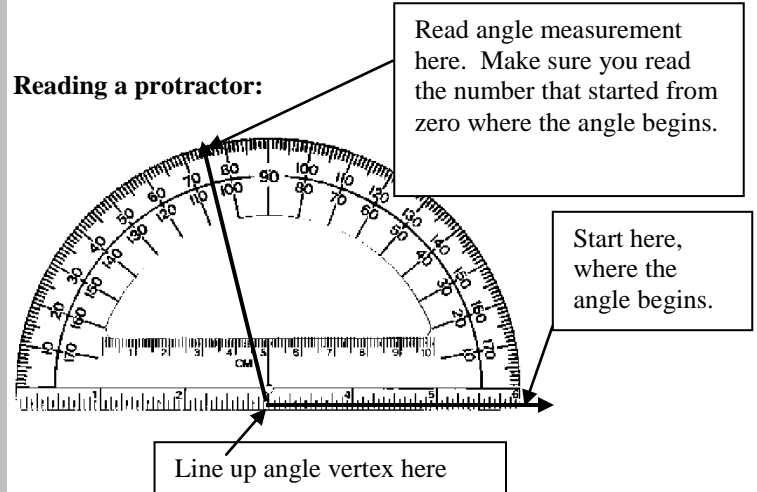
The protractor reading is 75°, but that reading is the complement of the desired angle. Subtracting 75° from 90° results in an angle of 15°. 75° is the angle opposite the longest leg.

**PA Core Standard:** CC.2.3.HS.A.14

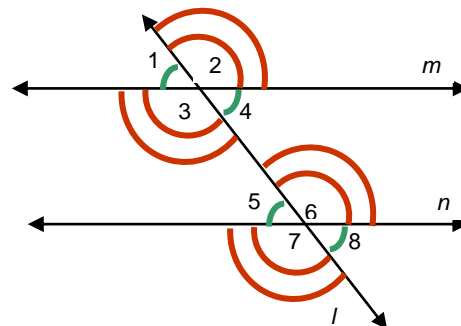
**Description:** Apply geometric concepts to model and solve real world problems.

**Math Associated Vocabulary:**  
ANGLE, DEGREES, INTERIOR ANGLES, EXTERIOR ANGLES, VERTICAL ANGLES, CORRESPONDING ANGLES, PARALLEL, TRANSVERSAL

**Formulas and Procedures:**



**Two parallel lines cut by a transversal:**



Angles 1&4, 2&3, 5&8, 6&7 are vertical angles.  
Angles 1&5, 2&6, 3&7, 4&8 are corresponding angles.  
If lines m and n are parallel then corresponding angles are congruent, Alternate Interior angles are congruent, and Alternate Exterior angles are congruent.  
Vertical angles are always congruent.

**Example 1:** If angle 1 = 40°, what is the measure of angle 8?  
Angle 8 must measure 40°, since ∠1 and ∠8 are alternate exterior angles.

**Example 2:** If  $m \angle 2 = 3x + 4$ , and  $m \angle 3 = x + 8$ , solve for x.  
(Vertical angles are equal.)  
 $3x + 4 = x + 8$  (subtract x from both sides)  
 $2x + 4 = 8$  (subtract 4 from both sides)  
 $2x = 4$  (divide both sides by 2)  
 $x = 2$

**Instructor’s Script – Comparing and Contrasting**

Knowing the correct angle patterns can help you find missing angles and other missing dimensions in many situations. Machine tool technology gives some great examples of this. Also, in real life applications, sometimes the machinist needs to use more accurate measures of angles and needs to measure the angles in degrees, minutes, and seconds.

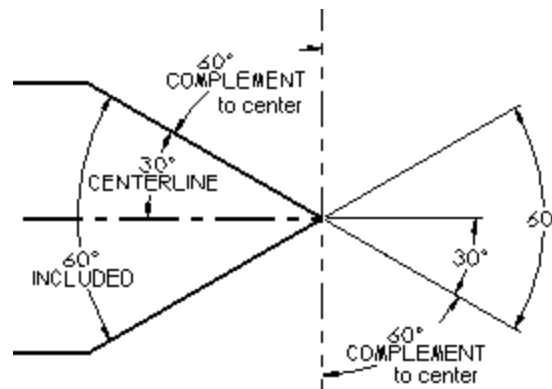
**Common Mistakes Made By Students**

- Reading the wrong indicator on the protractor (bottom number versus top number, or vice-versa).
- Confusing supplementary and complementary angles. Supplementary angles have a sum of 180 degrees and complementary angles have a sum of 90 degrees.
- Having trouble seeing the patterns if the parallel lines are in a picture with other lines.
- Students also sometimes use the properties of parallel lines when the lines are not parallel.
- Some students do not realize that vertical angles are always congruent.
- When working with degrees, minutes, and seconds, students sometimes do not realize that 1 degree is 60 minutes, and 1 minute is 60 seconds. If they do not realize this when converting, they may use a different scale.

**CTE Instructor’s Extended Discussion**





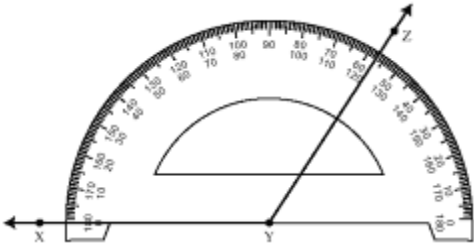
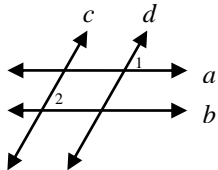
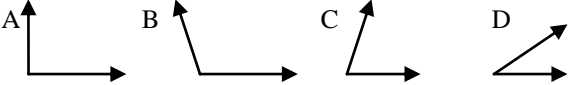
Students need to understand the difference between centerline and included angles as well as complementary and vertical angles.


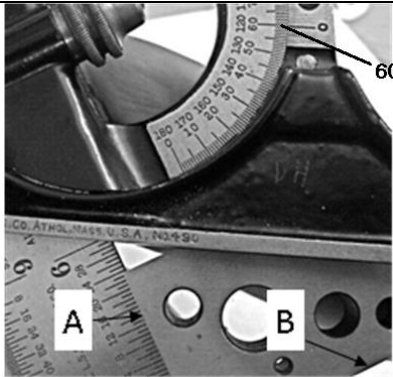
An included angle is the entire angle and a centerline angle is the angle from a centerline to one side of an angle.


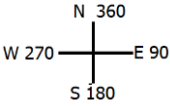
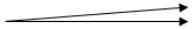


In the sketch, the two 30° angles are vertical and therefore equal. The two 60° angles are also vertical and equal.

If the 30° centerline angle were measured from a vertical line, its dimension would be 60°, the complement of 30°.

Problems	Career and Technical Math Concepts	Solutions
1. What are the centerline and included angles of the drill in the picture?		
2. What is the angle of the part opposite side "A"?		
3. What is the angle opposite side "B"?		
Problems	Related, Generic Math Concepts	Solutions
4. Which angle would you estimate to be the interior angle of the hairpin shown here? How would you describe a "hairpin turn" in the road? a) $10^\circ$ b) $45^\circ$ c) $90^\circ$ d) $120^\circ$		
5. Your GPS indicates that you are traveling in a direction (bearing) that is determined to be $270^\circ$ . If $90^\circ$ is east, in which direction are you traveling?		
6. To be wheelchair accessible, the grade of a ramp must not exceed 1 foot of rise per 12 feet of run. This equates approximately to a $5^\circ$ angle. Use a protractor to draw this angle measure.		
Problems	PA Core Math Look	Solutions
7. What is the angle measure of $\angle XYZ$ ? a) $57^\circ$ b) $63^\circ$ c) $123^\circ$ d) $137^\circ$		
8. Given: $a \parallel b, c \parallel d$ If $m\angle 1 = 2x + 16$ and $m\angle 2 = x + 14$ , then what is the value of $x$ ?		
9. Which of the angles on the right is closest to $76^\circ$ ?		

Problems	Career and Technical Math Concepts	Solutions
1. What are the centerline and included angles of the drill in the picture?		Centerline angle is 59°. $59 \times 2 = 118$ Included angle is 118°.
2. What is the angle of the part opposite side “A”?	30° $90 - 60 = 30$	
3. What is the angle opposite side “B”?	60°	

Problems	Related, Generic Math Concepts	Solutions
4. Which angle would you estimate to be the interior angle of the hairpin shown here? How would you describe a “hairpin turn” in the road? a) 10° b) 45° c) 90° d) 120° 		The correct answer is “a” because a 10° interior angle turn would very nearly turn a driver back in the direction from which s/he came. Hairpin turns get their name because they have interior angles similar to a real hairpin.
5. Your GPS indicates that you are traveling in a direction (bearing) that is determined to be 270°. If 90° is east, in which direction are you traveling?		You are traveling west (when your bearing is 270°). 90° is East; 180° is South; 360° is North. 
6. To be wheelchair accessible, the grade of a ramp must not exceed 1 foot of rise per 12 feet of run. This equates approximately to a 5° angle. Use a protractor to draw this angle measure.		This is what the angle should resemble if drawn by a protractor.

Problems	PA Core Math Look	Solutions
7. What is the angle measure of $\angle XYZ$ ? a) 57° b) 63° c) 123° d) 137°	c) 123°	
8. Given: $a \parallel b, c \parallel d$ If $m\angle 1 = 2x + 16$ and $m\angle 2 = x + 14$ , then what is the value of x?	Angles 1 and 2 are congruent angles so, $2x + 16 = x + 14$ $x + 16 = 14$ $x = -2$ (Subtract x from each side, then subtract 16 from each side).	
9. Which of the angles on the right is closest to 76°?	