

Determine volumes = Explain volume formulas and use them to solve problems

Program Task: Install hot water heater.

Program Associated Vocabulary:

AREA, VOLUME, LENGTH, WIDTH, HEIGHT, RECTANGULAR, ROUND, CYLINDRICAL, BASE, RADIUS

Program Formulas and Procedures:

HVAC work deals constantly with fluids. Fluids may be liquid or gaseous, and while a given mass does not change, volume and density do tend to vary, depending on ambient pressures and temperatures. Usually, fluids are contained in devices that have definite volumes, although some cleverly designed devices are able to change their shape and in doing so, their volume changes as well.

While the shapes of devices vary, they frequently, but not always, involve cylinders because of the inherent strength of the cylinder.

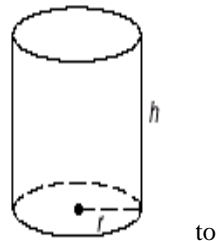
Volume is determined in some dimension cubed, it may be inches, feet, yards, meters, centimeters, etc. For the HVAC technician, finding volume is step 1 when determining some other weight or measure, such as pounds or gallons.

Example:

Let's take a look at how we would determine the capacity, in gallons, of a hot water heater, whose height is 4 feet and whose diameter is 1.5 feet.

NOTE THAT WE FIRST FIND VOLUME IN CUBIC FEET, AND THEN CONVERT THAT VOLUME INTO GALLONS.

$$\begin{aligned} \text{Cylinder Volume} &= \pi r^2 h \\ \text{Volume} &= 3.14 \times .75^2 \times 4 \\ \text{Volume} &= 7 \text{ ft.}^3 \end{aligned}$$



One cubic foot of water contains about 7.5 gallons of water, so we multiply volume in cubic feet by 7.5 find our capacity in gallons.

$$\begin{aligned} \text{Capacity (gals)} &= \text{VOL.}(\text{ft.}^3) \times 7.5(\text{gals}/\text{ft.}^3) \\ \text{Capacity (gals)} &= 7\text{ft.}^3 \times 7.5\text{gals}/\text{ft.}^3 \\ \text{Capacity (gals)} &= 52 \text{ gallons (approx.)} \end{aligned}$$

PA Core Standard: CC.2.3.HS.A.12

Description: Explain volume formulas and use them to solve problems.

Math Associated Vocabulary:

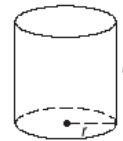
AREA, VOLUME, LENGTH, WIDTH, HEIGHT, RECTANGULAR, CYLINDER, BASE, RADIUS, RECTANGULAR PRISM

Formulas and Procedures:

Volume:

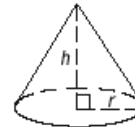
Cylinder:

$$V = \pi r^2 h$$



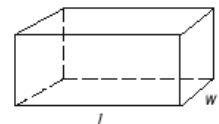
Cone:

$$V = \frac{1}{3} \pi r^2 h$$



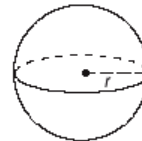
Rectangular Prism:

$$V = lwh$$



Sphere:

$$V = \frac{4}{3} \pi r^3$$

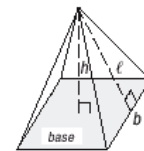


Pyramid:

$$V = \frac{1}{3} (\text{area of the base})h$$

h = height b = base

ℓ = slant height



Example:

How many cubic inches of air can a beach ball hold if it has a diameter of 14 inches? Round to the nearest whole number.

Steps to finding volume:

1. Identify the solid. (sphere)
2. Write the formula for calculating the volume of that solid using the formula sheet.
 $V = \frac{4}{3} \pi r^3$
3. Identify what information you are given in the example.
Given: diameter (d) = 14"
4. Solve for radius using the formula radius (r) = 1/2 (diameter).
 $r = \frac{1}{2} \times 14 = 7$
5. Perform the necessary mathematical operations to obtain your answer.
 $V = \frac{4}{3} \pi r^3 = \frac{4}{3} (3.14) (7^3) = 1,436 \text{ in.}^3$
6. Write the appropriate unit after your answer.
1,436 in.³

Instructor's Script - Comparing and Contrasting

HVAC technicians must typically find the volumes of cylinders and rectangular prisms, but students are expected to be able to also find the volumes of spheres, pyramids, and cones. HVAC technicians must recognize that the height of a cylinder, h , may also be referred to as length as in the case of piping and tubing. HVAC technicians must be able to extend their mathematical knowledge in finding volumes by also converting units and finding percent capacities.

Common Mistakes Made By Students

Students may use an incorrect formula to solve a problem: To rectify these errors have the students correctly identify the type of object they are dealing with and use the appropriate formula. Frequently two formulas may be needed for complex problems.

Using consistent units: If the problem asks for the answer in square feet instead of square inches, be sure to either convert your given measurements into feet first ($\text{inches} \div 12 = \text{feet}$) or convert your square inch answer into square feet ($\text{sq. inches} \div 144 = \text{sq. feet}$).

CTE Instructor's Extended Discussion

This is a great opportunity to discuss with HVAC students other important application regarding volume. HVAC topics that require an understanding of volume, as well as the ability to calculate volume in a variety of scenarios, would include:

- Volume of compressor cylinders (think Boyle's Law).
- Volume of air in a building (think formulas for calculating required fresh air / make up air).
- Tank volume (propane, butane, oxygen, acetylene, nitrogen, diesel, etc., differences between gas / liquid uses).
- Tubing and piping systems (think water capacity to size expansion tank, think additional refrigerant needed to weigh in a charge, think sizing of liquid line receivers for pump downs, think drain back of condensate piping into Deaerator tanks in High-Low (hospital) pressure steam systems, think drain down of solar panel systems, etc.).
- Complete hydronic systems (think calculating anti-freeze charge in commercial closed loop systems).
- Collapsible gas storage tanks (think cities - natural gas reserves with collapsible dome – pressurized tanks).
- Thermal storage banks (solar panel heat storage).
- Swimming pools (think volume of water to boiler or solar panel sizing).
- How many examples can you add?

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Determine the capacity, in gallons, of a hot water heater, whose height is 2 feet and whose diameter is 2 feet. (formulas given on page 1)		
2. Determine the safe fill capacity, in pounds of propane product, of a propane tank whose height is 4 feet and whose diameter is 2 feet. You will need the following formulas: Volume = $\pi r^2 h$ Capacity (gals) = Vol(ft. ³) \times 7.5gals/ft. ³ Capacity (water gals) to Propane lbs. = Gals. \times 4.24 Safe fill level = Tank Capacity \times 80%		
3. How many gallons of water are in a rectangular swimming pool that is 40 feet long, 30 feet wide has an average depth of 5 feet?		
Problems	Related, Generic Math Concepts	Solutions
4. One soup can has a d = 3" and h = 4"; another soup can has a d = 4" and h = 3". Which can holds more soup?		
5. A size 7 regulation basketball has a d = 9.39". What is the volume of the basket ball?		
6. How much water would you need to fill a rectangular fish tank with a height of 16.5", a length of 32", and a width of 8.5"?		
Problems	PA Core Math Look	Solutions
7. Find the volume of a cylinder d = 12.5' and h = 28.75'. Round your answer to the nearest thousandth.		
8. Find the volume of a sphere d = 27.75". Round your answer to the nearest hundredth.		
9. Find the volume of a pyramid with a square base with sides of 10" and a height of 25".		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Determine the capacity, in gallons, of a hot water heater, whose height is 2 feet and whose diameter is 2 feet.	Volume = $\pi \times r^2 \times h$ Volume = $3.14 \times 1 \times 2'$ Volume = 6.28 cubic feet Capacity (gals) = $6.28 \text{ ft.}^3 \times 7.5 \text{ gals/ft.}^3$ Capacity (gals) = 47 gallons (approx.)	
2. Determine the safe fill capacity, in pounds of propane product, of a propane tank whose height is 4 feet and whose diameter is 2 feet. You will need the following formulas: Volume = $\pi \times r^2 \times h$ Capacity (gals) = $\text{Vol. (ft.}^3) \times 7.5 \text{ gals./ft.}^3$ Capacity (water gals) to Propane lbs. = $\text{Gals.} \times 4.24$ Safe fill level = $\text{Tank Capacity} \times 80\%$	Volume = $3.14 \times 1 \times 4'$ Volume = 12.5 cubic feet Capacity (water gals.) = $12.5 \text{ ft.}^3 \times 7.5 \text{ gals/ft.}^3$ Capacity (water gals.) = 94 gallons (approx.) Propane weight capacity = $94 \text{ gals.} \times 4.24$ Propane weight capacity = 400 lbs. propane Safe fill level = $400 \text{ lbs.} \times 80\%$ Safe fill level = 320 lbs. propane	
3. How many gallons of water are in a rectangular swimming pool that is 40 feet long, 30 feet wide has an average depth of 5 feet?	Volume of rectangular prism = $w \times l \times h$ Volume = $40 \times 30 \times 5$ Volume = 6000 cubic feet Gallons = $\text{Volume} \times 7.5 \text{ gal/ft.}^3$ Gallons = $6000 \text{ ft.}^3 \times 7.5 \text{ gal/ft.}^3$ Pool holds 45,000 gallons of water	
Problems	Related, Generic Math Concepts	Solutions
4. One soup can has a $d = 3''$ and $h = 4''$; another soup can has a $d = 4''$ and $h = 3''$. Which can holds more soup?	$V = \pi r^2 h$ Can 1: $r = \frac{1}{2} (3)$ $V = \pi (1.5)^2 4$ $V \approx 28.26 \text{ in.}^3$ Can 2 holds more soup.	Can 2: $r = \frac{1}{2} (4)$ $V = \pi (2)^2 3$ $V \approx 37.68 \text{ in.}^3$
5. A size 7 regulation basketball has a $d = 9.39''$. What is the volume of the basketball?	Radius (r) = $\frac{9.39}{2} = 4.695$ $V = \frac{4}{3} \times \pi \times r^3$ $V \approx 1.333 \times \pi \times 103.5$	$V = 1.333 \times \pi \times 4.695^3$ $V \approx 433.21 \text{ in.}^3$
6. How much water would you need to fill a rectangular fish tank with a height of 16.5'', a length of 32'', and a width of 8.5''?	$V = lwh$ $V = (32)(8.5)(16.5) = 4,488 \text{ in.}^3$	
Problems	PA Core Math Look	Solutions
7. Find the volume of a cylinder $d = 12.5'$ and $h = 28.75'$. Round your answer to the nearest thousandth.	$V = \pi r^2 h$ $r = \frac{1}{2} (12.5) = 6.25$ $V = \pi \times 6.25^2 \times 28.75$ $V \approx 3,526.37 \text{ ft.}^3$	
8. Find the volume of a sphere $d = 27.75''$. Round your answer to the nearest hundredth.	$V = \frac{4}{3} \times \pi \times r^3$ $V \approx 11,180.44 \text{ in.}^3$	$V = 1.333 \times \pi \times 13.875^3$
9. Find the volume of a pyramid with a square base with sides of 10'' and a height of 25''.	$V = \frac{1}{3} (\text{area of base}) h$ $V = \frac{1}{3} (100) (25) \approx 833.33 \text{ in.}^3$	Area of base = $10 \times 10 = 100$