

Identify, measure, calculate angles = Apply geometric concepts to model and solve real world problems

Program Task: Install solar heat collector panel.

PA Core Standard: CC.2.3.HS.A.14

Program Associated Vocabulary:

ANGLE, PERPENDICULAR, VERTEX, LATITUDE, INCLINATION, DECLINATION, RAY

Description: Apply geometric concepts to model and solve real world problems.

Math Associated Vocabulary:

ANGLE, DEGREES, INTERIOR ANGLES, EXTERIOR ANGLES, VERTICAL ANGLES, CORRESPONDING ANGLES, PARALLEL, TRANSVERSAL

Program Formulas and Procedures

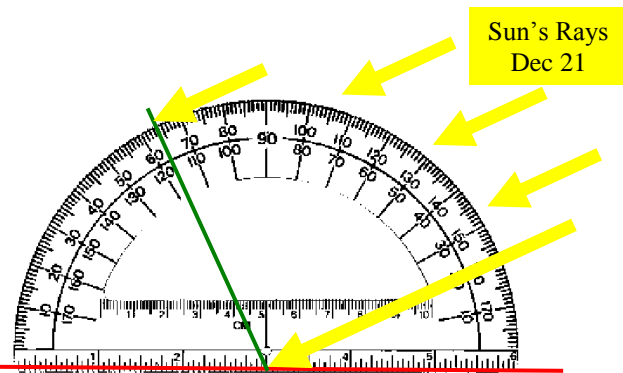
In order to optimize solar energy collection, solar panels must be adjusted to the angle that most allows them to face directly into the Sun. Since the Sun's height (inclination) changes with latitude and season, a formula is used to determine its range of inclination.

Sun's Inclination = $90^\circ - \text{Latitude}^\circ + 23.45^\circ(\text{summer})$ or $-23.45^\circ(\text{winter})$

The average latitude in Pennsylvania is about 41° North. That means that at its low point (Dec. 21), the Sun's inclination is about 26° ($(90^\circ - 41^\circ) - 23.45^\circ$). It's high point (June 21) is about 72° ($(90^\circ - 41^\circ) + 23.45^\circ$).

To optimize collection of solar energy, we position panels perpendicular to the Sun's rays. In the illustration below, the **Green** line represents the solar panel, positioned for optimum collection on December 21. On this date, the Sun is at its lowest position, 25° above the horizon. To find the panel's angle, we add 90° to the Sun's inclination, making the panel's angle 115° from the ground (sunny side). Notice that the **green** line begins at the vertex and passes through the protractor at 115° , while the lower **yellow** line (*ending at the vertex*) passes through the protractor at 25° .

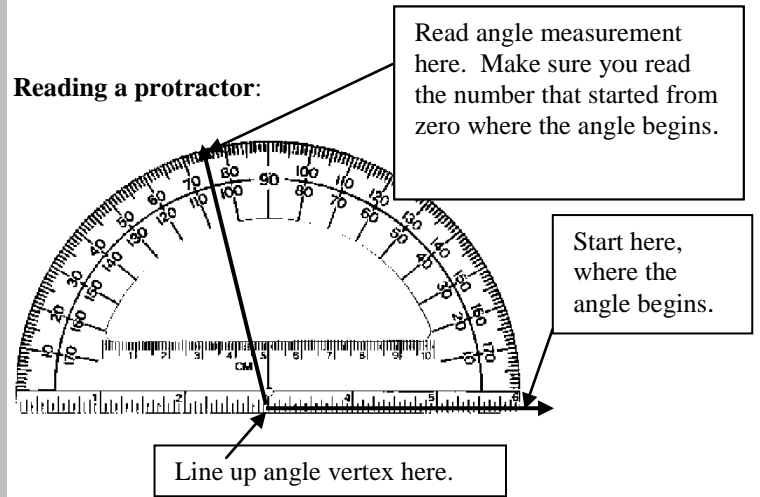
$25^\circ + 90^\circ = 115^\circ$



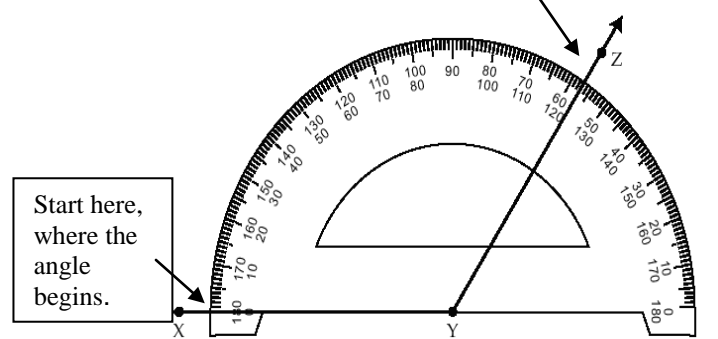
The **red** line represents the ground as a reference only, since the panel is likely to be roof mounted.

Formulas and Procedures:

Reading a protractor:



Read angle measurement here. Make sure you read the number that started from zero where the angle begins.

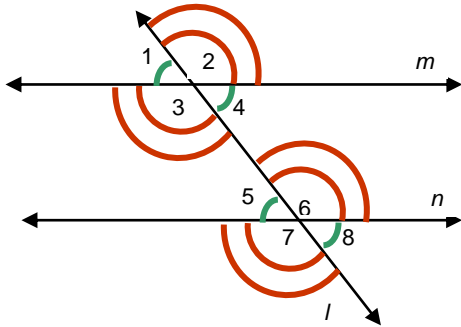


Instructor's Script - Comparing and Contrasting

HVAC technicians must be able to identify and measure angles. Although the eligible content requires the student to measure and/or compare angles in degrees, other Keystone exam items require the student to understand and use angle relationships formed when two parallel lines are cut by a transversal. Part of the eligible content requires that the student use properties of corresponding angles, vertical angles, alternate interior angles, and alternate exterior angles.

The information below is needed to solve problem #9 on pages 3 and 4. While this specific technical task (install solar heat collector panel) may not use the concept shown below, students will see questions related to this information on the Keystone Geometry exam.

Two parallel lines cut by a transversal



Angles **1&4, 2&3, 5&8, 6&7** are **vertical angles**.

Angles **1&5, 2&6, 3&7, 4&8** are **corresponding angles**.

If lines m and n are parallel then **corresponding angles** are congruent. **Alternate Interior** angles are congruent, and **Alternate Exterior** angles are congruent.

Vertical angles are always congruent.

Examples:

- If angle $1 = 40^\circ$, what is the measure of angle 8?
Angle 8 must measure 40° , since $\angle 1$ and $\angle 8$ are alternate exterior angles.
- If $m \angle 2 = 3x + 4$, and $m \angle 3 = x + 8$, solve for x
 $3x + 4 = x + 8$ (subtract x from both sides)
 $2x + 4 = 8$ (subtract 4 from both sides)
 $2x = 4$ (divide both sides by 2)
 $x = 2$

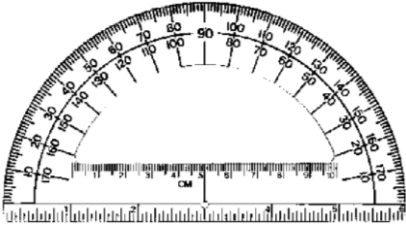

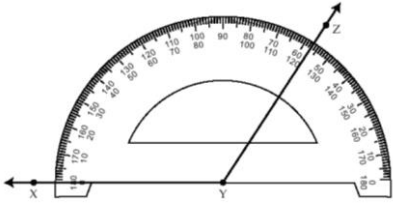
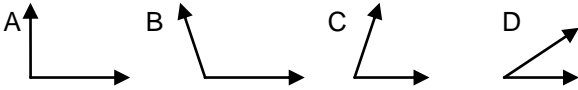
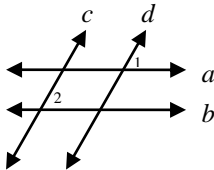
Common Mistakes Made By Students

- Not aligning the index line (line along the bottom of the protractor) with one side of the angle in question
- Not placing the vertex of the angle at the hole or point at the bottom-center of the protractor
- Not clearly specifying a reference or starting point for an angle
- Reading the wrong indicator on the protractor (bottom number versus top number, or vice-versa).

CTE Instructor's Extended Discussion

Angles are identified, measured, and calculated regularly in HVAC work. This is a good time to discuss with your students some examples of how the HVAC professional must be comfortable working with protractors and other tools to determine angles. Listed here are few examples of the use of angles in HVAC. A proficient technician must be comfortable with the concrete and abstract uses for angles. This list is by no means complete, and no doubt, you can add to it!

- Oil Burner Nozzle spray patterns (30° , 45° , 60° , 70° , 80° and 90° spray patterns are available). Discuss with your students how the shape of the combustion chamber determines the nozzle selection, from the mobile home furnace with a 90° nozzle, to an old Bethlehem Dynatherm boiler with a 30° nozzle.
- Variable pitch angle axial fans have blades whose angle (relative to the hub) may be adjusted (manually or automatically) to achieve the exact amount of airflow needed in an application (e.g., water cooling towers).
- Pipe fittings, valves, and other components in 30° , 45° , 60° , 90° and other custom configurations
- Duct fittings: take-offs, bends, etc.
- Calibrated tubing and conduit benders
- OSHA (Ladder Safety angles)
- References in tool sharpening guides (drill bits, drivers, knives, chisels, etc.)

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Using the formula defined on page 1, determine the best angle at which to position a solar panel during the summer months in Texas (average latitude 30°). Calculate for Sun at its highest arc. Show Sun's summer inclination. Show panel's summer angle.		
2. Using the formula defined on page 1, determine the best angle at which to position a solar panel during the summer months in Texas (average latitude 30°). Calculate for Sun at its highest arc. Show Sun's winter inclination (angle). Show panel's winter angle.		
3. Using the formula defined on page 1, determine the best angle at which to position a solar panel during the summer months in Pennsylvania (calculate for Sun at its highest arc). What are the Sun's summer inclination (angle) and the panel's winter angle?		
Problems	Related, Generic Math Concepts	Solutions
4. Which angle would you estimate to be the interior angle of the hairpin shown here? How would you describe a "hairpin turn" in the road? a) 10° b) 45° c) 90° d) 120°		
5. Your GPS indicates that you are traveling in a direction (bearing) that is determined to be 270°. If 90° is east, in which direction are you traveling?		
6. To be wheelchair accessible, the steepness of ramps must not exceed 1 foot of rise per 12 feet of run. This equates approximately to a 5° angle. Use a protractor to draw this angle measure.		
Problems	PA Core Math Look	Solutions
7. What is the angle measure of $\angle XYZ$? a) 57° b) 63° c) 123° d) 137°		
8. Which of the angles on the right is closest to 76°?		
9. Given: $a \parallel b, c \parallel d$ If $m\angle 1 = 2x + 16$ and $m\angle 2 = x + 14$, then what is the value of x ?		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Using the formula defined on page 1, determine the best angle at which to position a solar panel during the summer months in Texas (average latitude 30°) Calculate for Sun at its highest arc. Show Sun's summer inclination. Show panel's summer angle.		$\text{Sun's Summer Incline} = 90^\circ - \text{Latitude} + 23.45^\circ$ $\text{Sun's Summer Incline} = 90^\circ - 30 + 23.45^\circ = 83.45^\circ$ $\text{Panel's Summer Angle} = 83.45 + 90 = 173.45^\circ$
2. Using the formula defined on page 1, determine the best angle at which to position a solar panel during the summer months in Texas (average latitude 30°) Calculate for Sun at its highest arc. Show Sun's winter inclination. Show panel's winter angle.		$\text{Sun's Winter Incline} = 90^\circ - \text{Latitude} - 23.45^\circ$ $\text{Sun's Summer Incline} = 90^\circ - 30 - 23.45^\circ = 36.55^\circ$ $\text{Panel's Winter Angle} = 36.55 + 90 = 126.55^\circ$
		<p>Yellow is Summer, Green is Winter</p>
3. Using the formula defined on page 1, determine the best angle at which to position a solar panel during the summer months in Pennsylvania (calculate for Sun at its highest arc). What are the Sun's summer inclination (angle) and the panel's winter angle?		$\text{Sun's Summer Incline} = 90^\circ - \text{Latitude} + 23.45^\circ$ $\text{Sun's Summer Incline} = 90^\circ - 41 + 23.45^\circ = 72.45^\circ$ $\text{Panel's Summer Angle} = 72.45 + 90 = 162.45^\circ$

Problems	Related, Generic Math Concepts	Solutions
4. Which angle would you estimate to be the interior angle of the hairpin shown here? How would you describe a "hairpin turn" in the road? a) 10° b) 45° c) 90° d) 120°		The correct answer is "a" because a 10° interior angle turn would very nearly turn a driver back in the direction from which s/he came. Hairpin turns get their name because they have interior angles similar to a real hairpin.
5. Your GPS indicates that you are traveling in a direction (bearing) that is determined to be 270°. If 90° is east, in which direction are you traveling?		You are traveling west (when your bearing is 270°) 90° is east; 180° is south; 360° is north.
6. To be wheelchair accessible, the steepness of ramps must not exceed 1 foot of rise per 12 feet of run. This equates approximately to a 5° angle. Use a protractor to draw this angle measure.		This is what the angle should resemble if drawn by a protractor.

Problems	PA Core Math Look	Solutions
7. What is the angle measure of $\angle XYZ$? a) 57° b) 63° c) 123° d) 137°		c) 123°
8. Which of the angles on the right is closest to 76°?		
9. Given: $a \parallel b, c \parallel d$ If $m\angle 1 = 2x + 16$ and $m\angle 2 = x + 14$, then what is the value of x?		Angles 1 and 2 are congruent angles so, $2x + 16 = x + 14 \rightarrow x + 16 = 14 \rightarrow x = -2$ (Subtract x from each side, then subtract 16 from each side.)