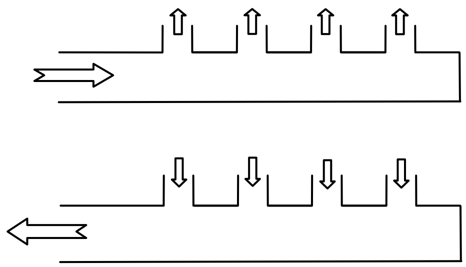
HVAC (47.0201) T-Chart

Determine cross sectional areas = Apply geometric concepts to model and solve real-world problems

Program Task: Design and build manifolds for hyrdonic and other fluid systems.

Program Associated Vocabulary:
DIAMETER, RADIUS, AREA, VOLUME, FLOW, RESTRICTION, RESISTANCE, FRICTION

Program Formulas and Procedures:
A manifold is a pipe or chamber having multiple apertures for making connections.



Manifolds are distribution systems for any material that has the ability to flow including refrigerants, hot water, steam, chilled water, drinking water, air, oils, propane, natural gas, and steam. Manifolds demonstrate how small changes in diameter can have a dramatic effect on area.

Depending on the application, a manifold may need to be designed to handle the full flow of all of its branches at one time. In the example above, it would be easy to incorrectly multiply the number of branches by their diameters to determine a manifold body size (e.g., four 1" inch branches = one 4" manifold body). That would be a mistake!

Correctly we need to add the cross sectional area of all of the branches by first finding the area of one of the branches.

$A = \pi r^2 = 3.14(.5)^2 = 0.785$ (round up to 0.8 in.^{2})

So four 1" branches would total (4 x 0.8) or 3.2 in.^{2}.
If we had used the 4" manifold body, its area would have been (3.14 x 2 x 2) or 12.6 in.^{2}. That's nearly 4 times bigger than it needs to be! Instead, we use the sum of the branch areas (3.2 in.^{2}) and work backwards to find the diameter of a pipe with that area.

$$A = \pi r^2 \quad \rightarrow \quad 3.2 = 3.14 r^2$$

$$\frac{3.2}{3.14} = \frac{3.14 r^2}{3.14}$$

$$1.0191 = r^2$$

$$\sqrt{1.0191} = r \quad \rightarrow \quad 1.00 \text{ in.} = r$$

The correct manifold body size, based on equal areas, is 2 inch diameter, not 4 inch!

PA Core Standard: CC.2.3.HS.A.14

Description: Apply geometric concepts to model and solve real-world problems.

Math Associated Vocabulary:
LINEAR DIMENSION, PERIMETER, CIRCUMFERENCE, AREA, VOLUME, DOUBLE, TRIPLE, CUBIC, SQUARE

Formulas and Procedures:
Students are often asked to evaluate the impact on perimeter, circumference, area, or volume when one of the linear dimensions, such as length, height or radius is increased. Often, the questions involve doubling or multiplying the dimension by a certain value.

In order to solve these problems, students should solve the problem using the original and larger linear dimension and then compare the results.

Example 1*:* If the radius of a circle is doubled, by how much is the area increased?

Step 1: Since a value has not been assigned to the original figure, assign any value to the original circle and solve for the area. For this example, use a value of 10 inches for the radius.

$$A = \pi r^2$$

$$A = 3.14(10)(10) = 314 \text{ in.}^2$$

Step 2: Double the radius and solve for the area.

$$A = \pi r^2$$

$$A = 3.14(20)(20) = 1256 \text{ in.}^2$$

Step 3: Compare the two areas. Did they double also or increase by some other multiplier? Hint: Divide the larger area by the smaller area.

1256 is 4 times larger than 314 because 1256/314 = 4

Example 2: If the side of a cube is increased from 3 in. to 6 in., by how much is the volume increased?

Step 1: Since a value has been assigned to the original figure, use this value (3 in.) and solve for the volume.

$V = l \times w \times h$ (the length, width, and height all equal 3 in.)

$$V = 3 \times 3 \times 3 = 27 \text{ in.}^3$$

Step 2**:** Find the volume of the larger figure.

$$V = 6 \times 6 \times 6 = 216 \text{ in.}^3$$

Step 3: Compare the two volumes. Did they double also or increase by some other multiplier? Hint: Divide the larger area by the smaller area.

216 is 8 times larger than 27 because 216/27 = 8

Instructor's Script - Comparing and Contrasting
In some instances, the old and the new linear dimensions are given (as shown in problem # 2 on page 3). In more complex cases, students are merely told that one of the linear dimensions has been doubled or tripled, or increased by a certain value. In these cases, it is very important for the student to be able to choose values that meet the given constraints (as shown in problem # 1 on page 3).

Common Mistakes Made By Students
Students often do not recognize that values may be substituted into the formulas to evaluate the effects of changing the linear dimensions. For instance, if the problem asks how the volume of a cube is affected if the lengths of the sides are doubled, the student could use two fictitious values to test, like 2 and 4 or 3 and 6.

CTE Instructor's Extended Discussion
Geometry is an integral part of our work that we use at times to resolve equipment, building shape and space issues.

HVAC professionals carry volume, area, perimeter and other formulas in their heads just as surely as they carry gauges, meters, and hand tools in their belts and boxes. As teachers, our goal is to help our students get to that same place. Become that well-rounded teacher, do the math, make it your business to reach the comfort level necessary for teaching the math concepts and formulas that make HVAC the profitable and satisfying career that we all know it can be.

Problems	Occupational (Contextual) Math Concepts Solutions
1. A 10 inch smoke header currently has one 5 inch smoke vent and two 6 inch smoke vents feeding into it. One of the 6 inch vents is removed and the customer would like the header size reduced. Determine the new header diameter.	
2. You are calculating the perimeter of a rectangular room (12 ft. x 16 ft.) in order to install perimeter heat. The architect decides that one end of the room will have its 12 foot flat wall replaced with a semi-circular window wall. The diameter of the half circle is 12 feet. How many additional feet of perimeter will the room have?	
3. To defeat downdrafts, a 16 foot metal chimney is extended 4 more feet. If the guy wire's lower shackle remains where it had been, 20 feet from the chimney base, what are the perimeters of the old and new triangles formed by roof line (base), guy wires (hypotenuse), and chimney (altitude)? How long must the new guy wire be to reach the top of the extended chimney?	
Problems	**Related, Generic Math Concepts Solutions**
4. A soup company would like to change the design of their cans so they hold more soup. If they triple the radius of the can, by how much will they increase the amount of soup each can can hold?	
5. Jake and Jenny each have a beach ball, but Jenny's ball has twice the diameter of Jake's ball. How many times more volume of air can Jenny's ball hold?	
6. A family decides to extend their garden to make it bigger. It originally had a width of 10 feet and a length of 13 feet. They must keep the length the same, but plan to increase the width of the garden to 12 feet. How much more fencing will they need to enclose the garden?	
Problems	**PA Core Math Look Solutions**
7. By how much does the area of a rectangle increase if the width remains the same but the length is doubled?	
8. The perimeter of a rectangle is 30 feet when the width is 5 ft. and the length is 10 ft. Find the perimeter if the length is 15 ft.	
9. By how much does the volume of a cylinder increase if the radius remains the same, but the height doubles?	

Problems	Occupational (Contextual) Math Concepts Solutions
1. A 10 inch smoke header currently has one 5 inch smoke vent and two 6 inch smoke vents feeding into it. One of the 6 inch vents is removed and the customer would like the header size reduced. Determine the new header diameter based on equal areas of vents.	$\text{Total Vent Area} = 3.14(2.5)^2 + 3.14(3)^2 = 47.885\,\text{in.}^2$ $A = \pi r^2 \rightarrow 47.885 = 3.14r^2$ $\frac{47.885}{3.14} = \frac{3.14r^2}{3.14} \rightarrow 15.25 = r^2$ $\sqrt{15.25} = r \rightarrow 3.91\,\text{in.} = r \rightarrow 7.81\,\text{in.} = d$ The diameter would be rounded to 8 in.
2. You are calculating the perimeter of a rectangular room (12 ft x 16 ft) in order to install perimeter heat. The architect decides that one end of the room will have its 12 foot flat wall replaced with a semi-circular window wall. The diameter of the half circle is 12 feet. How many additional feet of perimeter will the room have?	$\text{old } P = 2L + 2W = 2(12) + 2(16) = 56\,\text{in.}$ $C \text{ of semicircle} = (1/2)\pi d = (1/2)(3.14)(12) = 18.84\,\text{in.}$ $\text{new } P = L + 2W + \text{semicircle} = 12 + 2(16) + 18.84 = 62.84\,\text{in.}$ $62.84 - 56 = 6.84$ inches extra
3. To defeat downdrafts, a 16 foot metal chimney is extended 4 more feet. If the guy wire's lower shackle remains where it had been, 20 feet from the chimney base, what are the perimeters of the old and new triangles formed by roof line (base), guy wires (hypotenuse), and chimney (altitude)? How long must the new guy wire be to reach the top of the extended chimney?	$c = \sqrt{a^2 + b^2} \rightarrow c = \sqrt{20^2 + 16^2} \rightarrow c = \sqrt{656} \rightarrow c = 25.61\,\text{ft.}$ $\text{old } P = 20 + 16 + 25.61 = 61.61\,\text{ft.}$ $c = \sqrt{a^2 + b^2} \rightarrow c = \sqrt{20^2 + 20^2} \rightarrow c = \sqrt{800} \rightarrow c = 28.28\text{ft.}$ $\text{new } P = 20 + 20 + 28.28 = 68.28\,\text{ft.}$ change in guidewire $= 28.28 - 25.61 = 2.67$ ft.
Problems	**Related, Generic Math Concepts Solutions**
4. A soup company would like to change the design of their cans so they hold more soup. If they triple the radius of the can, by how much will they increase the amount of soup each can can hold?	$V = \pi r^2 h$ Method 1: replace r with 3r (tripled) $V = \pi(3r)^2 h = \pi(9r^2)h$ The volume is nine times larger. Method 2: Compare the volumes using r = 1 and r = 3, h = 1. $V = \pi r^2 h \quad V = (3.14)(1)(1)(1) = 3.14$ $V = \pi r^2 h \quad V = (3.14)(3)(3)(1) = 28.26$ Divide the two values to find the factor by which the volume increases. $28.26 \div 3.14 = 9$
5. Jake and Jenny each have a beach ball, but Jenny's ball has twice the diameter of Jake's ball. How many times more volume of air can Jenny's ball hold?	$V_1 = \frac{4}{3}\pi r^3 \quad V_2 = \frac{4}{3}\pi(2r)^3 \quad V_2 = \frac{4}{3}\pi(8r^3)$ Jenny's ball holds 8 times more air.
6. A family decides to extend their garden to make it bigger. It originally had a width of 10 feet and a length of 13 feet. They must keep the length the same, but plan to increase the width of the garden to 12 feet. How much more fencing will they need to enclose the garden?	Original perimeter = 2(10) + 2(13) = 46 ft. New perimeter = 2(12) + 2(13) = 50 ft. They will need 4 more feet of fencing.
Problems	**PA Core Math Look Solutions**
7. By how much does the area of a rectangle increase if the width remains the same but the length is doubled?	$A_1 = lw \quad A_2 = 2lw$ The area doubles.
8. The perimeter of a rectangle is 30 feet when the width is 5 ft. and the length is 10 ft. Find the perimeter if the length is 15 ft.	Perimeter = 2(5) + 2(15) = 40 feet.
9. By how much does the volume of a cylinder increase if the radius remains the same, but the height doubles?	If the height doubles, the volume doubles. $V_1 = \pi r^2 h,\ V_2 = \pi r^2 2h$