Automotive Technology (47.0604) T-Chart

Determine fluid service intervals	=	Construct and compare linear, quadratic, and exponential models to solve problems

Program Task: Perform under hood, under car vehicle maintenance.

Program Associated Vocabulary:
MEASURE, RATE, CHANGE, GROWTH, DEGRADATION, INCREASE, DECREASE

Program Formulas and Procedures:
Additives and compounds in automotive fluids, such as anti-freeze (coolant) and brake fluid break down (deteriorate) after time. Deterioration not only stops protecting specific parts and components; it actually begins to corrode those same parts. This can lead to expensive repairs, which could result in accidents if parts fail while driving. That is why it is important to change/flush fluids based on time as well as mileage.

Example 1:
New brake fluid contains 10,000 PPM of a particular anti-corrosion inhibitor that keeps the brake lines from rusting. If the inhibitor breaks down 40% a year, how much of the additive will be present after 2, 4 & 10 years?

NOTE: In this example r will be negative because amount of additive is decreasing.

$$IV \times (1 - r)^t = NV$$

Where : IV = Initial value, NV = New value,

t = time, r = rate of change

$$10,000 \times (1 - .40)^t = NV$$

$$10,000 \times (.60)^2 = 3600 \text{ ppm after 2 - years}$$

$$10,000 \times (.60)^4 = 1296 \text{ ppm after 4 - years}$$

$$10,000 \times (.60)^{10} = 60.5 \text{ ppm after 10 - years}$$

Where: IV = Initial value, NV = New value, t = time, r = rate of change

Example 2:
A particular brand of brake fluid contains 200 PPM of water when first produced. Brake fluid can absorb water at a rate of 12% a year. How much water will be absorbed by the brake fluid in 1, 3 and 7 years

NOTE: In this example r will be positive because the amount of water is increasing.

$$IV \times (1 + r)^t = NV$$

$$200 \times (1 + .12)^t = NV$$

$$200 \times 1.12^1 = 224 \text{ ppm after 1 year}$$

$$200 \times (1 + .12)^3 = 280 \text{ ppm after 3 years}$$

$$200 \times (1 + .12)^7 = 442 \text{ ppm after 7 years}$$

PA Core Standard: CC.2.2.HS.C.5

Description: Construct and compare linear, quadratic, and exponential models to solve problems.

Math Associated Vocabulary:
RATE, COMPOUNDED GROWTH, EXPONENTIAL GROWTH, INITIAL VALUE, *e*, CONSTANT, HALF-LIFE

Formulas and Procedures:
Imagine walking towards an open door, but reducing the size of each step 50%. Would you ever get completely past the door?

Even though the *rate* of change in step size is constant (-50%), the change in the size of each step is not constant. This is due to compounding (50% of 50% of 50% of…).

Exponential growth and decay formulas come in different forms, but they usually follow one of the following:

where **A**=Amount of material, at start **(A_0)** and time t **(A_t)**

$A_t = A_0(1+r)^n$	$A_t = A_0 e^{kt}$
Non-continuous compounding (growth at set intervals) **r** = growth rate (amount gained/lost per time) **n** = num. of growth periods	Continuous compounding (growing all the time) **e**: 2.7182818… (an irrational number like ***π***) **k** = growth constant **t** = elapsed time

r and **k** can be positive (if growing) or negative (if decaying)

Example:
The charge on a capacitor loses approximately 9% charge every second. If the capacitor initially starts with a 2 coulomb charge, estimate how much charge is remaining after 5 seconds?

A_0 = 2 coulombs starting amount
r = –9% = -0.09 growth rate

$$\mathbf{n} = \frac{5\text{ sec}}{1\text{ sec}} = 5 \text{ growth periods}$$

$$A_t = (2)(1 + -0.09)^5$$

$$A_t = (2)(0.91)^5$$
$$= (2)(0.624)$$
$$= 1.248 \text{ Coulomb}$$

The continuous growth formula often produces a more accurate result, but requires calculation of **k** = -.105

$$A_t = A_0 e^{kt} = 2e^{(-.105)(5)} = 2e^{-.525} = 1.18 \text{ Coulomb}$$

Instructor's Script – Comparing and Contrasting
The growth rate (**r**) is the fractional amount added or removed within a given time period. If the amount is increasing, **r** will be positive; if decreasing, **r** will be negative. Many times, the growth rate is given as a percentage and must be converted to a decimal: (70% = 70 ÷ 100 = 0.70).

For half-life problems, use a growth rate of -50% or -0.50 (1 + r = 1 - 0.50 = 0.50)
For doubling problems, use a growth rate of 100% or +1.00 (1 + r = 1 + 1 = 2)

The growth period (**n**) is calculated by dividing the elapsed time by the time indicated in the growth rate.

Example: If 100 bacteria double every 30 minutes, how many would you have in 135 minutes?

r = +100% per 30 minutes, so **n** = $\frac{135\text{ min.}}{30\text{ min.}}$ = 4.5 growth periods (bacteria will increase 100% 4.5 times)

For half-life problems, the growth rate time is the half-life, so $n = \frac{t}{t_{half}}$.

The exponential growth and decay formulas can be used in many different types of applications (finance, biology, radioactivity, etc.), but in the technical trades, persons will often encounter the formulas in problems about material or substance growth and decay. Since growth or decay of these materials is continuous, the continuous growth formula should be used when accuracy of the result is very important.

e, aka Euler's constant, is an irrational number, like π, that can be found on most scientific calculators.

Calculating **k** (the growth constant): When using the continuous growth formula, **k** must be calculated from the information given. To do so, the natural logarithm function **(ln)** is used (a calculator function): $k = \frac{\ln(1+r)}{t}$

Example: The half-life of radium is 1800 years (50% loss in 1800 years)

r = -0.50, t = 1800, $k = \frac{\ln(1-0.50)}{1800} = \frac{\ln(0.50)}{1800}$ = -0.000385

Common Mistakes Made By Students
- Not performing the order of operations correctly: Parentheses (1 + r), Exponent (^{n}), Multiplication (A_o).
- Setting **r** to the amount of material remaining after a change instead of the growth or decay change ("1 + r" is the amount remaining after growth).
- Setting the sign of r incorrectly ("1 + r" should be greater than 1.0 if growing and less than 1.0 if decaying).

CTE Instructor's Extended Discussion
Technical tasks are usually not presented using this model. Therefore, it is important that technical instructors demonstrate to students how these math concepts link to and are relevant in their technical training. Automotive Technology instructors should help their students become proficient in working with numbers in as many aspects of the trade as possible. The most proficient and successful technicians, engineers, and automotive repair/service business owners will be those who are comfortable working with and communicating using numbers and formulas.

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Problems	Career and Technical Math Concepts Solutions
1. The horsepower in a particular engine produces 105 Hp when cold. The Hp increases 5% every minute it is running. Estimate how much Hp the engine is producing after running 15 minutes and 25 minutes. **NOTE:** We **MUST ADD r** because the Hp is increasing.	
2. New gasoline contains 35,000 PPM of a particular anti-varnish agent. The agent breaks down 32% each year. How much of the agent remains after 1 year, after 6 years?	
3. A particular coolant solution contains 8 quarts of Ethylene Glycol (EC). EC has a half-life of 10 years in this solution. How much EC will be present after 4, 6 and 15 years? (**r** = -50%)	

Problems	Related, Generic Math Concepts Solutions
4. Jon has 15,600 cars in his salvage (junk) yard. The number of vehicles in the yard doubles every 10 years. How many cars will Jon have in 18 years, 24 years and 35 years? Keep in mind r = 100% and $n = \frac{\text{elapsed time}}{\text{growth time}}$	
5. A town with a population of 100,000 residents is growing at a rate of 4.75% per year. What will the estimated population be in 60 years? Use the non-continuous exponential growth formula: $NV = IV(1 \pm r)^t$	
6. An antibiotic has a half-life of 12 hours in the bloodstream (k = -.0578). A 10 mg. injection of the antibiotic is given at 1:00 p.m. How much remains in the blood at 9:00 p.m. (Use the continuous growth formula)?	

Problems	PA Core Math Look Solutions
7. Initially, there is 1250 ml. of solvent A in a solution and 20% breaks down every 6 months. Each year enough solvent is added to bring the level back to 1250 ml. How much will need to be added in one year?	
8. A bottle of spring water contains 3 ppm of particular bacteria. Once you open the bottle and place in the refrigerator, the bacteria will grow at a rate of 10% per day. How many bacteria will there be in 7 and 30 days?	
9. There is 100g of Isotope A with a half-life of 3 months (k = -.231). There is 500g of Isotope B with a half-life of 1 month (k = -.693). Will there still be more Isotope B in four months?	

Automotive Technology (47.0604) T-Chart

Problems	Occupational (Contextual) Math Concepts	Solutions
1. The horsepower in a particular engine produces 105 Hp when cold. The Hp increases 5% every minute it is running. Estimate how much Hp the engine is producing after running 15 minutes and 25 minutes. **NOTE:** We **MUST ADD r** because the Hp is increasing.		$IV \times (1+r)^t = NV$ or $105 \times (1+.05)^{15} = NV$ $105 \times 1.05^{15} = 218$ Hp after running 15 minutes $105 \times 1.05^{25} = 355$ Hp after running 25 minutes
2. New gasoline contains 35,000 PPM of a particular anti-varnish agent. The agent breaks down 32% each year. How much of the agent remains after 1 year, after 6 years?		IV = 35,000, r = -0.32, t = 1 and 6 $NV = 35000(1-0.32)^1 = 35000(.68) = 23800$ppm $NV = 35000(1-0.32)^6 = 35000(.68)^6 = 3460$ppm
3. A particular coolant solution contains 8 quarts of Ethylene Glycol (EC). EC has a half-life of 10 years in this solution. How much EC will be present after 4, 6 and 15 years? (**r** = -50%)		$NV = IV(1-0.5)^{(t/Thalf)}$ $NV = 8 \times (0.5)^{(4/10)}$ $NV = 6.06$gl $NV = 8 \times (0.5)^{(6/10)}$ $NV = 5.27$gl $NV = 8 \times (0.5)^{(15/10)}$ $NV = 2.82$gl
Problems	**Related, Generic Math Concepts**	**Solutions**
4. Jon has 15,600 cars in his salvage (junk) yard. The number of vehicles in the yard doubles every 10 years. How many cars will Jon have in 18 years, 24 years and 35 years? Keep in mind r = 100% and $n = \frac{\text{elapsed time}}{\text{growth time}}$		$NV = 15600(1+1)^n$ 18 YEARS $\rightarrow 15,600 \times 2^{1.8} = 54,322$ 24 YEARS $\rightarrow 15,600 \times 2^{2.4} = 82,337$ 35 YEARS $\rightarrow 15,600 \times 2^{3.5} = 176,494$
5. A town with a population of 100,000 residents is growing at a rate of 4.75% per year. What will the estimated population be in 60 years? Use the non-continuous exponential growth formula: $NV = IV(1 \pm r)^t$		IV = 100,000, r = +0.0475, t = 60 $NV = 100000(1+0.0475)^{60}$ $NV = 100000(1.0475)^{60} = 1,460,248$
6. An antibiotic has a half-life of 12 hours in the bloodstream (k = -.0578). A 10 mg. injection of the antibiotic is given at 1:00 p.m. How much remains in the blood at 9:00 p.m. (Use the continuous growth formula)?		t = 8 hours, so $A_8 = 10e^{(-.0578)(8)} = 6.3$mg
Problems	**PA Core Math Look**	**Solutions**
7. Initially, there is 1250 ml. of solvent A in a solution and 20% breaks down every 6 months. Each year enough solvent is added to bring the level back to 1250 ml. How much will need to be added in one year?		IV = 1250, r = -0.20, $n = \frac{12 \text{ mo.}}{6 \text{ mo.}} = 2$ $NV = 1250(1-0.20)^2 = 800$ ml. remaining Solvent to add = 1250 ml. – 800 ml. = 450 ml.
8. A bottle of spring water contains 3 ppm of particular bacteria. Once you open the bottle and place in the refrigerator, the bacteria will grow at a rate of 10% per day. How many bacteria will there be in 7 and 30 days?		$IV \times (1+r)^t$ 7 days $\rightarrow 3\text{ ppm} \times (1+0.10)^7 = 5.85$ ppm 30 days $\rightarrow 3\text{ ppm} \times (1+0.10)^{30} = 52.35$ ppm
9. There is 100g of Isotope A with a half-life of 3 months (k = -.231). There is 500g of Isotope B with a half-life of 1 month (k = -.693). Will there still be more Isotope B in four months?		$A_t = A_0 e^{kt}$ Isotope A: $A_4 = 100e^{(-.231)(4)} = 39.7$g Isotope B: $A_4 = 500e^{(-.693)(4)} = 31.3$g No, there is more A