

Belt and Pulley Ratios –Inverse and Direct Proportions

Program Task: Combination pulleys are used to make the job of lifting heavy objects easier. In the past, workers would need to use their own force to move objects. The use of pulleys, and now motors to run the pulleys, has made work easier for all maintenance professionals.

Program Associated Vocabulary

PULLEY, RATIO, RPM (REVOLUTIONS PER MINUTE), DIAMETER, ANGULAR VELOCITY

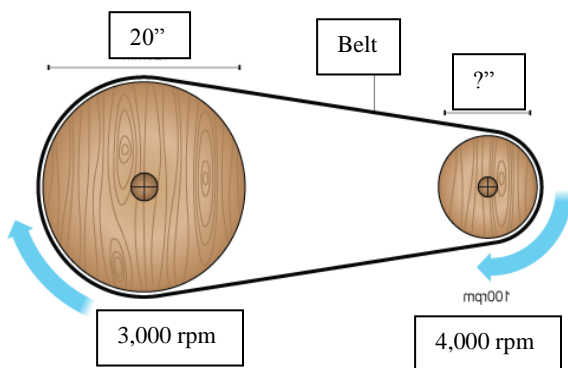
Program Formulas and Procedures

Pulleys work on a ratio and proportion concept. Looking at the ratio of RPM (revolutions per minute) in relation to diameter shows students that if the original pulley diameter and rpms stay the same, as the RPM's increase on the second pulley, the diameter of the second pulley will decrease. The formula is:

$$\text{Belt \& Pulley Ratio} \rightarrow \frac{\text{Angular Velocity (Pulley 1)}}{\text{Angular Velocity (Pulley 2)}} = \frac{\text{Diameter (Pulley 2)}}{\text{Diameter (Pulley 1)}}$$

This ratio is the basis for our example below.

Example: A maintenance supervisor must set up a pulley system in order to lift an item from the ground floor of the apartment complex to the fourth floor. He has a system that looks like this one in the diagram. If Pulley A is moving at 3,000 revolutions per minute (rpm) and has a diameter of 20", what diameter of a secondary pulley would be needed to have that pulley run at 4,000 rpm?



Step 1: Set up the formula for Belt and Pulley Ratio

$$\text{Belt \& Pulley Ratio} \rightarrow \frac{\text{Angular Velocity (Pulley 1)}}{\text{Angular Velocity (Pulley 2)}} = \frac{\text{Diameter (Pulley 2)}}{\text{Diameter (Pulley 1)}}$$

Step 2: Define the variables in the equation and solve.

$$\text{Belt \& Pulley Ratio} \rightarrow \frac{3,000 \text{ rpm}}{4,000 \text{ rpm}} = \frac{x''}{20''}$$

$$3,000(20) = 4,000x$$

$$\frac{60,000}{4,000} = \frac{4,000x}{4,000}$$

$$15'' = x \quad \text{Pulley 2 needs to have a 15'' diameter.}$$

Use units as a way to understand and solve problems.

PA Core Standard: CC.2.1.HS.F.4 Use units to understand and solve multi-step problems.

Description: Use units as a way to understand problems and to guide the solution of multi-step problems.

Math Associated Vocabulary

INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT

Formulas and Procedures

Direct Proportions

Two quantities, A and B, are directly proportional if by whatever factor A changes, B changes by the same factor.

Example 1: Take the formula distance = rate x time. If the rate remains constant, 30 miles per hour, then the time and distance are directly proportional.

$$\begin{aligned} d &= 30t \\ \text{when } t &= 2, d = 60 \\ \text{when } t &= 4, d = 120 \end{aligned}$$

*Note that when the time doubles, so does the distance.

Example 2: If speed is directly proportional to distance and a car can travel 100 miles at 50 miles per hour. How far can that car travel during the same time if it travels 70 mph?

Step 1: Set up proportion.

$$\frac{50 \text{ mph}}{70 \text{ mph}} = \frac{100 \text{ mi.}}{x}$$

Step 2: Cross multiply and divide to solve.

$$50x = 70(100) \rightarrow 50x = 7000 \rightarrow x = 140 \text{ miles}$$

Indirect Proportions

Two quantities, A and B, are inversely proportional if by whatever factor A changes, B changes by the multiplicative inverse, or reciprocal of that factor.

Example 1: Take the formula distance = rate x time. If the distance is constant, 100 miles, then as the rate increases the time decreases.

$$\begin{aligned} 100 &= rt \\ \text{When } r &= 100, t = 1 \\ \text{When } r &= 50, t = 2 \end{aligned}$$

*Note that when the rate doubles, the time is halved.

Example 2: If the time needed to complete a job is inversely proportional to the number of people working, how long would it take 4 people to paint a room if 1 person needs 8 hours?

Step 1: Set up the proportion. $\frac{1 \text{ person}}{4 \text{ people}} = \frac{8 \text{ hours}}{x \text{ hours}}$

Step 2: Invert (flip A) one ratio $\frac{1 \text{ person}}{4 \text{ people}} = \frac{x \text{ hours}}{8 \text{ hours}}$

Step 3: Cross-multiply and divide to solve

$$4x = 8, x = 2 \quad 4 \text{ people can paint the room in 2 hours.}$$

Building Property Maintenance (46.0401) T-Chart

Teacher's Script - Comparing and Contrasting

Indirect and Direct proportional relationships are important when working with all forms of equations, both in the maintenance environment and in other real world applications. This combination pulley problem is a great practical use of ratios and proportions in a real world situation. It is important to know that there is an inverse relation between the two pulleys. If the rpms and the pulley diameter of the first pulley remain constant, any increase in the rpms of the second pulley will result in a decrease in the diameter.

Direct proportions basically mean that when one factor changes (increases) the other factor does the same (increases). It relates directly to the others actions.

Indirect proportions basically mean that when one factor changes (increases) the other factor does the opposite (decreases). It relates indirectly to the way the other one

Common Mistakes Made By Students

When students compare Direct and Inverse Proportional relationships, they may become confused and have difficulty differentiating one from the other.

1. Students don't set up the relation correctly and compare and analyze the results.

Example: If you have a maintenance man working on a job for 6 hours, how long will it take for 2 maintenance men to work on this same job?

You choose: $\frac{1 \text{ maint man}}{2 \text{ maint men}} = \frac{6 \text{ hours}}{x \text{ hours}}$ $12=x \text{ hours}$ $x=12 \text{ hours}$

Now – ask yourself, “Does this make sense?” The answer should be, “If I add a maintenance man to help work at this job, then the hours should decrease. In this problem they increased. It is an indirect proportion, so I need to flip and solve:

$\frac{1 \text{ maint. man}}{2 \text{ maint. men}} = \frac{x \text{ hours}}{6 \text{ hours}}$ $6=2x \text{ hours}$ $x=3 \text{ hours}$ Now say, “1 maintenance man takes 6 hours, if I add a second maintenance it takes 3 hours.” This makes sense.

2. Choosing information from a written situation:

Many students do not read and “pull out” information from the problem correctly. This is something that every teacher could work on with any situation. Many students know how to “do the math”, but if the problem is not set up properly or information is not correct, good math calculations will still result in incorrect results.

Lab Teacher's Extended Discussion

It is important for students to understand the relationship of pulleys and drive belts. There are times when you are on the job where machines break and you need to do things manually. It is a real world problem that gets overlooked due to today's automation. A mechanic may need to do calculations and replace the part within an hour; therefore, knowing how to calculate a diameter based upon the rpms needed would be imperative.

Many students have trouble with ratio and proportions. They do not understand the common sense behind the fact that the faster the rpms in a combination pulley system, the smaller the diameter of the pulley. It is of value to set up a lab to demonstrate how this ratio actually works between two pulleys.

Building Property Maintenance (46.0401) T-Chart

Problems	Occupational (Contextual) Math Concepts	Solutions
1. If Pulley A is moving at 3,000 revolutions per minute (rpm) and has a diameter of 20", what diameter of a secondary pulley would you need to have that one run at 7,000 rpm?		
2. The maintenance department has two pulleys. The larger one runs at 3,000 rpms and has a diameter of 20". If the other one has a diameter of 12", what are the rpm's for this pulley?		
3. A maintenance apprentice was sent to pick up some pulleys for a job. The store has 3 pulleys for \$ 145.59. If he needs 8 pulleys and can buy them for the same price as the 3 for \$145.59, what is the cost of 8 pulleys?		
Problems	Related Generic Math Concepts	Solutions
4. If it takes 12 eggs to make 1 dozen, how many eggs will be needed to make 9 dozen?		
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm, what would the pressure be of 0.060 m ³ of the same gas at the same temperature?		
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?		
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional and y = 2 when x = 5, find the value of y when x = 15.		
8. Given that y and x are inversely proportional and y = 2 when x = 5, find the value of y when x = 15.		
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same 20 carrots?		

Building Property Maintenance (46.0401) T-Chart

Problems	Occupational (Contextual) Math Concepts	Solutions
1. If Pulley A is moving at 3,000 revolutions per minute (rpm) and has a diameter of 20", what diameter of a secondary pulley would you need to have that one run at 7,000 rpm?	Belt & Pulley Ratio $\rightarrow \frac{3,000 \text{ rpm}}{7,000 \text{ rpm}} = \frac{x''}{20''}$ $3,000(20) = 7,000x$ $60,000 = 7,000x$ $8.57'' = x$	Pulley 2 needs to have a 8.57" diameter.
2. The maintenance department has two pulleys. The larger one runs at 3,000 rpms and has a diameter of 22 ½". If the other one has a diameter of 12", what are the rpm for this pulley?	Belt & Pulley Ratio $\rightarrow \frac{3,000 \text{ rpm}}{x \text{ rpm}} = \frac{12''}{22.5''}$ $3,000(22.5) = 12x$ $67,500 = 12x$ $5,625 \text{ rpm} = x$	Pulley 2 would be running at 5,625 rpms.
3. A maintenance apprentice was sent to pick up some pulleys for a job. The store has 3 pulleys for \$ 145.59. If he needs 8 pulleys and can buy them for the same price as the 3 for \$145.59, what is the cost of 8 pulleys?	Belt & Pulley Purchase® $\frac{\$ 145.59}{3 \text{ pulleys}} = \frac{\$ x}{8 \text{ pulleys}}$ $\$145.59(8)=3x$ $1,164.72 = 3x$ $\$388.24= x$	It will cost him \$ 388.24 for 8 pulleys.
Problems	Related, Generic Math Concepts	Solutions
4. If it takes 12 eggs to make 1 dozen, how many eggs will be needed to make 9 dozen?	(Direct)	$\frac{12\text{eggs}}{x\text{eggs}} = \frac{1\text{dozen}}{9\text{dozen}} \rightarrow 1x = 12(9) \rightarrow x = 108\text{eggs}$
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm, what would the pressure be of 0.060 m ³ of the same gas at the same temperature?	(Inverse)	$\frac{0.24\text{m}^3}{0.060\text{m}^3} = \frac{0.5\text{atm}}{x\text{atm}}$ (Invert one ratio since, it's an inverse proportion) $\frac{0.24\text{m}^3}{0.060\text{m}^3} = \frac{x\text{atm}}{0.5\text{atm}} \rightarrow 0.24(0.5) = 0.060x \rightarrow x = 2\text{atm.}$
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?	(Direct)	$\frac{10\text{castings}}{14\text{castings}} = \frac{26\text{lbs.}}{x\text{lbs.}} \rightarrow 10x = 26(14) \rightarrow x = 36.4\text{lbs.}$
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional and y = 2 when x = 5, find the value of y when x = 15.	(Direct)	$\frac{5}{15} = \frac{2}{y} \rightarrow 5y = 2(15) \rightarrow y = 6$
8. Given that y and x are inversely proportional and y = 2 when x = 5, find the value of y when x = 15.	(Inverse)	$\frac{5}{15} = \frac{y}{2} \rightarrow 15y = 2(5) \rightarrow y = 0.667$
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same 20 carrots?	(Inverse)	$\frac{1}{5} = \frac{x}{15} \rightarrow 5x = 1(15) \rightarrow x = 3 \text{ hours}$