

Determine surface areas = Apply geometric concepts to model and solve real world problems

Program Task: Explain system operation and maintenance requirements.

Program Associated Vocabulary:
 SURFACE AREA (SA), CIRCUMFERENCE, π , LENGTH, O.D. (OUTSIDE DIAMETER), I.D. (INSIDE DIAMETER), EXCHANGE AREA, TRANSFER AREA, RECTANGULAR PRISM

Program Formulas and Procedures:
 Cylinders may stand upright or lay sideways; they may include two, one, or no ends. They may be cut along any center line to form tubs, arches, or troughs.

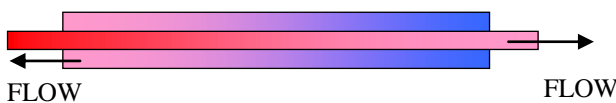
To calculate surface area (SA) when requisitioning insulating materials that will *minimize* the transfer of heat across the SA of a liquid storage tank, such as a HW Heater, a Feedwater Deaerator Tank, or a Chilled Water Storage Tank, use this formula:
 $SA = 2\pi r^2 + 2\pi rh$
 (cylinder, with 2 ends, closed tanks)



Use the formula below when determining the Wetted SA of an upright liquefied petroleum gas tank (i.e., propane or butane). Wetted SA is critical because LPG's must boil in order to produce gas pressure, and the heat required to make the liquid boil must pass through the wetted SA of the tank.
 $SA = \pi r^2 + 2\pi rh$
 (cylinder, with 1 end closed)



Use the formula below to determine the SA of tubes and pipes. SA is important when designing heat exchangers using tubes and pipes since heat transfer is a product of SA and other factors.
 $SA = 2\pi rh$
 (open cylinders, tubing, pipe, tube-in-tube heat exchangers)



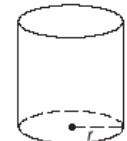
PA Core Standard: CC.2.3.HS.A.14

Description: Apply geometric concepts to model and solve real world problems.

Math Associated Vocabulary:
 AREA, CROSS SECTION, LENGTH, WIDTH, ROUND, BASE, HEIGHT, RADIUS, RECTANGULAR PRISM

Formulas and Procedures:

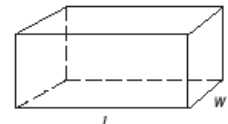
Surface Area:
Cylinder:
 $SA = 2\pi r^2 + 2\pi rh$



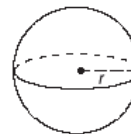
Cone:
 $SA = \pi r^2 + \pi r \sqrt{r^2 + h^2}$



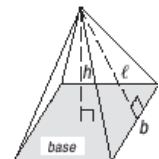
Rectangular Prism:
 $SA = 2lw + 2wh + 2hl$



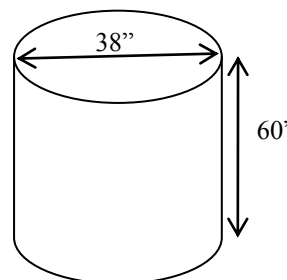
Sphere:
 $SA = 4\pi r^2$



Pyramid:
 $SA = (\text{area of the base}) + \frac{1}{2} \ell (\text{perimeter of base})$
 b = base, h = height, ℓ = slant length



Example: Find the surface area of the cylinder below.



$$r = \frac{1}{2} \times 38'' = 19'' \quad h = 60''$$

$$\text{Cylinder SA} = 2\pi r^2 + 2\pi rh$$

$$SA = 2\pi(19)''^2 + 2\pi(19)''(60)$$

$$SA = 722\pi + 2,280\pi$$

$$SA = 3,002\pi$$

$$SA \approx 9,426.28 \text{ in.}^2$$

Instructor's Script - Comparing and Contrasting

It is essential that HVAC technicians be able to calculate the surface area, or partial surface area of cylinders. The formula presented in HVAC for finding the surface area of a cylinder states that the area is equal to the length times the circumference ($2\pi rh$) plus the area of the two circular bases ($2\pi r^2$). For example the formula: Cylinder SA = $2\pi r^2 + 2\pi rh$. ($2\pi r^2$ = area of both circular bases and $2\pi rh$ = length times the circumference of the base).

The HVAC formula changes slightly depending if both ends are included, only one end is included, or neither end is included). In the tubing example, the HVAC technician is essentially using only the formula $2\pi rh$, where h is the length of the pipe and r is 1/2 of the diameter of the pipe. In the propane tank example, the technician is including the base since this cylinder is not hollow.

Common Mistakes Made By Students

Using incorrect formula: Students may use an incorrect formula to solve a problem. To rectify these errors have the students correctly identify the type of object they are dealing with and use the appropriate formula. Frequently two formulas may be needed for complex problems.

Using consistent units: If the problem asks for the answer in square feet instead of square inches, be sure to either convert your given measurements into feet first (inches \div 12 = feet) or convert your square inch answer into square feet (sq. inches \div 144 = sq. feet).

Not “removing” unnecessary surface areas from calculations: Depending on the problem, not all surface areas included in formula may be needed. Identify the areas that are required for the calculation and remove from formula as needed.

CTE Instructor’s Extended Discussion

Shapes addressed in geometry books can be found in the HVAC industry. Cylindrical shapes are probably the most frequently seen. This T-Chart focuses on determining the surface area of cylinders and partial cylinders. They may appear as pipes, tubes, hoses, conduits, ducts, tanks, and bottles, as well as integral parts of tools and machines including compressor cylinders, shafts of motors and pumps, and linkage devices.

The cylindrical shape is probably most important for its inherent strength, but its surface area may also be a component of system performance. One frequent use of the cylinder is in heat exchanger applications. These devices are able to maximize the transfer of heat by maximizing the contact or surface area between two masses.

Butane and Propane (LPG) tanks make a good example of the relationship between heat transfer and surface area. LPG must boil in order to produce a pressurized gas. The heat of vaporization must pass through the tank material to the liquid inside the tank. LPG suppliers may install multiple tanks to increase the critical surface area (wetted surface area) needed to boil the liquefied gases.

Or consider the tube-in-tube heat exchanger: its surface area is easily extended by adding to the length, the diameter, or the number of cylinders (tubes) used. Similar cases can be made for tubes used in boilers (fire or water tube boilers), furnaces, evaporators, condensers, and other equipment used for heating and cooling structures and machines. Surface area is almost always a crucial factor when determining the capacity and performance of a heat transferring device. Whatever the function of the cylinder may be, the formula for finding the surface area of a cylinder remains the same: *Area = Height x Circumference*. When working with pipe and tubing, *Height* typically refers to the length of pipe or tubing. Variations of this formula can be used as the cylinder shape varies to fit the application.

This PA Core Standard should be revisited often by the teacher in order to address the many different geometric shapes found in the HVAC industry. Examples include but are not limited to spherical shaped liquid storage tanks, cylinders whose ends are dome shaped (horizontal LPG tanks), parabolic shapes (solar dish collectors), rectangular prisms (plenums), and many other HVAC installations.

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A fire tube boiler has 120 fire tubes. Each tube is 20 feet long and has an I.D. of 1.9375 inches. What is the area in square feet of the fire side (interior) of the tubes?		
2. A propane (LPG) customer with a 125 gallon tank is having problems maintaining gas pressure during cold weather when the liquid level drops to 20%. The LP Company replaces the 125 gallon tank with 2 - 60 gallon tanks. If the dimensions of the 125 gallon tank are 30" Dia. × 54.5" Height, and each of the 60 gallon tanks are 24" Dia. × 43" Height, how many more square inches of wetted surface area (at 20% full) are obtained by using the two smaller tanks?		
3. One quart of rust proofing paint will cover 75 square feet of surface area. How many quarts of paint are needed to rust proof a water tank that is 16 feet tall and 4 feet in diameter?		
Problems	Related, Generic Math Concepts	Solutions
4. You need fabric to cover a 4-sided pyramid with base sides of 12' & slant length of 20'. How many square feet of fabric will you need to cover all sides of the pyramid? How many square yards? Note: 1 yd. ² = 9 ft. ²		
5. One soup can has a radius = 3" and height = 4"; another soup can has a radius = 4" and height = 3". Which can has a greater total surface area?		
6. A size 7 regulation basketball has a d = 9.39". A size 6 regulation basketball has a d = 9.07". What is the surface area of each basketball?		
Problems	PA Core Math Look	Solutions
7. Find the surface area of a cylinder with a diameter of 13.75' and a height of 28.45'.		
8. Find the surface area of a sphere that has a diameter of 27.75".		
9. Find the total surface area of cone with base diameter = 15.50" and a height of 22".		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A fire tube boiler has 120 fire tubes. Each tube is 20 feet long and has an I.D. of 1.9375 inches? What is the area in square feet of the fire side (interior) of the tubes?	Radius = Diameter ÷ 2 = 1.9375 ÷ 2 = 0.98 Convert feet to inches = 20 × 12 = 240"	Int. SA of one tube = 2πrh = 2 × 3.14 × 0.98 × 240 = 1477.056 in ² Int. SA of all 120 tubes = 1477.056 × 120 = 177246.72 in. ² Convert sq.in to sq. = 177246.72/144= 1230.88 ft. ²
2. A propane (LPG) customer with a 125 gallon tank is having problems maintaining gas pressure during cold weather when the liquid level drops to 20%. The LP Company replaces the 125 gallon tank with 2 - 60 gallon tanks. If the dimensions of the 125 gallon tank are 30" Dia. × 54.5" Height, and each of the 60 gallon tanks are 24" Dia. × 43" Height, how many more square inches of wetted surface area (at 20% full) are obtained by using the two smaller tanks?	1 - 125 Gallon Tank wetted area @ 20% liquid level: Liquid level = 0.2 × 54.5 = 10.9 SA = πr ² + 2πrh = (3.14 × 15 ²) + (2 × 3.14 × 15 × 10.9) = 706.5 + 1026.78 = 1733.28 in. ² wet surface area 2 - 60 Gallon Tanks wetted area @ 20% liquid level: Liquid level = 0.2 × 43 = 8.6 SA = πr ² + 2πrh = (3.14 × 12 ²) + (2 × 3.14 × 12 × 8.6) = 452.16 + 648.096 = 1100.256 in. ² wet surface area each = 1100.256 × 2 = 2200.512 in. ²	Though the smaller tanks hold fewer gallons, combined they offer an additional 467.232 in. ² of wet SA at 20% full.
3. One quart of rust proofing paint will cover 75 square feet of surface area. How many quarts of paint are needed to rust proof a water tank that is 16 feet tall and 4 feet in diameter?	SA = 2πr ² + 2πrh SA = (2 × 3.14 × 2 ²) + (2 × 3.14 × 2 × 16) SA = 25.12 ft. ² + 200.96 ft. ² = 226.08 ft. ² (approx.) 226 / 75 = 3.013 quarts You will need 4 quarts of paint	
Problems	Related, Generic Math Concepts	Solutions
4. You need fabric to cover a 4-sided pyramid with base sides of 12' & slant length of 20'. How many square feet of fabric will you need to cover all sides of the pyramid? How many square yards? Note: 1 yd. ² = 9 ft. ²	Pyramid: SA = (base area) + ½ l (number of base sides)(b) SA = 144 + ½ (20)(4)(12) SA = 144 + 480 SA = 624 ft. ² SA = 624 ft. ² ÷ 9 = 69.3 yd. ²	
5. One soup can has a radius = 3" and height = 4"; another soup can has a radius = 4" and height = 3". Which can has a greater total surface area?	Can 1: SA = 2π(3 ²) + 2π(3)(4) SA = 57 + 75 SA = 132 in. ²	Can 2 has the greater surface area. SA = 2π(4 ²) + 2π(4)(3) SA = 101 + 75 SA = 176 in. ²
6. A size 7 regulation basketball has a d = 9.39". A size 6 regulation basketball has a d=9.07". What is the surface area of each basketball?	Ball 1: r = 4.695 SA = 4π(4.695 ²) SA = 4π(22.04) SA = 277 in. ²	Ball 2: r = 4.535 SA = 4π(4.535 ²) SA = 4π(20.57) SA = 259 in. ²
Problems	PA Core Math Look	Solutions
7. Find the surface area of a cylinder with a diameter of 13.75' and a height of 28.45'.	Cylinder SA = 2πr ² + 2πrh radius = ½ d = 6.875' SA = 2π(6.875) ² + 2π(6.875)(28.45) SA = 94.53125π + 391.1875π SA = 485.71875π SA = 1525.9 ft. ²	
8. Find the surface area of a sphere that has a diameter of 27.75".	One Sphere SA = 4πr ² Radius = 27.75/2 = 13.875" SA = 4π(13.875) ² SA = 770.0625 π SA ≈ 2,419.2 in. ²	
9. Find the total surface area of cone with base diameter =15.50" and a height of 22".	Cone: SA = π r ² + πr√(r ² +h ²) SA = π(7.75) ² + π(7.75)√((7.75) ² + 22 ²) SA = 60.0625π + π(7.75)√60.0625 + 484 SA = 60.0625π + π(7.75)√544.0625 SA = 60.0625π + π(7.75)(23.325) SA = 60.0625π + π(180.769) SA = 240.83π SA ≈ 756.2 in. ²	