

Calculate volume = Explain volume formulas and use them to solve problems

Program Task: Calculate volumes of various shapes.

Program Associated Vocabulary:

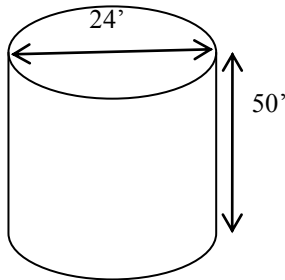
AREA, VOLUME, LENGTH, WIDTH, HEIGHT, RECTANGULAR, ROUND, CYLINDRICAL, BASE, RADIUS

Program Formulas and Procedures:

Both architectural and mechanical drafters often will need to calculate the volume of a specific shape that holds a liquid or gas. This may be required if you are designing a metal tank and need to calculate the capacity of a the tank, or if you design a swimming pool and need to calculate the amount of water required to fill it.

Example:

Calculate how many gallons of water the illustrated tank will hold. One cubic foot contains approximately 7.5 gallons.



Solution:

$$V = \pi r^2 h$$

$$V = 3.14 \times 12^2 \times 50'$$

$$V = 3.14 \times 144 \times 50'$$

$$V = 22,608 \text{ cubic feet}$$

$$22,608 \times 7.5 = 169,256 \text{ gallons}$$

PA Core Standard: CC.2.3.HS.A.12

Description: Explain volume formulas and use them to solve problems.

Math Associated Vocabulary:

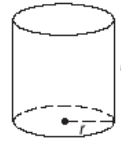
AREA, VOLUME, LENGTH, WIDTH, HEIGHT, RECTANGULAR, ROUND, CYLINDRICAL, BASE, RADIUS, RECTANGULAR PRISM, CYLINDER, CONE, SPHERE, PYRAMID

Formulas and Procedures:

Volume:

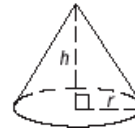
Cylinder:

$$V = \pi r^2 h$$



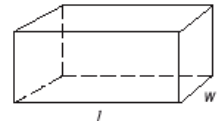
Cone:

$$V = \frac{1}{3} \pi r^2 h$$



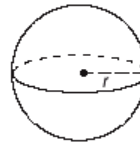
Rectangular Prism:

$$V = lwh$$



Sphere:

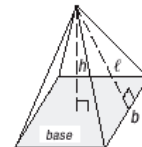
$$V = \frac{4}{3} \pi r^3$$



Pyramid:

$$V = \frac{1}{3} (\text{area of the base})h$$

b = base ℓ = slant length



Example:

How many cubic inches of air can a beach ball hold if it has a diameter of 14 inches? Round to the nearest whole number.

Steps to finding volume:

1. Identify the solid. (sphere)
2. Write the formula for calculating the volume of that solid using the formula sheet.
 $V = \frac{4}{3} \pi r^3$
3. Identify what information you are given in the example.
Given: diameter (d) = 14"
4. Solve for radius using the formula radius (r) = $\frac{1}{2}$ (diameter).
 $r = \frac{1}{2} \times 14 = 7$
5. Perform the necessary mathematical operations to obtain your answer.
 $V = \frac{4}{3} \pi r^3 = \frac{4}{3} (3.14) (7^3) = 1,436 \text{ in}^3$
6. Write the appropriate unit after your answer.
1,436 in³

Instructor's Script - Comparing and Contrasting

This PA Common Core Standard includes using volume and volume formulas to “work backward” to find a missing dimension. Many students are able to calculate volumes but are unable to manipulate the formulas to find missing dimensions. Teaching these two concepts together will help the student gain a deeper understanding of the concept of volume. In many real-world examples, students must also be able to convert the cubic linear measurement into gallons.

Common Mistakes Made By Students

Students may use an incorrect formula to solve a problem: To rectify these errors have the students correctly identify the type of object they are dealing with and use the appropriate formula. Frequently two formulas may be needed for complex problems.

Using consistent units: If the problem asks for the answer in square feet instead of square inches, be sure to either convert your given measurements into feet first ($\text{inches} \div 12 = \text{feet}$) or convert your square inch answer into square feet ($\text{sq. inches} \div 144 = \text{sq. feet}$).

CTE Instructor's Extended Discussion

Understanding how to calculate volumes is used across many disciplines of drafting. Mechanical designers often design liquid storage vessels such as hot water storage tanks. HVAC designers need to be able to calculate volumes of spaces in order to determine air volume requirements for heating, cooling, and ventilation.

Problems	Career and Technical Math Concepts	Solutions
1. You design a swimming pool for a client. The pool is 32' long, 16' wide, and 8' deep. How many gallons of water are required to fill it? Use 3.14 for π and round to the nearest whole number. A cubic foot contains approximately 7.5 gallons of water.		
2. You are asked to design a cylindrical water storage tank that will hold 200 gallons. There is only enough space for a 2' diameter tank. How long must it be? Use 3.14 for π and convert your final answer into feet and inches. A cubic foot contains approximately 7.5 gallons of water.		
3. A client has a 10 foot diameter round tank (sphere). He needs you to tell him how much water it will hold. Use 3.14 for π and round to the nearest tenth if necessary. A cubic foot contains approximately 7.5 gallons of water.		
Problems	Related, Generic Math Concepts	Solutions
4. One soup can has a $d = 3''$ and $h = 4''$; another soup can has a $d = 4''$ and $h = 3''$. Which can holds more soup? Use π and round to the nearest hundredth.		
5. A size 7 regulation basketball has a $d=9.39''$. A size 6 regulation basketball has a $d=9.07''$. What is the volume of each basketball to the nearest whole number?		
6. How much water would you need to fill a rectangular fish tank with a height of 16.5'', a length of 32'', and a width of 8.5''?		
Problems	PA Core Math Look	Solutions
7. Find the volume of a cylinder if $d = 12.5'$ and $h = 28.45'$. Round your answer to the nearest thousandth.		
8. Find the volume of a sphere if $d = 27.75''$. Round your answer to the nearest hundredth.		
9. Find the volume of a regular pyramid with a square base with sides of 10'' and a height of 25''.		

Problems	Career and Technical Math Concepts	Solutions
1. You design a swimming pool for a client. The pool is 32' long, 16' wide, and 8' deep. How many gallons of water are required to fill it? Use 3.14 for π and round to the nearest whole number. A cubic foot contains approximately 7.5 gallons of water.	$V = lwh$ $V = 32 \times 16 \times 8 = 4096 \text{ ft}^3$ $4096 \text{ ft}^3 \times 7.5 \text{ gals per ft}^3 = 30,720 \text{ gallons of water.}$	
2. You are asked to design a cylindrical water storage tank that will hold 200 gallons. There is only enough space for a 2' diameter tank. How long must it be? Use 3.14 for π and convert your final answer into feet and inches. A cubic foot contains approximately 7.5 gallons of water.	$V = \pi r^2 h$ $200 \text{ gallons} / 7.5 \text{ gallons per cubic foot} = 26.67 \text{ ft}^3$ $26.67 = 3.14 \times 1^2 \times y$ $26.67 = 3.14 \times 1 \times y$ $26.67 = 3.14y$ $26.67 / 3.14 = 8.49 \text{ feet}$ $.49 \times 12 = 6 \rightarrow 8 \text{ft } 6 \text{in}$ Check: $26.67 = 3.14 \times 1 \times 8.49 = 26.66$ Rounding off, the tank would be 8'-6" long	
3. A client has a 10 foot diameter round tank (sphere). He needs you to tell him how much water it will hold. Use 3.14 for π and round to the nearest tenth if necessary. A cubic foot contains approximately 7.5 gallons of water.	$V = 4/3\pi r^3$ $V = 4/3 \times 3.14 \times 5^3$ $V = 4/3 \times 3.14 \times 125$ $\frac{4 \times 3.14 \times 125}{3} = \frac{1570}{3} = 523 \text{ ft}^3$ $523 \text{ ft}^3 \times 7.5 \text{ gallons per ft}^3 = 3922.5 \text{ gallons}$	
Problems	Related, Generic Math Concepts	Solutions
4. One soup can has a $d = 3''$ and $h = 4''$; another soup can has a $d = 4''$ and $h = 3''$. Which can holds more soup? Use π and round to the nearest hundredth.	$V = \pi r^2 h$ Can 1: $V = \pi(1.5)^2 4$ $V = 28.27 \text{ in.}^3$	Can 2: $V = \pi(2)^2 3$ $V = 37.70 \text{ in.}^3$
5. A size 7 regulation basketball has a $d=9.39''$. A size 6 regulation basketball has a $d=9.07''$. What is the volume of each basketball to the nearest whole number?	Size 7 $V = \frac{4}{3} \pi r^3 = 1.333 \times \pi \times 4.695^3 = 432 \text{ in.}^3$ Size 6 $V = \frac{4}{3} \pi r^3 = 1.333 \times \pi \times 4.535^3 = 391 \text{ in.}^3$	
6. How much water would you need to fill a rectangular fish tank with a height of 16.5'', a length of 32'', and a width of 8.5''?	$V = (32)(8.5)(16.5) = 4488 \text{ in.}^3$	
Problems	PA Core Math Look	Solutions
7. Find the volume of a cylinder if $d = 12.5'$ and $h = 28.45'$. Round your answer to the nearest thousandth.	$V = \pi r^2 h$ $r = \frac{1}{2} (12.5) = 6.25$ $V = \pi \times 6.25^2 \times 28.75$ $V \approx 3,526.37 \text{ ft.}^3$	
8. Find the volume of a sphere if $d = 27.75''$. Round your answer to the nearest hundredth.	$V = \frac{4}{3} \times \pi \times r^3$ $V = 1.333 \times \pi \times 13.875^3$ $V \approx 11,180.44 \text{ in.}^3$	
9. Find the volume of a regular pyramid with a square base with sides of 10'' and a height of 25''.	$V = \frac{1}{3} (\text{area of base}) h$ Area of base = $10 \times 10 = 100$ $V = \frac{1}{3} (100) (25) \approx 833.33 \text{ in}^3$	