

Determine transformer calculations = Use reasoning to solve equations and justify the solution method

Program Task: Identify the relationship between primary and secondary turns, primary and secondary voltage, and primary and secondary currents.

Program Associated Vocabulary

PRIMARY WINDING, SECONDARY WINDING, TURNS, CURRENT (or AMPERAGE), RATIO, VOLTAGE, VOLTAMPS (VA) RATING

Program Formulas and Procedures

A transformer is an electrical device that transforms an applied voltage to a higher or lower output voltage. The voltage in:out ratio is equal to the ratio of turns in the windings (primary:secondary).

$V = \text{Voltage}$ $N = \text{Number of Turns}$

From Faraday's Law : $V_2 = V_1 (N_2/N_1)$ – Direct Relation
Transformers are rated in voltamps, or VA's (volts x amps).
Input VA's are equal output VA's, which means the output voltage is inversely proportional to the output current.
Knowing these relationships makes calculating without formulas possible.

$V = \text{Voltage}$ $I = \text{Current (amperes)}$

From conservation of energy: $V_1 I_1 = V_2 I_2$
Then: $I_2 = I_1 (N_1/N_2)$ – Inverse Relation

Example 1: A transformer has 300 turns in the primary, 50 turns in the secondary, and 120 volts applied to the primary. What is the voltage of the secondary?

Step 1: Set up the proportion Step 2: Invert ratio?
 $\frac{300 \text{ turns}}{50 \text{ turns}} = \frac{120 \text{ Volts}}{x \text{ volts}}$ (no, it's a direct proportion)

Step 3: Cross-multiply and divide to solve
 $50(120) = 300x$, $x = 20$ volts

Example 2: A transformer has 300 turns in the primary, 50 turns in the secondary, and 1.2 amperes applied to the primary. What is the current of the secondary?

Step 1: Set up the proportion Step 2: Invert ratio
 $\frac{300 \text{ turns}}{50 \text{ turns}} = \frac{1.2 \text{ Amperes}}{x \text{ Amperes}}$ $\frac{300 \text{ turns}}{50 \text{ turns}} = \frac{x \text{ Amperes}}{1.2 \text{ Amperes}}$

Step 3: Cross-multiply and divide to solve
 $300(1.2) = 50x$, $x = 7.2$ Amperes

PA Core Standard: CC.2.2.HS.D.9

Description: Use reasoning to solve equations and justify the solution method.

Math Associated Vocabulary

INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT

Formulas and Procedures

Direct Proportions

Two quantities, A and B, are directly proportional if by whatever factor A changes, B changes by the same factor.

Example 1: Take the formula distance = rate x time. If the rate remains constant, 30 miles per hour, then the time and distance are directly proportional.

$d = 30t$
when $t = 2$, $d = 60$
when $t = 4$, $d = 120$

*Note that when the time doubles, so does the distance.

Example 2: If speed is directly proportional to distance and a car can travel 100 miles at 50 miles per hour. How far can that car travel during the same time if it travels 70 mph?

Step 1: Set up proportion.

$\frac{50\text{mph}}{70\text{mph}} = \frac{100\text{mi.}}{x}$

Step 2: Cross multiply and divide to solve.

$50x = 70(100) \rightarrow 50x = 7000 \rightarrow x = 140$ miles

Indirect Proportions

Two quantities, A and B, are inversely proportional if by whatever factor A changes, B changes by the multiplicative inverse, or reciprocal of that factor.

Example 1: Take the formula distance = rate x time. If the distance is constant, 100 miles, then as the rate increases the time decreases.

$100 = rt$
When $r = 100$, $t = 1$
When $r = 50$, $t = 2$

*Note that when the rate doubles, the time is halved.

Example 2: If the time needed to complete a job is inversely proportional to the number of people working, how long would it take 4 people to paint a room if 1 person needs 8 hours?

Step 1: Set up the proportion

$\frac{1 \text{ person}}{4 \text{ people}} = \frac{8 \text{ hours}}{x \text{ hours}}$

Step 2: Invert (flipA) one ratio

$\frac{1 \text{ person}}{4 \text{ people}} = \frac{x \text{ hours}}{8 \text{ hours}}$

Step 3: Cross-multiply and divide to solve

$4x = 8$, $x = 2$ 4 people can paint the room in 2 hours.

Teacher's Script - Comparing and Contrasting

Transformers provide a real-world example of direct and inverse proportional relationships at work.

Common Mistakes Made By Students

When students compare Direct and Inverse Proportional relationships, they may become confused and have difficulty differentiating one from the other. One way to keep them straight is to:

1. Set up one pair of values on the same line, e.g., $\frac{12''}{24''} = \frac{100 \text{ lbs.}}{x \text{ lbs.}}$ (from problem #1)
2. Beneath that line, place the other pair of values, $\frac{12''}{24''} = \frac{100 \text{ lbs.}}{x \text{ lbs.}}$
3. Cross multiply (24 times 100) and (12 times x), but first determine if you have to invert one ratio.
4. If you have to invert one ratio, then it is an inverse proportion.
5. If need be, set up the problem and do it both ways to see which answer makes sense! We know in *problem #9*, for example, that it won't take 5 rabbits more time than it took 1 rabbit to eat 20 carrots, so it must be an inverse proportion.

Lab Teacher's Extended Discussion

None

Problems	Occupational (Contextual) Math Concepts	Solutions
1. I have a primary input of 120v and 161 turns; the secondary is 88 turns. What is the secondary voltage?		
2. Given a primary voltage of 120 volts, a primary current of 7.5 Amperes, and a secondary voltage of 16, find the current.		
3. Given a primary voltage of 120 volts, a primary current of 7.5 Amperes, and secondary amperage of 45 amps, find the secondary voltage.		
Problems	Related, Generic Math Concepts	Solutions
4. If it takes 12 eggs to make 1 dozen, how many eggs will be needed to make 9 dozen?		
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm (atmospheres), what would the pressure be of 0.060 m ³ of the same gas at the same temperature?		
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?		
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional and y = 2 when x = 5, find the value of y when x = 15.		
8. Given that y and x are inversely proportional and y = 2 when x = 5, find the value of y when x = 15.		
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same 20 carrots?		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. I have a primary input of 120v and 161 turns; the secondary is 88 turns. What is the secondary voltage?	*turns and voltage are directly proportional	<p><u>Step 1: Set up the proportion</u></p> $\frac{120 \text{ volts}}{x} = \frac{161 \text{ turns}}{88 \text{ turns}}$ <p><u>Step 2: Cross-multiply and divide to solve</u></p> $120(88) = 161x \rightarrow x = 65.6 \text{ Volts}$
2. Given a primary voltage of 120 volts, a primary current of 7.5 Amperes, and a secondary voltage of 16, find the current.	*voltage and current are inversely proportional	<p><u>Step 1: Set up the proportion</u> <u>Step 2: Invert one ratio</u></p> $\frac{120 \text{ volts}}{16 \text{ volts}} = \frac{7.5 \text{ Amps}}{x \text{ Amps}} \quad \frac{120 \text{ volts}}{16 \text{ volts}} = \frac{x \text{ Amps}}{7.5 \text{ Amps}}$ <p><u>Step 3: Cross-multiply and divide to solve</u></p> $120(7.5) = 16x, x = 56.25 \text{ Amps}$
3. Given a primary voltage of 120 volts, a primary current of 7.5 Amperes, and secondary amperage of 45 amps, find the secondary voltage.	*voltage and current are inversely proportional	<p><u>Step 1: Set up the proportion</u> <u>Step 2: Invert one ratio</u></p> $\frac{120 \text{ volts}}{x \text{ volts}} = \frac{7.5 \text{ Amps}}{45 \text{ Amps}} \quad \frac{120 \text{ volts}}{x \text{ volts}} = \frac{45 \text{ Amps}}{7.5 \text{ Amps}}$ <p><u>Step 3: Cross-multiply and divide to solve</u></p> $120(7.5) = 45x, x = 20 \text{ volts}$
Problems	Related, Generic Math Concepts	Solutions
4. If it takes 12 eggs to make 1 dozen, how many eggs will be needed to make 9 dozen?	(Direct)	$\frac{12 \text{ eggs}}{x \text{ eggs}} = \frac{1 \text{ dozen}}{9 \text{ dozen}} \rightarrow 1x = 12(9) \rightarrow x = 108 \text{ eggs}$
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm (atmospheres), what would the pressure be of 0.060 m ³ of the same gas at the same temperature?	(Inverse)	$\frac{0.24\text{m}^3}{0.060\text{m}^3} = \frac{0.5 \text{ atm}}{x \text{ atm}} \quad (\text{Invert one ratio since it is an inverse proportion.})$ $\frac{0.24\text{m}^3}{0.060\text{m}^3} = \frac{x \text{ atm}}{0.5 \text{ atm}} \rightarrow 0.24(0.5) = 0.060x \rightarrow x = 2 \text{ atm}$
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?	(Direct)	$\frac{10 \text{ castings}}{14 \text{ castings}} = \frac{26 \text{ lbs}}{x \text{ lbs.}} \rightarrow 10x = 26(14) \rightarrow x = 36.4 \text{ lbs}$
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional and y = 2 when x = 5, find the value of y when x = 15.	(Direct)	$\frac{5}{15} = \frac{2}{y} \rightarrow 5y = 2(15) \rightarrow y = 6$
8. Given that y and x are inversely proportional and y = 2 when x = 5, find the value of y when x = 15.	(Inverse)	$\frac{5}{15} = \frac{y}{2} \rightarrow 15y = 2(5) \rightarrow y = 0.667$
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same 20 carrots?	(Inverse)	$\frac{1}{5} = \frac{x}{15} \rightarrow 5x = 1(15) \rightarrow x = 3 \text{ hours}$