

Use estimated values = Apply the properties of rational and irrational numbers to solve real-world or mathematical problems

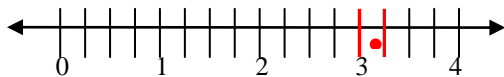
**Program Task:** Determine materials needed to meet job specifications.

**Program Associated Vocabulary:**  
ESTIMATE, TRUNCATE

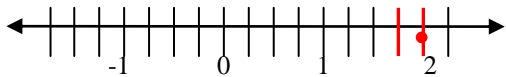
**Program Formulas and Procedures:**  
HVAC professionals don't always think of numbers in terms of rational and irrational. They simply truncate repeating decimals and provide technical solutions that are appropriate for a given situation. Whether we are conscious of their irrational classification or not, we routinely attribute concrete values to these numbers.

Anytime we use ( $\pi$ ) or a number such as  $\sqrt{3}$ , we are working with irrational numbers. Since we derive concrete measurements from irrational numbers, one could argue that in effect, we have located them on a number line (or tape measure/ruler).

$\pi$  (3.141592...) is about midway between 3 and 3.25



$\sqrt{3}$  (1.7320508075...) is slightly less than 1.75



**Example:** A pump curve indicates that schedule 40 iron pipe must have a cross sectional area not less than 9.4 square inches in order to maintain the desired flow rate. To find the diameter of the pipe needed, we could use these 2 formulas (use also for air ducts).

1.  $r^2 = \text{Area} / \pi$  (since  $\text{Area} = \pi \times r^2$ )
2.  $\text{Diameter} = 2 \times \sqrt{r^2}$  (since  $\sqrt{r^2} = \text{radius}$ )

$r^2 = \text{Area} / \pi$   
 $r^2 = 9.4 \text{ sq.in.} \div 3.14$  ( $\pi$  is truncated)  
 $r^2 = 3$  (round up from 2.99)  
 $r = 1.73$  (square root of 3 is truncated)

$\text{Diameter} = 2 \times \sqrt{r^2}$   
 $\text{Diameter} = 2 \times \sqrt{3}$   
 $\text{Diameter} = 2 \times 1.73$   
 $\text{Diameter} = 3.46''$

Using a pipe chart, we see that 3.46" I.D. schedule 40 pipe does not exist. 3.5" is available though, so that is our choice. 3.5" I.D. is the closest size pipe that is equal to or larger than the specified 9.4 square inches.

Note that rounding is usually upward when materials choices are the consideration –the actual pipe size (in this case, 3.5") will typically never be less than the derived number (in this case, 3.46").

**PA Core Standard:** CC.2.1.HS.F.2

**Description:** Apply the properties of rational and irrational numbers to solve real-world or mathematical problems.

**Math Associated Vocabulary:**  
IRRATIONAL NUMBER, SQUARE ROOT, PI

**Formulas and Procedures:**  
**Irrational Number:** a non-repeating & non-terminating decimal number that cannot be written as a fraction.

$\pi$ : The number  $\pi$  is a mathematical constant, commonly approximated as 3.14159.

**Square Root:** The square root of a number is a number which, when multiplied by itself, yields that original number.

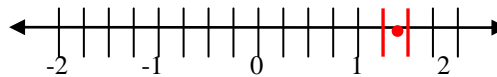
General steps to locate a number on a number line:

1. Rewrite the number as a decimal to the nearest tenth or hundredth digit.
2. Use marks (1/4, 1/2, 3/4,) to approximate the location on the number line.

**Example:** Identify  $\sqrt{2}$  on the number line below:

$$\sqrt{2} = 1.41421356237... \approx 1.41$$

1.41 is between 1.25 and 1.5, but closer to 1.5.



**Instructor's Script - Comparing and Contrasting**

Although it is not necessary for an HVAC technician to recognize that non-repeating, non-terminating decimal numbers are irrational, it is important for an HVAC technician to be able to round these irrational numbers to use them to find measurements. In a math classroom, students would be asked to identify the number on a traditional number line and in the HVAC classroom, the student would be asked to identify the number on a measuring tape or ruler.

**Common Mistakes Made By Students**

**Taking the square root of a number:** This mostly occurs when the student is unfamiliar with a calculator. Some calculators require the student to press the number then the square root button; others require that the square root button is pressed before the number. It may be important to show students how to take the square root of 4, using both methods to evaluate which order gives the correct answer of 2.

**Using the appropriate rounding technique for the given situation:** In most cases, it is beneficial to round the number to the nearest hundredth. If the number line is broken into quarters, thirds, or tenths, then rounding the number to the closest hundredth would provide the information necessary to correctly identify the number's location.

**Being able to partition a number line and identify the location of the decimal number:** Sometimes the number line uses integer values only (...,-2, -1, 0, 1, 2, 3...). In this case, the student must be able to mentally divide the space between the integers into quarters or thirds to best approximate the location of the irrational number.

**CTE Instructor's Extended Discussion**

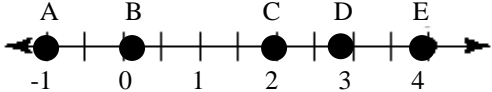
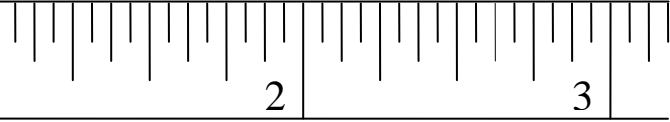
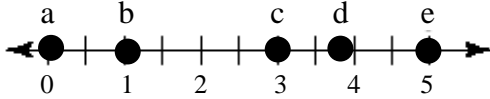
Your HVAC students may want to avoid dealing with the abstract nature of the irrational number. "How is it possible," the observant student may ask, "that we can measure the circumference of a circle, but we can't nail down a number that when multiplied by the diameter, produces that measured circumference?"

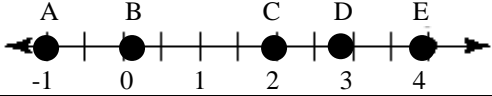
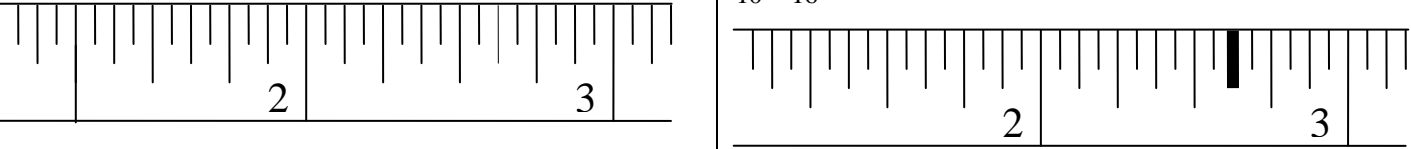
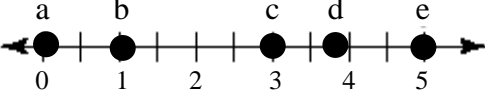
It doesn't seem to make sense, it's irrational!

It's easy to see how such abstract values can frustrate those whose comfort is more aligned with the concrete nature of rational numbers. It may have been this seemingly untidy bit of mathematical reasoning that led an engineer to say "Measure it with a micrometer, mark it with a pencil, and cut it with an axe!"

Hopefully the HVAC professional will not take that advice literally. Just know that a tape measure is a number line, and when we calculate a dimension that involves irrational numbers, then measure, make a mark, cut a material and install it, we are demonstrating proficiency in PA Core Standard CC.2.1.HS.F.2!

**Value Engineering** is defined as the **ratio of Function to Cost**. This is a good time to discuss this concept with your students. Mention the fact that when estimating for materials, one does not typically round down because waste and performance are real issues that must never be ignored. However, if the derived pipe size on page one had been 3.02", an HVAC installer would most likely have chosen 3 inch diameter pipe. There is room for judgment and some compromise. That is the nature of value engineering.

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A square duct has a cross sectional area of 11 square feet, what measurement do you expect to see on any side?		
2. Air flow velocity (V) in feet per minute = the square root of Velocity Pressure (P <sub>v</sub> ) × 4005. What is the Velocity in FPM if the P <sub>v</sub> = 0.7”?		
3. Pi (π) is an irrational number that we sometimes use to find a measurement (think of a tape measure as a number line). Problem: A roll of warning decal is 40 feet long. How many rolls are needed to wrap once around a chemical storage tank whose diameter is 38 feet?		
Problems	Related, Generic Math Concepts	Solutions
4. The location of $\sqrt{8}$ is closest to which point on the number line below? 		
5. What can't the square root of π be a rational number?		
6. Using the Pythagorean Theorem, a student finds that she needs $\sqrt{7}$ inches of material. Identify the location of this measurement on the measuring tape below. 		
Problems	PA Core Math Look	Solutions
7. Which of the following numbers would be located between 9 and 10 on the number line? a) $2\pi$ b) $3\pi$ c) $2\sqrt{5}$ d) $5\sqrt{2}$		
8. The location of $\sqrt{13}$ is closest to which point on the number line below? 		
9. 9) Which of the following would be closest to the value of $\sqrt{8}$ ? a) $2\frac{3}{4}$ b) $3\frac{1}{4}$ c) 4 d) $2\frac{1}{2}$		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A square duct has a cross sectional area of 11 square feet, what measurement do you expect to see on any side?		$\sqrt{11} = 3.316$ feet (3' 3 3/4")
2. Air flow velocity (V) in feet per minute = the square root of Velocity Pressure (P <sub>v</sub> ) × 4005. What is the Velocity in FPM if the P <sub>v</sub> = 0.7"?		Velocity = $\sqrt{.7} \times 4005$ FPM Velocity = 3,350.8 FPM
3. Pi (π) is an irrational number that we sometimes use to find a measurement (think of a tape measure as a number line). Problem: A roll of warning decal is 40 feet long. How many rolls are needed to wrap once around a chemical storage tank whose diameter is 38 feet?		Circumference = $\pi \times D$ Circumference = $3.1415 \times 38$ Circumference = 119.38 feet $119.38 \div 40 = 2.98$ rolls (3 rolls @ 40' each = 120 linear feet).
Problems	Related, Generic Math Concepts	Solutions
4. The location of $\sqrt{8}$ is closest to which point on the number line below? 		d) $\sqrt{8} = 2.828$
5. What can't the square root of π be a rational number?		Because π is an irrational number, and any rational number squared would produce a rational number.
6. Using the Pythagorean Theorem, a student finds that she needs $\sqrt{7}$ inches of material. Identify the location of this measurement on the measuring tape below. 		Since $\sqrt{7} = 2.645751\dots$ , We round to 2.6 inches. $\frac{6}{10} = \frac{x}{16} \rightarrow 6(16) = 10x \rightarrow 96 = 10x \rightarrow 9.6 = x, 2 \frac{10}{16} = 2 \frac{5}{8}$
Problems	PA Core Math Look	Solutions
7. Which of the following numbers would be located between 9 and 10 on the number line? a) $2\pi$ b) $3\pi$ c) $2\sqrt{5}$ d) $5\sqrt{2}$		b) $3\pi$
8. The location of $\sqrt{13}$ is closest to which point on the number line below? 		d.) $\sqrt{13} \approx 3.61$
9. Which of the following would be closest to the value of $\sqrt{8}$ ? a) $2 \frac{3}{4}$ b) $3 \frac{1}{4}$ c) 4 d) $2 \frac{1}{2}$		a) $2 \frac{3}{4}$ $\sqrt{8} \approx 2.828$