A-F.1.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

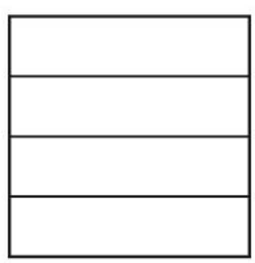
Solve addition and subtraction problems involving fractions (straight computation or word problems).

▶ Eligible Content

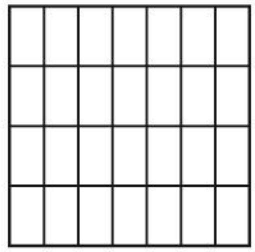
A-F.1.1.1 Add and subtract fractions (including mixed numbers) with unlike denominators. (May include multiple methods and representations.)

▶ Instructional Supports

- Give students two fractions with unlike denominators and ask them to use a rectangular model to create equivalent fractions with like denominators. For example, give students the fractions $\frac{1}{4}$ and $\frac{4}{7}$. Ask students to partition a rectangle with horizontal lines to represent the denominator of $\frac{1}{4}$ and to partition the same rectangle with vertical lines to represent the denominator of $\frac{4}{7}$. The total number of smaller rectangles created represents a common denominator. In this case, students partition a rectangle into fourths using horizontal lines.

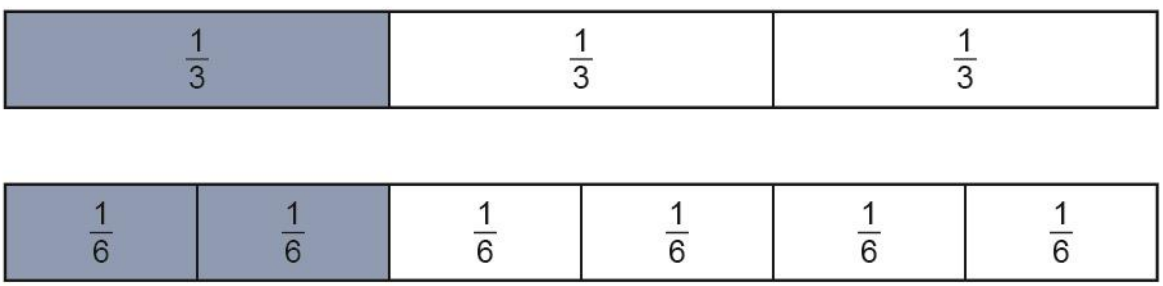


Students then partition the rectangle into sevenths using vertical lines.



The total number of small rectangles, 28, can be used as a common denominator for the fractions $\frac{1}{4}$ and $\frac{4}{7}$.

- Ask students to add or subtract fractions in which one denominator is a multiple of the other denominator. For example, ask students to determine the sum $\frac{5}{6} + \frac{1}{3}$. The number 6 is a multiple of 3, and it can be used as a common denominator for both fractions. Students can use fraction strips to model that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$.



Students use this fact to write the sum as $\frac{5}{6} + \frac{2}{6}$. Students add and determine a sum of $\frac{7}{6}$, or $1\frac{1}{6}$.

- Ask students to determine the sum or difference of fractions in which neither denominator is a multiple of the other. For example, give students the expression $\frac{1}{3} + \frac{1}{2}$. The numbers 3 and 2 are not multiples of each other, so equivalent fractions need to be determined for each fraction. Ask students to create fraction models to represent each fraction and determine an appropriate common denominator. Students should determine that 6 is an appropriate common denominator, rewrite the expression as $\frac{2}{6} + \frac{3}{6}$, and determine a sum of $\frac{5}{6}$.

- Ask students to determine a common denominator for two given fractions by making a partial list of the multiples of each denominator. For example, give students the expression $\frac{5}{6} - \frac{4}{9}$. Ask students to start a list of the multiples of 6 (6, 12, 18, 24, 30, 36, . . .) and the multiples of 9 (9, 18, 27, 36, 45, 54, . . .). The common multiples listed are 18 and 36. Discuss with students that any number that is in both lists works as a common denominator, but the least common multiple is the most convenient to use. Using the least common multiple, the problem can be rewritten as $\frac{15}{18} - \frac{8}{18} = \frac{7}{18}$.
- Ask students to determine a missing value in a partially solved problem involving the sum or difference of two fractions with unlike denominators. For example, given the problem $\frac{4}{11} - \frac{1}{7} = \frac{28 - 11}{?} = \frac{17}{?}$, students determine that each question mark represents 77.

► Identifying and Addressing Common Misconceptions

- Some students may not create a common denominator before adding or subtracting fractions. Ask students to add the fractions $\frac{3}{4}$ and $\frac{2}{3}$. Some students may give a sum of $\frac{5}{7}$. Ask students to determine whether the sum is greater than or less than 1 by comparing each fraction to $\frac{1}{2}$. Since each fraction is greater than $\frac{1}{2}$, the sum should be greater than 1; however, $\frac{5}{7}$ is less than 1. Remind students that adding the numerators of fractions is meaningful only when the denominators are the same. Ask students to create fraction models representing equivalent fractions for $\frac{3}{4}$ and $\frac{2}{3}$.
- Some students may think that multiplying the denominators is the only way to determine a common denominator. Ask students to add $\frac{5}{12}$ and $\frac{7}{18}$. Some students may give a sum of $\frac{174}{216}$. Ask students to write multiples of 12 and 18 to determine the first multiple these two numbers have in common. The first, or lowest, common multiple is 36. Help students recognize that while multiplying two unlike denominators will always result in a common denominator, it may also result in fractions with very large numerators and denominators. Ask them to add the original fractions using 36 as a common denominator and compare their result to the sum they determined using 216 as a common denominator.

- Some students may adjust only the denominator when writing equivalent fractions. Ask students to add $\frac{3}{4}$ and $\frac{1}{3}$. Some students may add $\frac{3}{12}$ and $\frac{1}{12}$ to get a sum of $\frac{4}{12}$. Ask students to create fraction bars with the common denominator of 12 and use them to compare $\frac{3}{12}$ and $\frac{3}{4}$. Help students recognize that the fraction bars do not represent the same area. Then ask students to compare $\frac{3}{4}$ to $\frac{9}{12}$ to see that they do represent the same area.

► Enrichment

- Ask students to determine fractions with different denominators that have a given sum or difference. For example, ask students to determine numbers to complete the number sentence $\frac{x}{7} - \frac{y}{5} = \frac{4}{35}$.
- Randomly select four numbers and ask students to place those numbers into the sum $\frac{a}{b} + \frac{c}{d}$ in a way that creates the greatest sum possible.
- Ask students to add three fractions and then compare the strengths and weaknesses of the following strategies:
  - determining a common denominator for all three fractions before adding
  - calculating a common denominator in order to add the first two fractions and then calculating a common denominator between the resulting sum and the third fraction

► Key Terms

addition, denominator, equation, equivalent fractions, fraction, mixed number, numerator, subtraction, visual model, common denominator

A-F.2.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

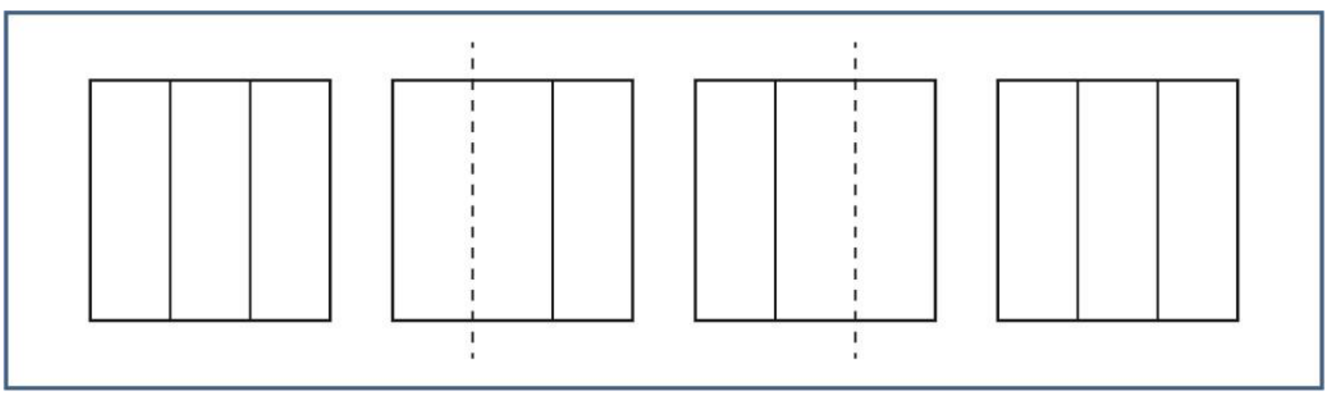
Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems).

► Eligible Content

A-F.2.1.1 Solve word problems involving division of whole numbers leading to answers in the form of fractions (including mixed numbers).

► Instructional Supports

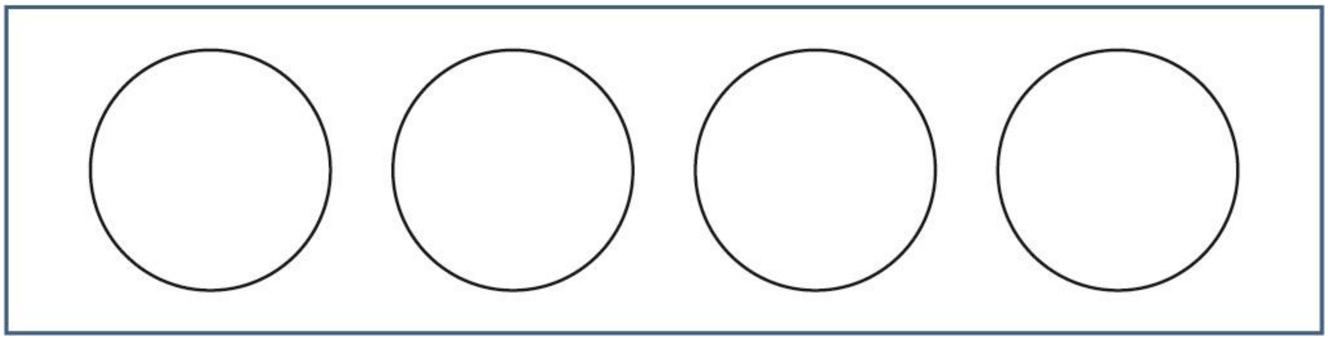
- Give students a visual model and ask them to interpret the model as a fraction and as a division statement in the form *a* ÷ *b*. For example, give students the following diagram:



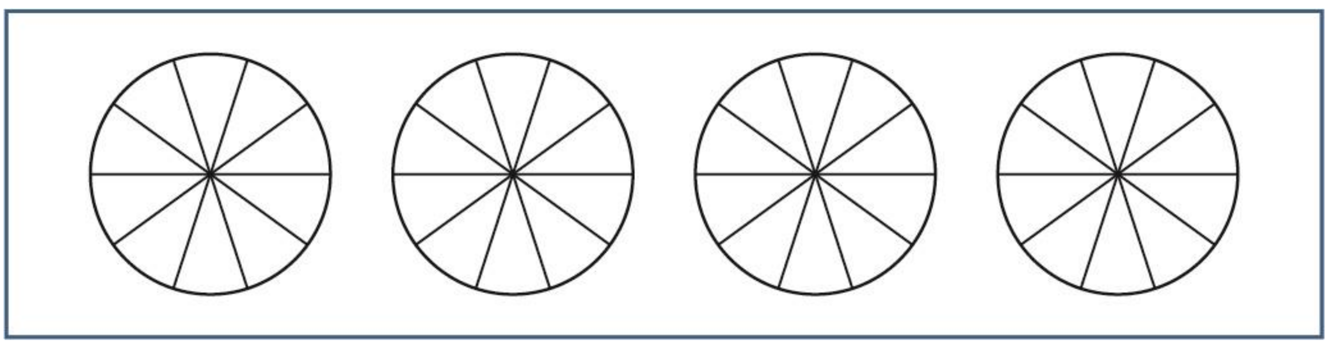
The diagram shows 4 large rectangles, or 4 “wholes.” The dashed lines show the 4 wholes being divided into 3 equal groups, which corresponds to the division statement
$4 \div 3$. Thinking fractionally, the dashed lines divide the 4 wholes into 3 equal groups, wherein each equal group consists of 4 parts of size $\frac{1}{3}$, or $\frac{4}{3}$. When 4 wholes are divided into 3 equal groups ($4 \div 3$), each group has size $\frac{4}{3}$, which implies $4 \div 3 = \frac{4}{3}$.

- Ask students to represent a given fraction using expressions involving addition of unit fractions, multiplication, and division. For example, students can represent $\frac{5}{9}$ as $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$, $5 \times \frac{1}{9}$, or $5 \div 9$.

- Ask students to solve word problems involving the division of whole numbers using a model, and ask students to verify answers using repeated addition or multiplication. For example, give students this problem: "A total of 4 whole pizzas are shared equally among 10 people. What fraction of a pizza does each person receive?" Students may start by creating a model of 4 circles, each representing 1 whole (pizza).

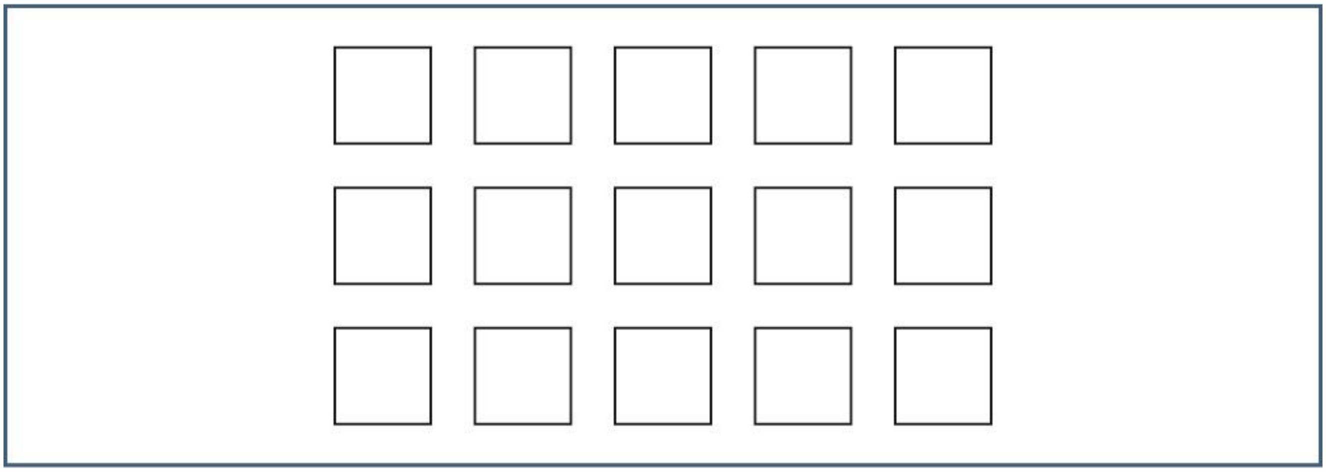


Students then divide each circle into 10 pieces to show the amount of a pizza each person receives.

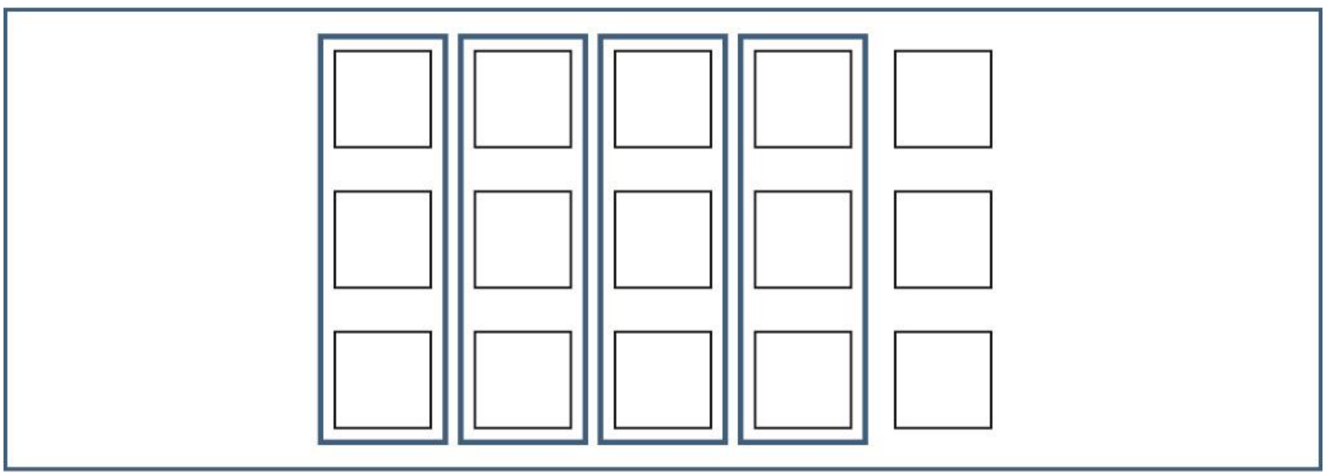


If each person receives 1 section from each pizza, then each person receives $\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}$, which is a total of $\frac{4}{10}$ of a pizza. Thus, $4 \div 10$ is equal to $\frac{4}{10}$ of a pizza. This can be verified by adding the amount each person receives together to ensure the total is 4 whole pizzas ($\frac{4}{10}+\frac{4}{10}+\frac{4}{10}+\frac{4}{10}+\frac{4}{10}+\frac{4}{10}+\frac{4}{10}+\frac{4}{10}+\frac{4}{10}+\frac{4}{10}=\frac{40}{10}=4$).

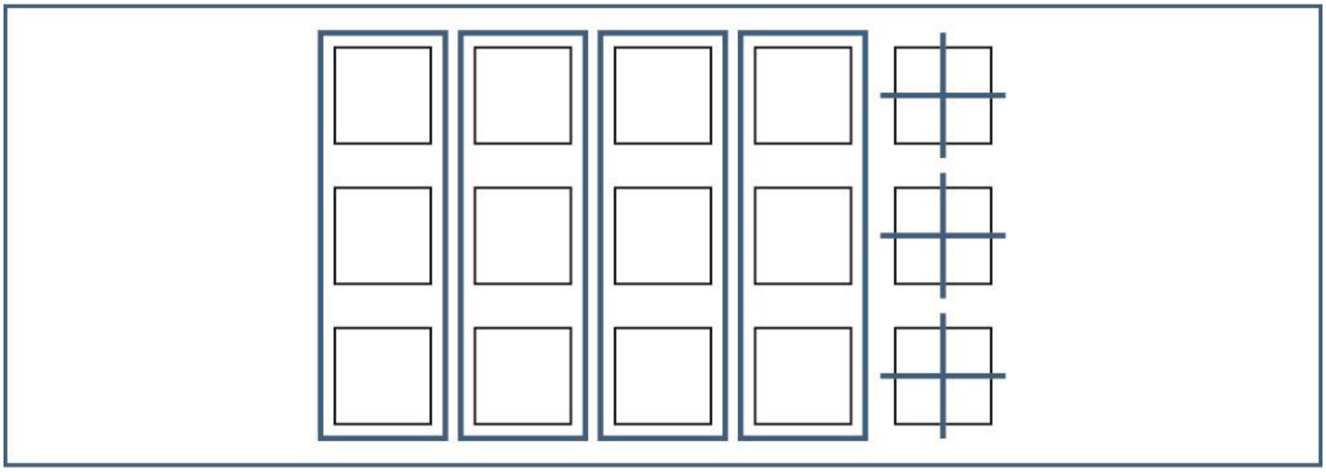
- Ask students to solve word problems involving a quotient that is greater than 1 and represent the word problem with a drawing. For example, give students this problem: "A baker uses a total of 15 cups of flour to make 4 identical loaves of bread. How many cups of flour are used for each loaf of bread?" Students can represent 15 cups of flour with 15 squares.



Then, students separate as many full squares as possible into 4 equal groups.

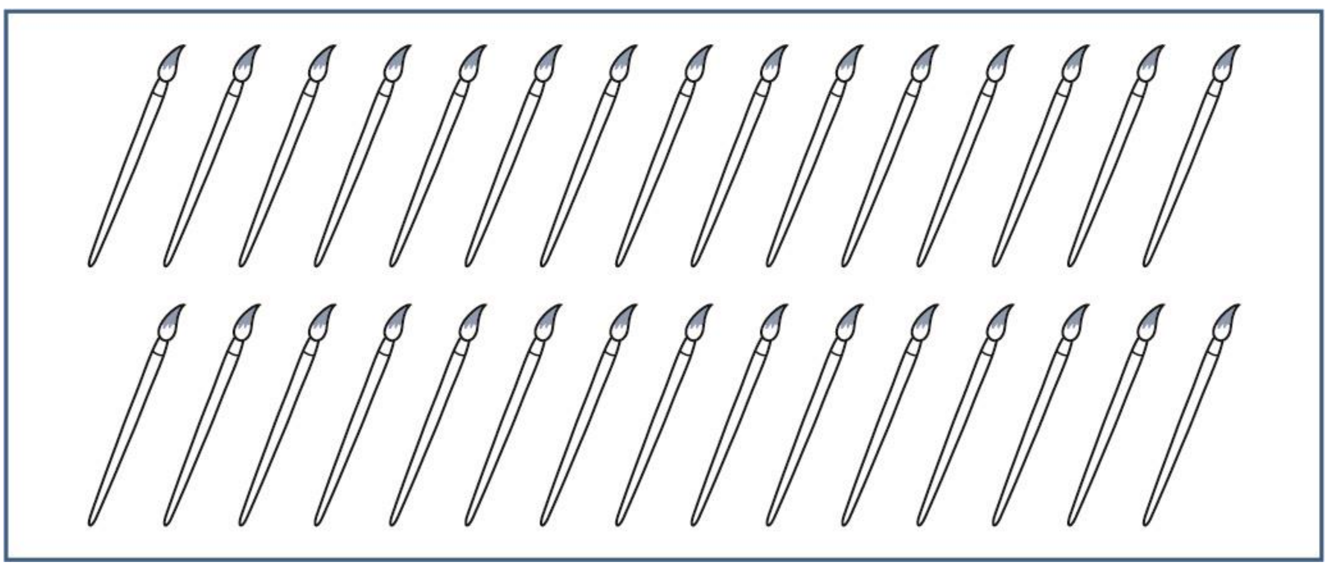


Then, students divide the remaining 3 squares each into 4 equal pieces. Each of the 4 groups receives a piece from each of the 3 remaining squares, and $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$.

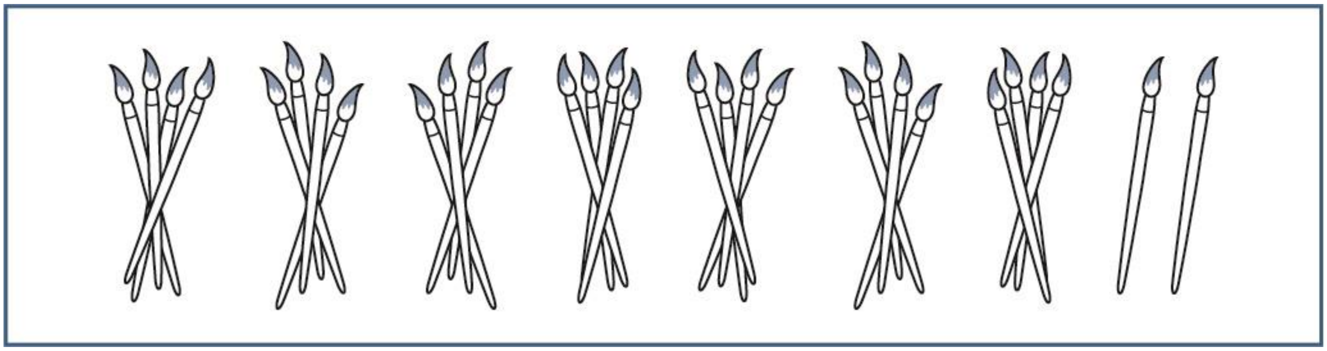


Therefore, the baker uses $\frac{15}{4}$, or $3\frac{3}{4}$, cups of flour for each loaf of bread.

- Ask students to solve problems involving the quotient of two whole numbers where the remainder must be explained. For example, give students this problem: "Mr. Martinez has 30 paintbrushes. He places the same number of paintbrushes on each of 7 tables. How many paintbrushes are on each table?" Students should recognize that the word problem is represented by the quotient 30 ÷ 7, or by the fraction $\frac{30}{7}$. Students then solve the problem by drawing an array that represents 30 paintbrushes.

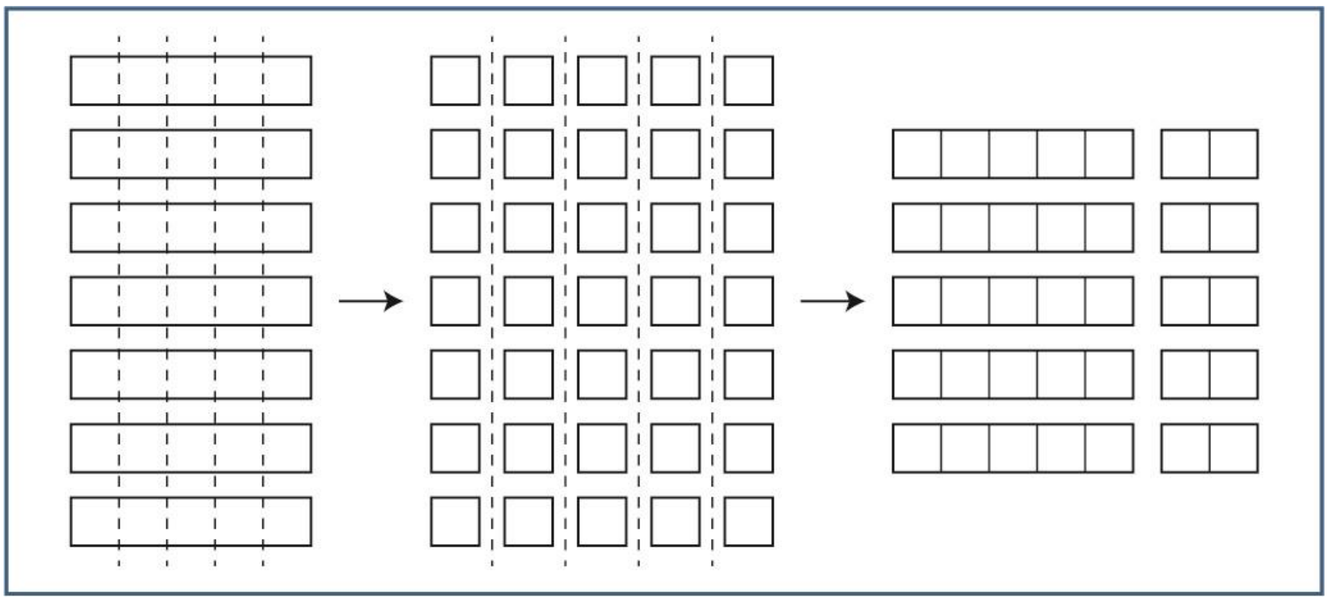


Next, students divide the paintbrushes into 7 equal groups of 4 paintbrushes.



In this context, the remaining 2 paintbrushes divided among 7 tables would be $\frac{2}{7}$. Since paintbrushes cannot be divided, instead of each table receiving $4\frac{2}{7}$ paintbrushes, each table receives 4 paintbrushes, and Mr. Martinez has 2 paintbrushes remaining.

- Ask students to interpret a given model involving the division of two whole numbers and create a corresponding word problem. For example, give students the following model.



Students observe that there are 7 "wholes" represented by the 7 rectangles on the left. Each rectangle is divided into 5 equal-sized squares, so each square has size $\frac{1}{5}$. The middle part of the model shows all the small squares divided into 5 groups each containing 7 squares of size $\frac{1}{5}$. The right part of the model reorganizes the 5 equal groups to show that each group contains 1 whole and $\frac{2}{5}$. A possible corresponding word problem is the following: "Alvin walks 7 miles in 5 days, walking the same number of miles each day. How many miles does Alvin walk each day?" The diagram shows that Alvin walks $1\frac{2}{5}$ miles each day.

► Identifying and Addressing Common Misconceptions

- Some students may switch the meaning of the numerator and denominator when writing a division problem from a fraction. Give students the fraction $\frac{8}{9}$ and ask them to represent it as a division expression. Some students may write the division expression $9 \div 8$. Remind students that the numerator represents the dividend while the denominator represents the divisor. Then ask students to create a visual model to represent $\frac{8}{9}$. A possible model shows 8 shaded sections of a fraction bar that is divided into 9 equal slices. Ask students to compare this model to a model of 8 wholes, each divided into 9 equal parts, and to a model of 9 wholes, each divided into 8 equal parts.
- Some students may misunderstand which piece of information represents the numerator and which represents the denominator when solving a word problem. Ask students to write a fraction and a division statement for this problem: "Three friends share 4 cups of popcorn. How much popcorn does each friend receive?" Some students may respond with $\frac{3}{4}$ and $3 \div 4$. Ask students to consider what is being shared: 3 friends or 4 cups of popcorn? Since it is the 4 cups of popcorn being shared, that value is represented by the numerator. Each friend receives $\frac{1}{3}$ of each cup of popcorn as there are 3 people sharing the popcorn, so 3 represents the denominator. Thus, the appropriate fraction and equivalent division expression are $\frac{4}{3}$ and $4 \div 3$.
- Some students may not consider whether an answer makes sense as a fraction. Give students this problem: "Matthew has 20 markers. He makes 3 equal piles of the markers. How many markers are in each pile?" Some students may give the answer of $6\frac{2}{3}$. Ask students what their answer represents and then ask them whether having $6\frac{2}{3}$ markers in each pile makes sense. Since a marker cannot be divided to get $\frac{2}{3}$ of a marker, each pile has 6 markers and there are 2 remaining markers.

► Enrichment

- Ask students to compare the difference in meaning between $\frac{a}{b}$ and $\frac{b}{a}$ by creating models of each and a situation to represent each.
- Ask students to solve problems involving the division of large whole numbers that would be harder to model. For example, "A school has 125 students who are divided into 4 equal-sized groups. How many students are in each group?"
- Ask students to write a list of situations where the remainder can be represented by a fraction and a list of situations where the remainder cannot be represented by a fraction.

► Key Terms

denominator, dividend, division, divisor, fraction, numerator, remainder

A-F.2.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems).

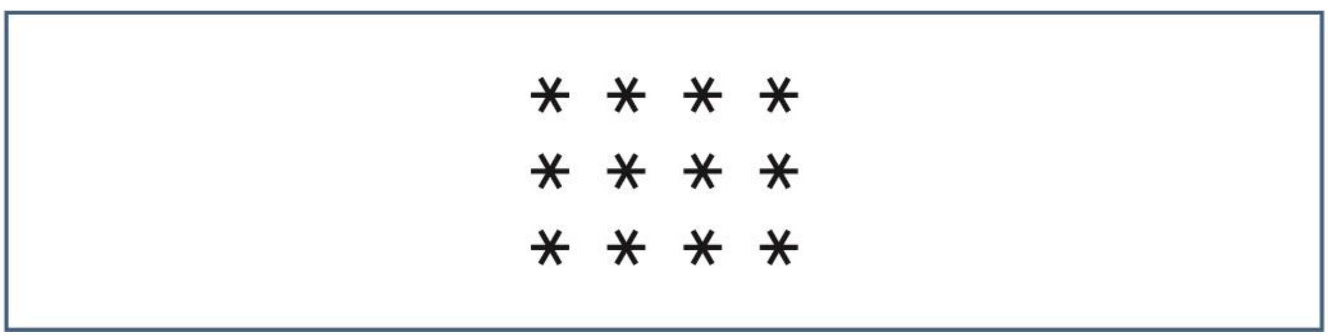
► Eligible Content

A-F.2.1.2 Multiply a fraction (including mixed numbers) by a fraction.

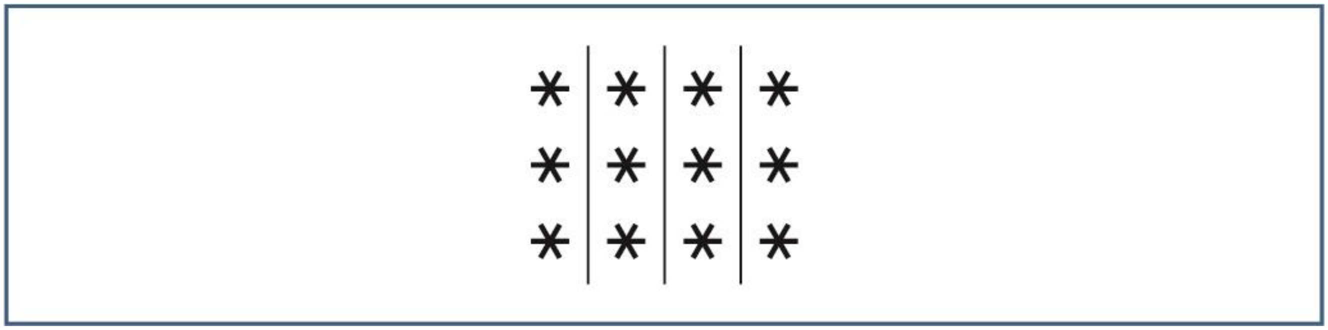
► Instructional Supports

Models for multiplying fractions

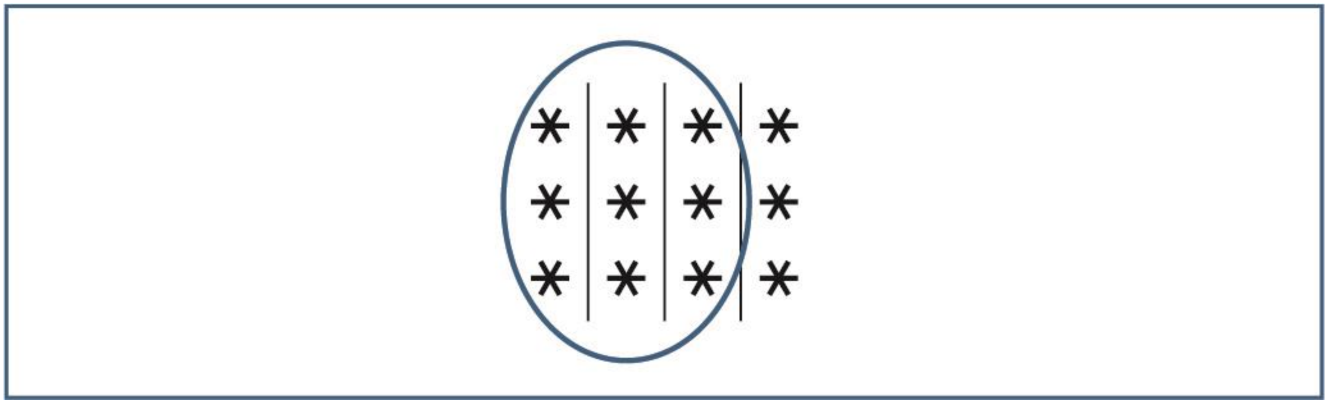
- Ask students to create a visual model of multiplication of a number by a fraction in the form $\frac{a}{b} \times q$. Some examples are shown.
    - Give students the multiplication expression $\frac{3}{4} \times 12$. First, students represent $q = 12$ as an array.



Then, students partition the array into $b = 4$ equal parts.

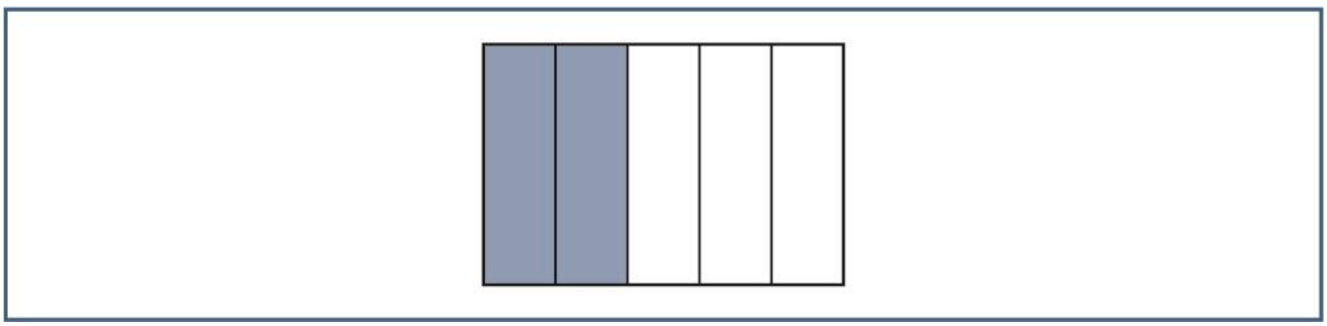


Finally, students circle $a = 3$ of the partitions.

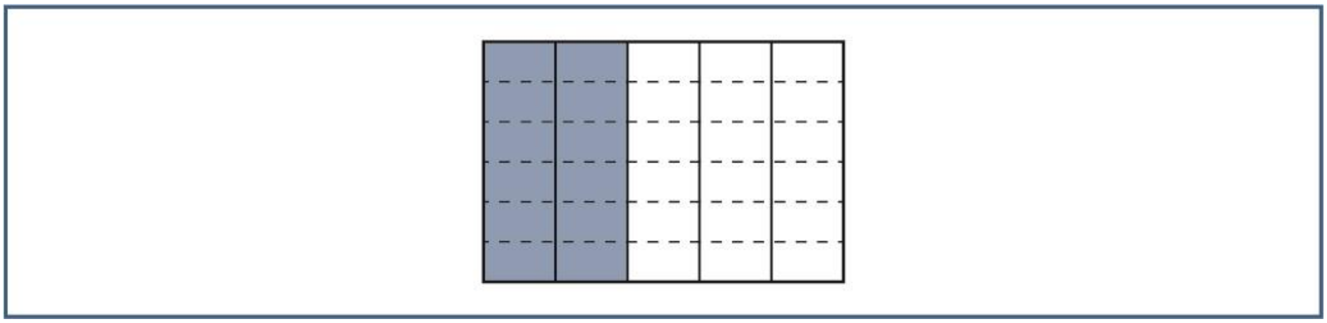


The circled portion of the array is the product of the expression (9). Discuss with students that 9 is $\frac{3}{4}$ of 12 and how that relationship is illustrated by the array.

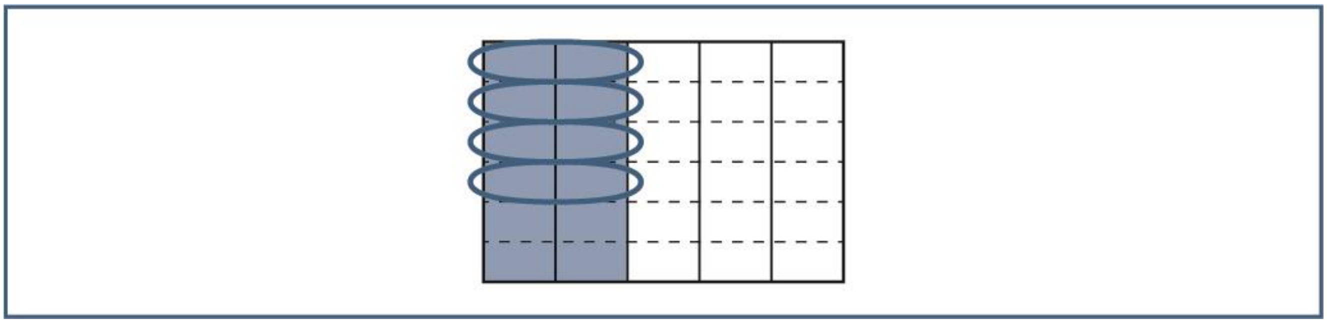
- Give students the multiplication expression $\frac{4}{6} \times \frac{2}{5}$. In this case, $a = 4$, $b = 6$, and $q = \frac{2}{5}$. First, students represent q using a visual model.



Then, students partition it into b equal parts. Note there are now 30 equal parts in the whole.



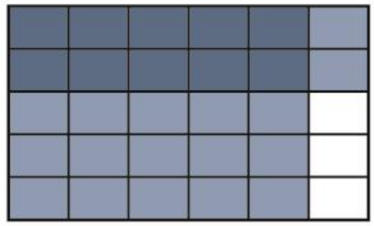
Finally, students circle *a* of the parts.



The circled portion of the fraction model represents a product of $\frac{8}{30}$.

Students should eventually notice that the product of two fractions can be determined by multiplying the numerators and the denominators separately.

- Ask students to multiply whole numbers and fractions by rewriting equivalent multiplication problems with only fractions. For example, provide students with expressions such as 4×9 and $\frac{1}{4} \times 8$. Students can rewrite them as $\frac{4}{1} \times \frac{9}{1}$ and $\frac{1}{4} \times \frac{8}{1}$. The products are $\frac{36}{1}$ and $\frac{8}{4}$.
- Ask students to solve real-world problems involving multiplication of fractions. For example, give students this problem: “Zachary has $\frac{1}{2}$ bag of flour. He uses $\frac{1}{3}$ of the flour to bake. What fraction of the whole bag of flour does Zachary use to bake?” Students determine the amount by multiplying $\frac{1}{2}$ by $\frac{1}{3}$ to get $\frac{1}{6}$. Zachary uses $\frac{1}{6}$ of the whole bag of flour.
- Ask students to determine a multiplication problem from a given model. For example, give students the multiplication model shown.



Students should interpret the model as having 6 vertical partitions, of which 5 are shaded, and 5 horizontal partitions, of which 2 are shaded. The overlap of the two shaded areas contains 10 sections out of a total of 30 sections. Therefore, the model may be represented by the equation $\frac{5}{6} \times \frac{2}{5} = \frac{10}{30}$.

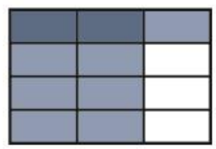
Real-world problems with fractions

- Ask students to use a visual model to solve real-world problems involving multiplication of fractions. For example, give students this problem: “Amelie painted $\frac{3}{4}$ of a wall. The following day, she repainted $\frac{1}{2}$ of what she had already painted. What fraction of the wall did Amelie paint twice?” Students use a model to show the painting Amelie did in the first day. The model is divided into 4 vertical partitions with 3 of them shaded.

Then the model is divided into 2 horizontal partitions with 1 of them shaded.

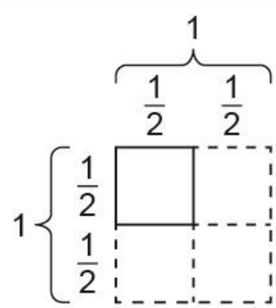
The area shaded twice represents the product of $\frac{3}{4} \times \frac{1}{2}$ and shows that Amelie painted $\frac{3}{8}$ of the wall twice.

- Give students a product of fractions and ask them to create and solve a word problem for the product. For example, give students the fractions $\frac{1}{4}$ and $\frac{9}{10}$ with which to create a problem. A possible student response is given: "In Mrs. Lewis's class, $\frac{9}{10}$ of the students watch TV. Of those students, $\frac{1}{4}$ of them watch TV every day. What fraction of Mrs. Lewis's class watches TV every day?" Students provide the solution of $\frac{1}{4} \times \frac{9}{10} = \frac{9}{40}$.
- Give students a multiplication model and ask them to create a word problem. For example, provide the following model.



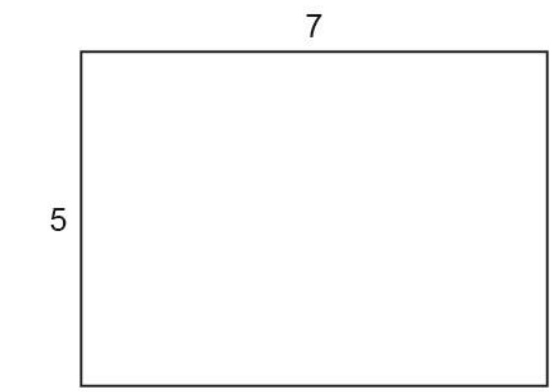
A possible student response is shown: "Malik mowed $\frac{2}{3}$ of his lawn. His brother put fertilizer on $\frac{1}{4}$ of the mowed lawn. What fraction of the lawn was mowed and fertilized?" Students use the expression $\frac{2}{3} \times \frac{1}{4}$ to determine that $\frac{2}{12}$ of the lawn was mowed and fertilized.

- Ask students to determine the area of a square with unit fraction side lengths. For example, ask students to find the area of a square with side lengths of $\frac{1}{2}$. Students can arrange four such squares into a unit square as shown.

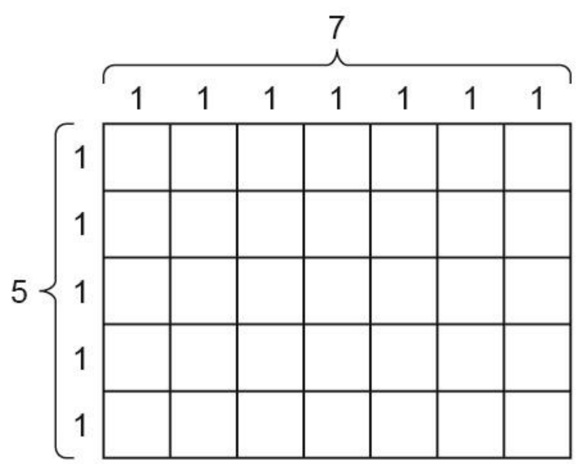


Because the area of each small square is $\frac{1}{4}$ the area of a unit square, which has an area of 1 square unit, the area of each small square is $\frac{1}{4}$ square unit. In general, given a square with a side length of $\frac{1}{n}$, $n \times n$ of these squares can be arranged to form a unit square. Therefore, a square with a side length of $\frac{1}{n}$ unit has an area of $\frac{1}{n} \times \frac{1}{n}$ square unit.

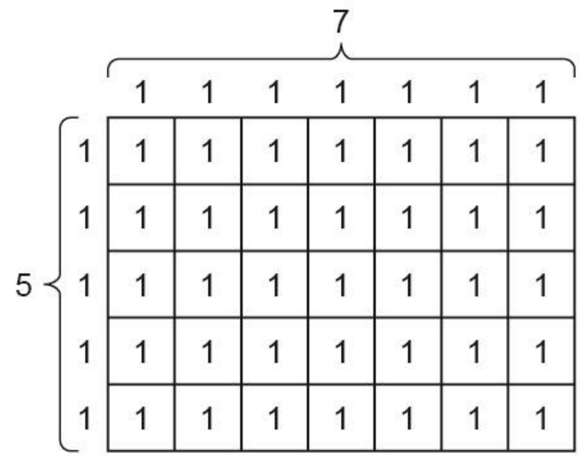
- Ask students to use the method of tiling squares to solve real-world or mathematical problems involving determining the area of rectangles with two whole-number side lengths. For example, ask students to determine the area of a rectangle with side lengths of 7 and 5.



Students divide the rectangle into unit squares by creating vertical and horizontal partitions.

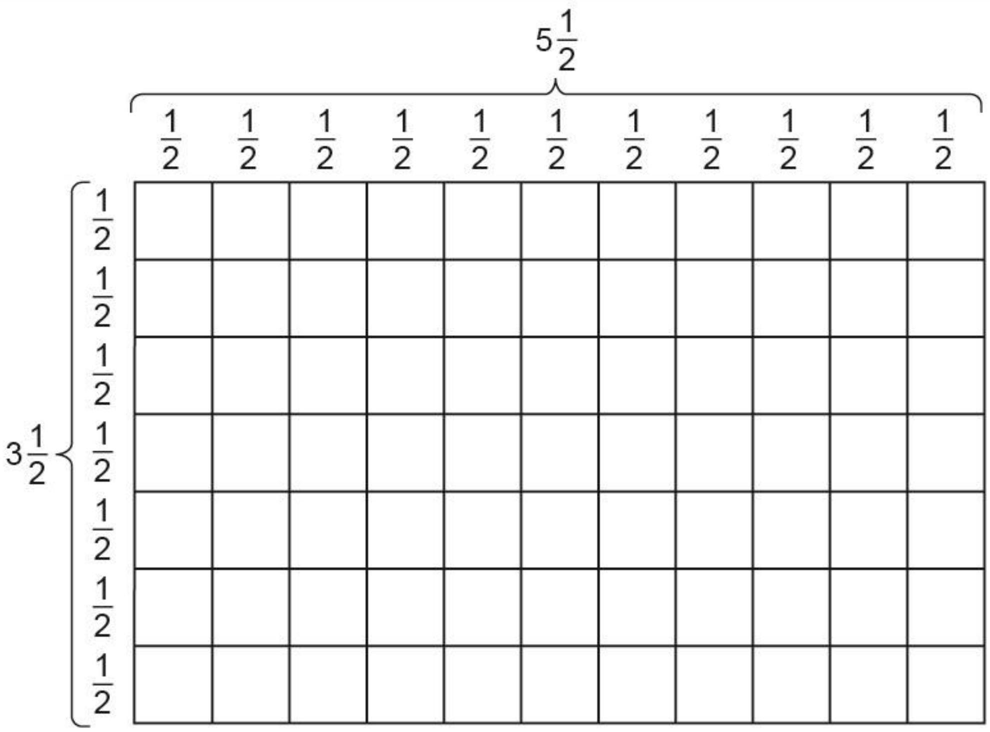


Each unit square measures 1 unit by 1 unit and has an area of 1 square unit (1 × 1 = 1).



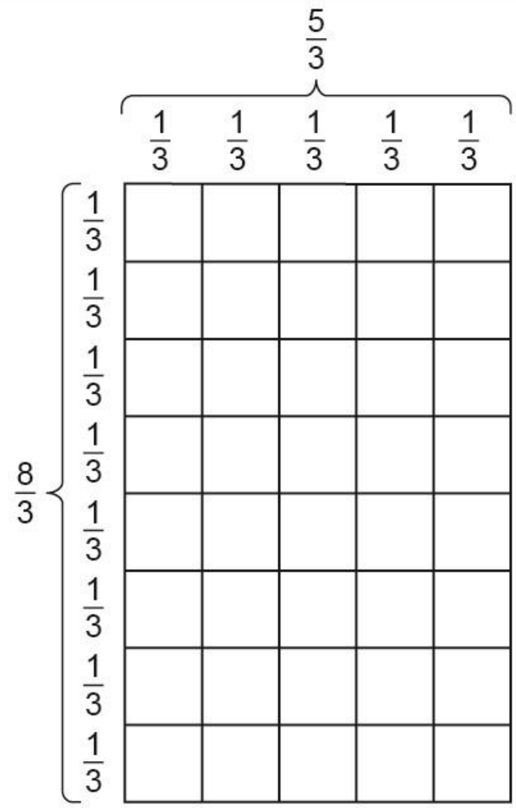
There are 35 unit squares. Students should recognize that this result can also be determined by solving 5 × 7 to get 35.

- Ask students to find the area of a rectangle with fractional side lengths by tiling it with smaller squares. For example, ask students to calculate the area a rectangular photograph with dimensions $5\frac{1}{2}$ inches by $3\frac{1}{2}$ inches. Students can calculate the area by tiling it with $\frac{1}{2}$-inch by $\frac{1}{2}$-inch squares as shown.



There are 11 squares in each row and 7 squares in each column for a total of 77 squares (because $7 \times 11 = 77$). Each square has an area of $\frac{1}{4}$ square inch, and the total area can be found by solving $77 \times \frac{1}{4}$ to get $\frac{77}{4}$ or $19\frac{1}{4}$ square inches.

- Ask students to use square tiles to explain why the area of a rectangle with fractional side lengths can be found by multiplying the dimensions the same way students multiply the dimensions when the side lengths are whole numbers. For example, give students a rectangle with a base of $\frac{5}{3}$ centimeters and a height of $\frac{8}{3}$ centimeters.

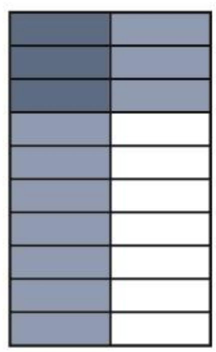


Students can tile the rectangle with squares that are $\frac{1}{3}$ inch on each side. Because $\frac{5}{3} = 5 \times \frac{1}{3}$, 5 small squares fit along the base of the rectangle. Similarly, 8 small squares fit along the height of the rectangle. There are 8×5 total small squares, each with an area of $\frac{1}{3} \times \frac{1}{3}$. Therefore, the total area is $8 \times 5 \times \frac{1}{3} \times \frac{1}{3}$, which is the same as the product of the original dimensions ($\frac{5}{3} \times \frac{8}{3}$).

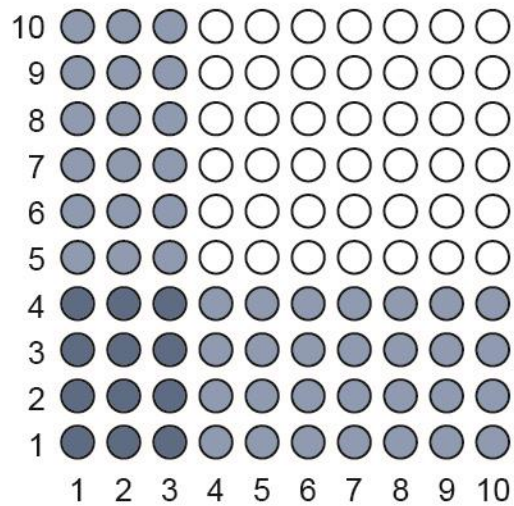
- Ask students to use the area formula $A = l \times w$ and the general rule $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ to solve real-world or mathematical problems that involve determining the areas of rectangles with fractional side lengths. For example, give students this problem: “Juliana plants a rectangular garden that is $7\frac{1}{3}$ feet long by $3\frac{1}{4}$ feet wide. What is the area, in square feet, of Juliana’s garden?” Students multiply $7\frac{1}{3}$ by $3\frac{1}{4}$, which is equivalent to multiplying $\frac{22}{3}$ by $\frac{13}{4}$, to determine an area of $\frac{286}{12}$ or $23\frac{10}{12}$ square feet.

► Identifying and Addressing Common Misconceptions

- Some students may not recognize that using the word "of" is a helpful substitute for "times" in multiplication, particularly when multiplying fractions. Ask students to calculate $\frac{4}{5}$ of $\frac{1}{2}$. Some students may not know to multiply. Help students recognize that the term "of" indicates multiplication by using concrete examples such as this one: "$\frac{1}{2}$ of a class with 30 students sings in the choir. How many students sing in the choir?" Help students understand that their answer is equivalent to multiplying 30 by $\frac{1}{2}$ and that the word "of" is the key word in the problem. Therefore, $\frac{4}{5}$ of $\frac{1}{2}$ can also be calculated by solving $\frac{4}{5} \times \frac{1}{2}$ to get $\frac{4}{10}$.
- Students may confuse which parts of visual models indicate the product of fraction multiplication. Give students the following model and ask them to interpret the model as a multiplication of two fractions.

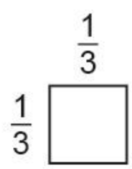


Some students may think that either the entire shaded region or the once-shaded regions represent the product. Ask students to consider array models for multiplying whole numbers. For example, 3 × 4, or 3 groups of size 4, can be modeled by the following array.

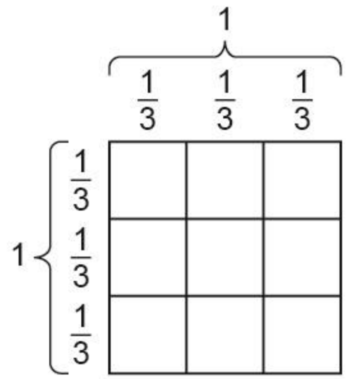


The array model has 3 shaded columns to represent 3 groups and 4 shaded rows to represent the size of each group. The product of 3×4 is twice-shaded; likewise, with fraction models, it is the twice-shaded portion that shows the product.

- Some students may think that multiplying fractional side lengths should result in an area that is greater than the measures of the individual sides. Ask students whether the area of the square shown is greater than $\frac{1}{3}$ square unit or less than $\frac{1}{3}$ square unit.



Some students may think the area is greater than $\frac{1}{3}$ square unit. Ask students to calculate the area of the square by creating the following diagram.



Since it takes 9 of the original squares to make up 1 unit square, the area of the original square is $\frac{1}{9}$ square unit, which is less than $\frac{1}{3}$ square unit.

► Enrichment

- Ask students to continuously divide a fraction into smaller parts and describe the process. For example, $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$, $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$, $\frac{1}{2}$ of $\frac{1}{8}$ is $\frac{1}{16}$, etc. Students may make connections between dividing by 2 and multiplying by $\frac{1}{2}$.
- Ask students to reason through multiplication of three or more fractions and brainstorm how to show it using a visual model. For example, give students the problem $\frac{9}{10} \times \frac{1}{8} \times \frac{5}{12}$. Students may reason that the product is less than any of the factors and that a three-dimensional model could be used to visualize the product similar to how a two-dimensional model is used to multiply two fractions.
- Ask students to use their experience with the areas of figures with fractional side lengths to develop hypotheses about calculating the volume of a rectangular prism with fractional side lengths.

► Key Terms

denominator, division, equation, expression, fraction, fraction model, multiplication, numerator, product, whole number, area, decomposing, length, rectangle, width

A-F.2.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

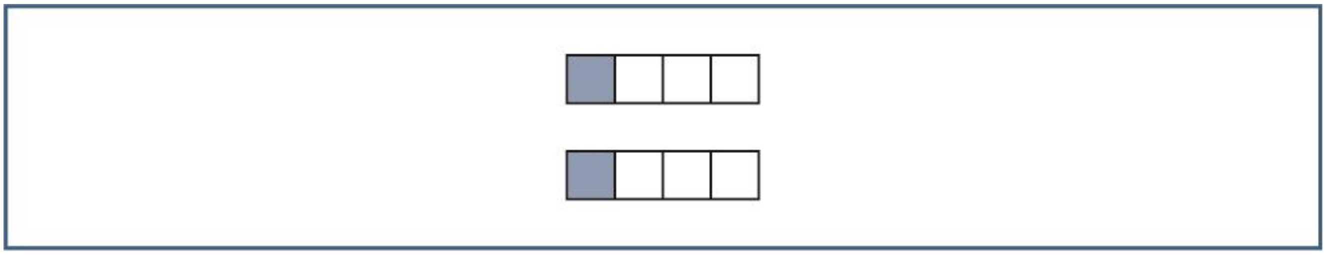
Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems).

► Eligible Content

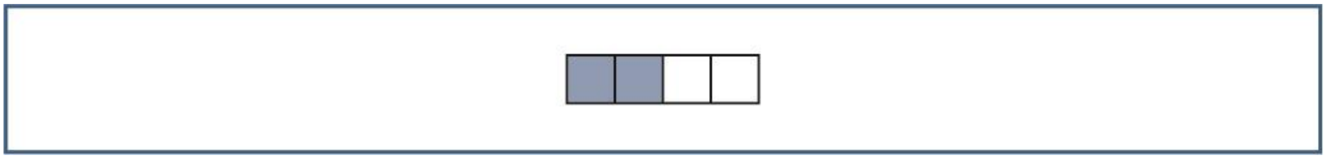
A-F.2.1.3 Demonstrate an understanding of multiplication as scaling (resizing).

► Instructional Supports

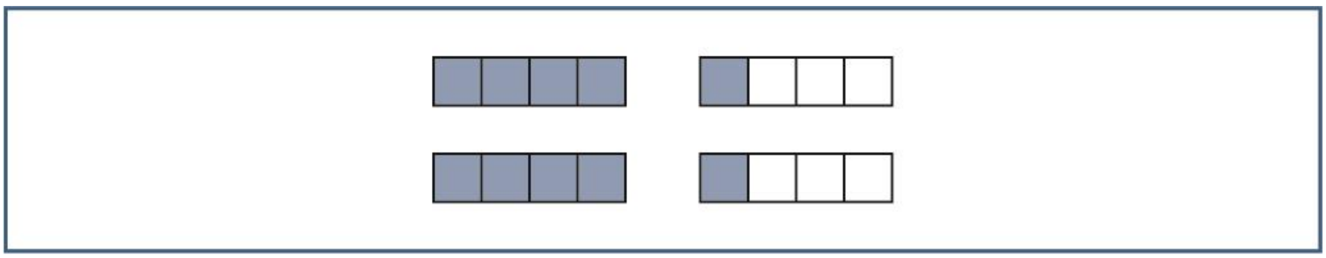
- Ask students to write comparative statements to compare the size of a product with the size of one of the factors in a multiplication equation. For example, given the equation $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$, students can write comparative statements such as "$\frac{1}{12}$ is $\frac{1}{3}$ of $\frac{1}{4}$" and "$\frac{1}{12}$ is $\frac{1}{4}$ of $\frac{1}{3}$."
- Ask students to write multiplication equations that match comparative statements. For example, given the comparative statement "8 is $\frac{1}{2}$ of 16," students write equations such as $8 = 16 \times \frac{1}{2}$ or $\frac{1}{2} \times 16 = 8$.
- Ask students to use models to compare the effect of multiplying a whole number by a fraction that is greater than 1 to the effect of multiplying a whole number by a fraction less than 1. For example, ask students to use models to compare the products $2 \times \frac{1}{4}$ and $2 \times \frac{5}{4}$.
    - Students create a model to show $2 \times \frac{1}{4}$.



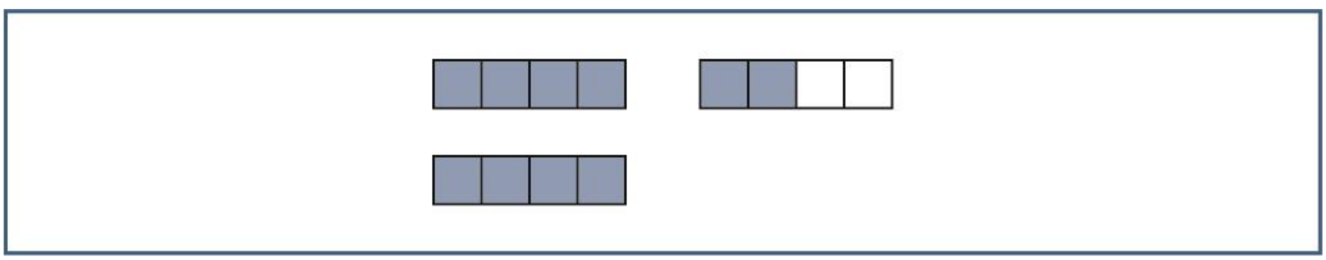
Two pieces of size $\frac{1}{4}$ are equivalent to $\frac{2}{4}$ or $\frac{1}{2}$, as shown.



- Students create a model to show $2 \times \frac{5}{4}$.

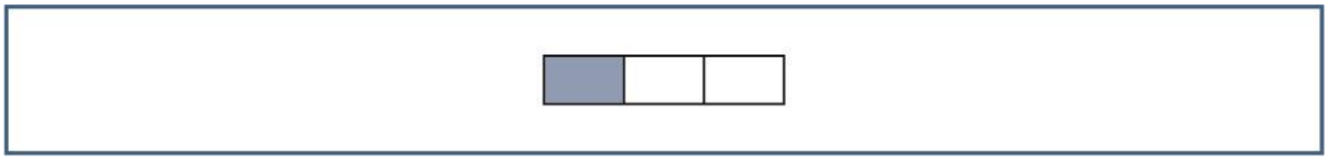


Two pieces of size $\frac{5}{4}$ are equivalent to $\frac{10}{4}$ or $2\frac{1}{2}$, as shown.

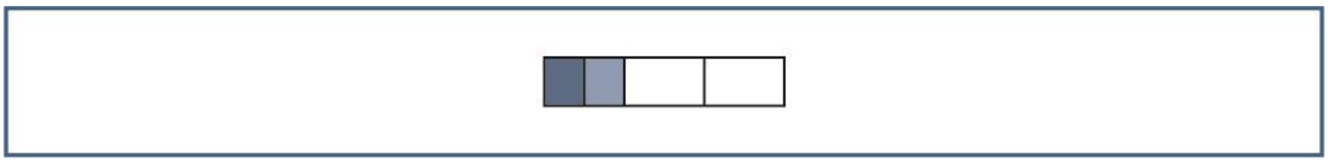


Discuss with students that since the fraction $\frac{1}{4}$ is less than 1 whole, multiplying $\frac{1}{4}$ by a whole number results in a product that is less than the whole number. Since the fraction $\frac{5}{4}$ is greater than 1 whole, multiplying $\frac{5}{4}$ by a whole number results in a product that is greater than the whole number.

- Ask students to use models to compare the effect of multiplying a fraction by another fraction greater than 1 to the effect of multiplying a fraction by another fraction less than 1. For example, ask students to compare the products $\frac{1}{2} \times \frac{1}{3}$ and $\frac{1}{2} \times \frac{4}{3}$.
  - To evaluate the product $\frac{1}{2} \times \frac{1}{3}$, students start by creating a model for $\frac{1}{3}$.

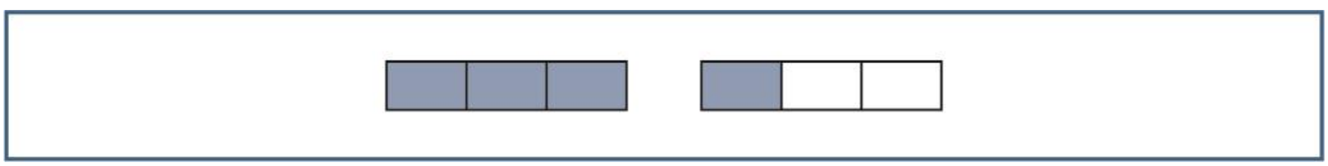


Students then change the model to show $\frac{1}{2}$ of the $\frac{1}{3}$.

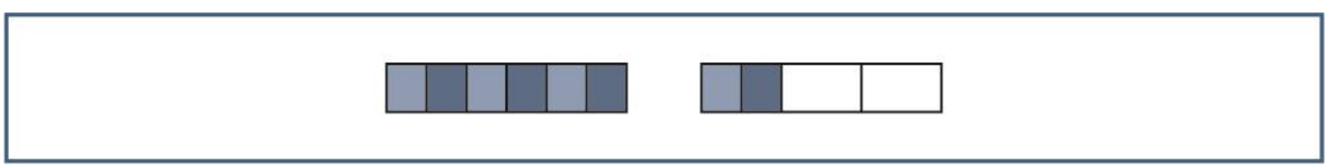


The resulting product of the two fractions is $\frac{1}{6}$, which is less than $\frac{1}{2}$.

  - To evaluate the product $\frac{1}{2} \times \frac{4}{3}$, students start by creating a model for $\frac{4}{3}$.



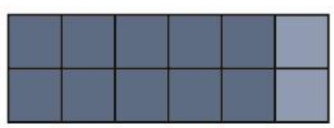
Students then change the model to show $\frac{1}{2}$ of the $\frac{4}{3}$.



The resulting product is $\frac{4}{6}$, which is greater than $\frac{1}{2}$.

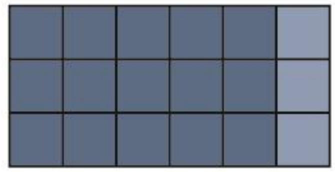
- Ask students to determine the possible values of missing numbers in inequalities. For example, give students the inequality $5 \times \frac{?}{7} > 5$ and ask them to determine values to replace the question mark that make the inequality true. In order to make the product greater than 5, the fraction must have a value greater than 1. Therefore, the missing value must be greater than 7.

- Ask students to select an inequality symbol to complete a given comparison without calculating the values involved. For example, give students the inequality $8 \times \frac{10}{9}$ □ 8. The greater than symbol (>) completes the inequality since $\frac{10}{9}$ is greater than 1 and multiplying a number (in this case 8) by a fraction greater than 1 results in a product greater than the number.
- Ask students to make inferences and draw conclusions relating the sizes of factors and products. For example, give students the mathematical statement $p \times \frac{q}{r} < p$. Students should be able to infer that in order for the inequality to be true, q must be less than r since the fraction must be less than 1 in order to result in a value less than p when the fraction is multiplied by p.
- Ask students to use models to show that multiplying a fraction by another fraction with the same numerator and denominator is equivalent to multiplying the original fraction by 1. This therefore results in a product that is equal to the original fraction.
    - Give students the problem $\frac{5}{6} \times \frac{2}{2}$. Students create the model shown.



A total of 5 of the 6 columns are shaded, and 10 of the 12 smaller squares are shaded. Because the shading is equal, $\frac{5}{6} \times \frac{2}{2}$ is equal to $\frac{10}{12}$, which is equivalent to $\frac{5}{6}$.

- Give students the problem $\frac{5}{6} \times \frac{3}{3}$.



Again, 5 of the 6 columns are shaded as well as 15 of the 18 smaller squares. Therefore, the model represents $\frac{15}{18}$, which is equivalent to $\frac{5}{6}$.

Noting the similarities between the models, students should conclude that multiplying by any fraction with the same numerator and denominator is equivalent to multiplying by 1 and will create an equivalent fraction.

- Ask students to sort multiplication problems into the categories of "greater than the whole number," "equal to the whole number," or "less than the whole number." For example, provide students with the following multiplication problems: $9 \times \frac{1}{10}$, $5 \times \frac{3}{2}$, $8 \times \frac{12}{12}$, $4 \times \frac{9}{8}$, and $2 \times \frac{4}{5}$. Students then sort the multiplication problems into the three categories as shown.

Greater Than the Whole Number	Equal to the Whole Number	Less Than the Whole Number
$4 \times \frac{9}{8}$ $5 \times \frac{3}{2}$	$8 \times \frac{12}{12}$	$9 \times \frac{1}{10}$ $2 \times \frac{4}{5}$

► Identifying and Addressing Common Misconceptions

- Some students may confuse the order of the values in multiplicative comparison statements. Ask students to write an equation for the statement "$\frac{1}{5}$ of $\frac{1}{6}$ is $\frac{1}{30}$." Some students may write equations such as $\frac{1}{30} \times \frac{1}{6} = \frac{1}{5}$. Help students recognize that taking one fraction "of" another number indicates the operation of multiplication. The word "is" generally indicates a statement of equality. As a result, $\frac{1}{5}$ and $\frac{1}{6}$ are multiplied (in any order) on one side of the equation and $\frac{1}{30}$ is alone on the other side of the equation.

- Some students may be confused about which factor is greater than or less than the product. Ask students to compare the size of $\frac{1}{2} \times \frac{5}{3}$ and $\frac{5}{3}$. Some students may notice that the product involves multiplying by $\frac{5}{3}$, which is greater than 1, and conclude that the product is greater than $\frac{5}{3}$. Rewrite the product as $\frac{5}{3} \times \frac{1}{2}$, ask students what the $\frac{5}{3}$ is being multiplied by, and ask them to explain what conclusions can be drawn as a result. When one factor is greater than 1 and one factor is less than 1, it is necessary to be careful which factor is being compared when estimating the size of the product. The product of $\frac{1}{2} \times \frac{5}{3}$ is greater than $\frac{1}{2}$ because the factor $\frac{5}{3}$ is greater than 1, but the product is less than $\frac{5}{3}$ because the factor $\frac{1}{2}$ is less than 1.
- Some students may incorrectly determine whether a fraction is greater than or less than 1. Ask students to find a numerator for the fraction $\frac{?}{100}$ that makes it greater than 1 and a numerator that makes it less than 1. Some students may think that a numerator of 18 makes the fraction greater than 1 because 18 is greater than 1. Some students may think the numerator must be a multiple of the denominator, such as 200, for the fraction to be greater than 1, and therefore think a numerator less than 200, such as 150, makes the fraction less than 1. Reiterate with visual models that the numerator needs to be greater than the denominator for the fraction to be greater than 1 and that the numerator needs to be less than the denominator for the fraction to be less than 1.

► Enrichment

- Ask students to write comparison statements with decimals and fractions. For example, provide a list of values such as 0.5, 1, 0.75, 9, 12, and $\frac{1}{2}$ and ask students to make at least one correct comparative statement or equation ("__ is __ of __" or "__ × __ = __") using the given values.
- Ask students to complete missing factor multiplicative comparisons. For example, provide equations such as $\frac{5}{10} \times \frac{?}{?} = \frac{35}{100}$ or statements such as "2 is $\frac{1}{3}$ of ?" and ask students to determine the missing factors.

- Ask students to order multiplication problems with mixed numbers, whole numbers, and fractions. For example, ask students to order the products $\frac{2}{3} \times 1\frac{1}{2}$, $\frac{2}{3} \times 3$, and $\frac{2}{3} \times \frac{9}{7}$ from least to greatest without computing the products.
- Ask students to explore not only whether a product is greater than or less than one of the factors but how much greater than or less than the product is than one of the factors. For example, give students the product $5 \times \frac{q}{r}$ and ask them to make the product as small as possible. Students should notice that not only is the product less than 5 as long as $\frac{q}{r}$ is less than 1, but the smaller the fraction $\frac{q}{r}$ is, the smaller the resulting product is. Students may also conclude that there is no smallest value product because there is no "smallest" fraction.
- Give students a product and ask them to draw conclusions about the sizes of possible factors. For example, ask students to find
    - a pair of numbers whose product is $\frac{1}{4}$ in which each number is greater than $\frac{1}{4}$,
    - a pair of numbers whose product is $\frac{1}{4}$ in which each number is less than $\frac{1}{4}$, and
    - a pair of numbers whose product is $\frac{1}{4}$ in which one number is greater than $\frac{1}{4}$ and one number is less than $\frac{1}{4}$.

  Students should conclude that the second task is impossible. Help students reason that if both numbers are less than $\frac{1}{4}$, then both numbers are less than 1. Therefore, their product is less than either number and (by the transitive property) is also less than $\frac{1}{4}$.
- Ask students to make and test conjectures about multiplying by decimals that are greater than 1 and multiplying by decimals that are less than 1.

► Key Terms

factor, product, resizing, scaling, equation, equivalent

A-F.2.1.4

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

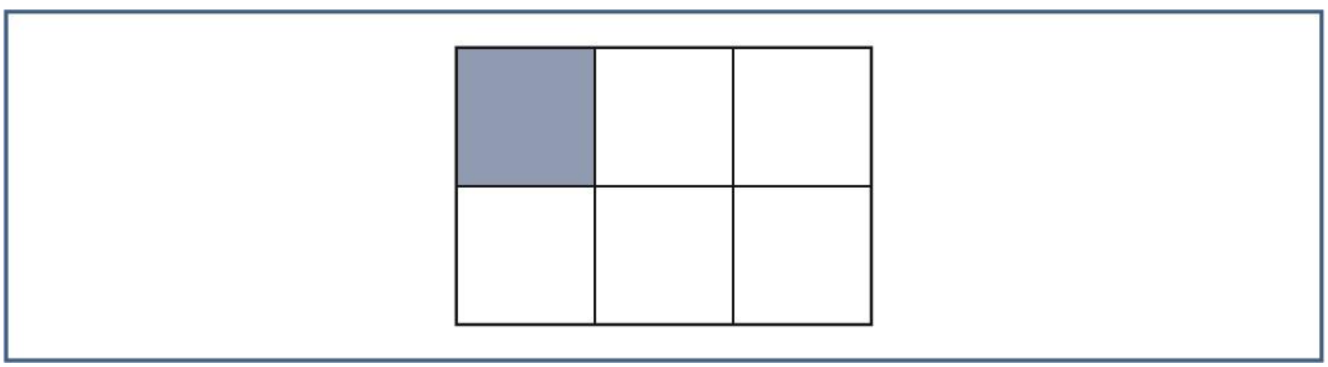
Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems).

▶ Eligible Content

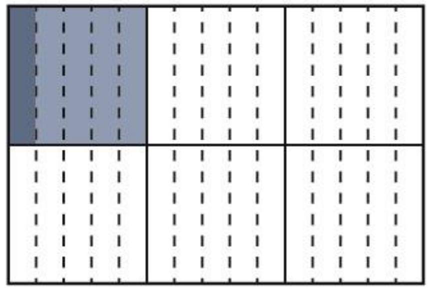
A-F.2.1.4 Divide unit fractions by whole numbers and whole numbers by unit fractions.

▶ Instructional Supports

- Ask students to use a model to divide a unit fraction by a whole number. For example, ask students to solve $\frac{1}{6} \div 5$. Students can create the following model for $\frac{1}{6}$.

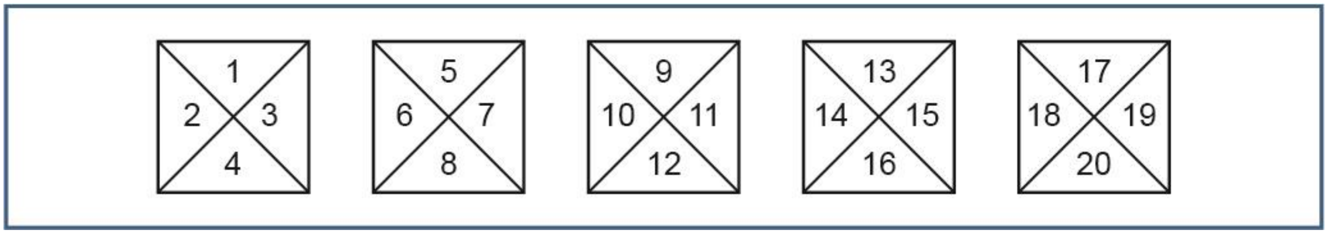


The shaded portion of the rectangle represents $\frac{1}{6}$ of a whole. Students represent $\frac{1}{6} \div 5$ by dividing one of the sixths into 5 equal parts. When each of the 6 parts of the whole is divided into 5 equal parts, there are 30 parts of the whole that are all the same size.



Each of the small parts represents $\frac{1}{30}$ of the whole; therefore, $\frac{1}{6} \div 5 = \frac{1}{30}$.

- Ask students to explain why dividing a unit fraction by a whole number produces a quotient that is less than the unit fraction. For example, ask students to explain why, in the equation $\frac{1}{5} \div 9 = m$, the unknown value represented by m must be less than $\frac{1}{5}$. When dividing a unit fraction by a whole number, each unit fraction is being further divided into equal parts. In this case, the $\frac{1}{5}$-size parts are being further divided into 9 even smaller parts, so there are now 45 smaller parts (because $5 \times 9 = 45$), each with a size of $\frac{1}{45}$. Therefore, $\frac{1}{5} \div 9 = \frac{1}{45}$ and $\frac{1}{45} < \frac{1}{5}$.
- Ask students to recognize the pattern that occurs when dividing a unit fraction by a whole number and develop a formula that could be used to easily calculate the appropriate quotients. For example, give students $\frac{1}{10} \div 3 = n$. Because each of the 10 equal parts of the whole is being divided into 3 parts each, there are now 10×3 equal parts. Therefore, $\frac{1}{10} \div 3 = \frac{1}{10 \times 3}$. One possible pattern is that the denominator can be multiplied by the divisor. Some students may also notice that $\frac{1}{10 \times 3} = \frac{1}{10} \times \frac{1}{3}$ and that dividing a unit fraction by 3 is equivalent to multiplying the unit fraction by $\frac{1}{3}$.
- Ask students to use a model to divide a whole number by a unit fraction. For example, ask students to divide 5 by $\frac{1}{4}$. A possible division model is shown.



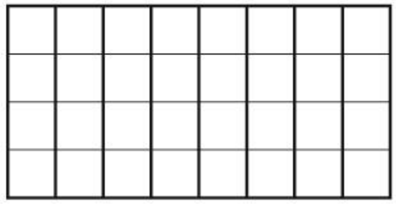
There are 5 squares, each divided into 4 equal parts. This represents the whole number 5 being divided into equal parts of size $\frac{1}{4}$, and there are a total of 20 parts. Therefore, 5 divided by $\frac{1}{4}$ is 20.

- Ask students to explain why dividing a whole number by a unit fraction produces a quotient that is greater than the whole number. For example, ask students to explain why, in the equation $15 \div \frac{1}{5} = m$, the unknown value represented by m must be greater than 15. The quotient $14 \div 2$ can be determined by dividing 14 wholes into equal parts of size 2 and counting the number of equal parts. In the case of $15 \div \frac{1}{5}$, each of the 15 wholes is being divided into equal parts of size $\frac{1}{5}$. Because each whole contains 5 such parts, the number of parts is greater than 15, the number of wholes.

- Ask students to recognize patterns that occur when dividing a whole number by a unit fraction and develop a formula that could be used to determine the appropriate quotients. For example, give students the quotient $3 \div \frac{1}{8} = n$. Each of the 3 wholes is being divided into 8 parts. Since each whole now has 8 parts, there are now 24 parts. In general, students may recognize that the quotient can be calculated by multiplying the whole number by the denominator of the unit fraction. Another pattern is the relationship between multiplication and division. The division equation $3 \div \frac{1}{8} = n$ represents the same relationship as the multiplication equation $n \times \frac{1}{8} = 3$. Students can use their fraction multiplication skills to deduce that $24 \times \frac{1}{8} = 3$, so $n = 24$.

- Ask students to use a model to solve a real-world problem involving division, a unit fraction, and a whole number. For example, give students this problem: “Jason has an 8-hour school day. What fraction of his 8-hour school day does he spend in his $\frac{1}{4}$-hour (15-minute) recess after lunch?” Students may create the following model.

8 Hours



Each column represents 1 hour, and each hour has been divided into 4 equal parts. There are 32 parts that each represent $\frac{1}{4}$ hour, and Jason's recess is represented by one of these parts. Thus, Jason's recess represents $\frac{1}{32}$ of his 8-hour school day.

- Ask students to use an equation to solve a real-world problem involving division, a unit fraction, and a whole number. For example, ask students to write and solve an equation to determine the number of servings of milk there are in 3 gallons of milk when each serving is 1 cup ($\frac{1}{16}$ gallon). Students set up the equation $3 \div \frac{1}{16} = s$, where s is the number of servings. Because $3 \div \frac{1}{16} = 3 \times 16 = 48$, there are 48 servings in the 3 gallons of milk.

► Identifying and Addressing Common Misconceptions

- Some students may divide a unit fraction by a whole number by dividing the denominator of the unit fraction by the whole number. Ask students to solve $\frac{1}{8} \div 4$. Some students may calculate $8 \div 4$ and give an answer of 2 or $\frac{1}{2}$. Ask students to explain the meaning of $\frac{1}{8} \div 4$. Dividing $\frac{1}{8}$ by 4 means dividing $\frac{1}{8}$ of a whole into 4 equal parts. Then ask students whether dividing a quantity into parts results in larger parts or smaller parts. Both 2 and $\frac{1}{2}$ are greater than $\frac{1}{8}$.
- Some students may reverse the dividend and divisor when dividing a unit fraction by a whole number. Ask students to solve $\frac{1}{6} \div 3$. Some students may interpret this as being the same as $3 \div \frac{1}{6}$ and come up with an answer of 18. Ask students if $12 \div 4$ is the same as $4 \div 12$. Help students understand that division is not commutative, and therefore they cannot reverse the dividend and divisor, even when the dividend is a unit fraction.

- Some students may divide a whole number by a unit fraction by dividing by the denominator of the unit fraction. Ask students to solve $6 \div \frac{1}{3}$. Some students may divide 6 by 3 and give an answer of 2 or $\frac{1}{2}$. Ask students to explain the meaning of $6 \div \frac{1}{3}$. Then help them create and use a visual model to determine the quotient.
- Some students may set up the equations for a real-world problem incorrectly. Give students this situation: "A group of 3 friends have $\frac{1}{5}$ of a pan of casserole to share equally. What fraction of the pan of casserole does each person get?" Some students may calculate $3 \div \frac{1}{5}$ and give an answer of 15. Ask students to assess the situation and evaluate whether an answer that is greater than 1 or an answer that is less than 1 makes more sense. In this situation, since the whole is 1 pan of casserole, each person should receive less than 1 whole pan of casserole.

► Enrichment

- Ask students to create a real-world situation that could be represented by a whole number divided by a fraction. For example, give students the quotient $\frac{1}{4} \div 5$. A possible response is given: "Toby has $\frac{1}{4}$ carton of milk remaining. He pours an equal amount of milk into each of 5 glasses. What fraction of the whole carton of milk is poured into each of the 5 glasses?"
- Give students a unit fraction and ask them to determine multiple ways to divide a unit fraction by a whole number that results in the given unit fraction. For example, give students the unit fraction $\frac{1}{18}$. Students determine the quotients $\frac{1}{1} \div 18$, $\frac{1}{2} \div 9$, $\frac{1}{3} \div 6$, $\frac{1}{6} \div 3$, $\frac{1}{9} \div 2$, and $\frac{1}{18} \div 1$. Students may notice that the number of quotients is the same as the number of factors of the denominator.
- Ask students to determine patterns in the sizes of unit fractions divided by whole numbers and make conjectures regarding the sizes of unit fractions divided by fractions. For example, ask students to compare $\frac{1}{5} \div 4$, $\frac{1}{5} \div 3$, $\frac{1}{5} \div 2$, and $\frac{1}{5} \div 1$. Students may notice that as the size of the divisor decreases, the size of the quotient increases. Students can then conjecture that dividing $\frac{1}{5}$ by a fraction less than 1 (e.g., $\frac{1}{5} \div \frac{1}{2}$) may result in a quotient that is greater than $\frac{1}{5}$.

- Ask students to create a real-world situation that could be represented by a whole number divided by a fraction. For example, give students the quotient $6 \div \frac{1}{2}$. A possible response is given: "Janice has 6 bags of apples. She uses $\frac{1}{2}$ bag of apples for each apple pie she bakes. How many apple pies can Janice bake using the 6 bags of apples?"
- Ask students to determine patterns when dividing by a product of whole numbers and extend these patterns to products of unit fractions. Give students the quotient $72 \div (2 \times 3) = 72 \div 6 = 12$. After several examples, students may notice and confirm that this is also the same as $72 \div 2 \div 3$. Likewise, students may notice and confirm that this pattern also holds when dividing by unit fractions. For example, $8 \div (\frac{1}{2} \times \frac{1}{3})$ can be determined by first dividing by $\frac{1}{2}$ and then dividing that quotient by $\frac{1}{3}$. Some students may also conjecture that the quotient $12 \div \frac{4}{7}$ can be determined by decomposing the fraction and changing the quotient to $12 \div (4 \times \frac{1}{7})$. The quotient can be calculated by first dividing by 4 and then dividing by $\frac{1}{7}$.
- Ask students to explain situations in which the quotient is greater than the dividend and situations in which the quotient is less than the dividend.
- Ask students to write (but not solve) equations that can be used to solve problems involving division of non-unit fractions.

► Key Terms

division, factor, multiplication, number line, unit fraction, whole number, dividend, divisor, equation, fractional pieces, quotient, rectangle, unit fraction, whole number

A-T.1.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Understand the place-value system.

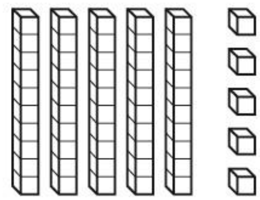
Demonstrate understanding of place-value of whole numbers and decimals, and compare quantities or magnitudes of numbers.

► Eligible Content

A-T.1.1.1 Demonstrate an understanding that in a multi-digit number, a digit in one place represents 1/10 of what it represents in the place to its left.

► Instructional Supports

- Provide students with a multi-digit number that has a missing digit and ask students to identify what the digit must be so that it represents 10 times as much as the digit in the place to the right. For example, give students the number 4?8 and ask them to find the missing digit that represents a value that is 10 times as great as the digit 8 in the ones place. Students should identify the value of the missing digit as 80 and the digit as 8.
- Provide students with a multi-digit number that has a missing digit and ask students to identify what the digit must be so that it represents $\frac{1}{10}$ as much as the digit in the place to the left. For example, give students the number 3,?76 and ask them to find the missing digit that represents the value that is $\frac{1}{10}$ the value of the digit 3 in the thousands place. Students should identify the value of the missing digit as 300, which is $\frac{1}{10}$ of 3,000, so the digit in the hundreds place is 3.
- Provide students with base-ten blocks and a two-digit number that is a multiple of 11. Ask students to represent the value of each digit separately using base-ten blocks. Next, have students compare the number of unit blocks in the representations of each of the two digits. For example, given the number 55, students represent the 5 in the tens place with five of the ten-blocks and represent the 5 in the ones place with five of the individual unit blocks as shown.



Students should observe that even though there are 5 tens and 5 ones, there are ten times as many unit blocks that represent the tens digit as represent the ones digit because each ten-block is a bundle of ten ones. Emphasize that the digit in the tens place represents the number of bundles of size ten.

► Identifying and Addressing Common Misconceptions

- Students may think that a digit has the same meaning in all situations. Ask students to identify the value that the digit 5 represents in both 5,867 and 3,592. Some students may think that both 5s represent a value of 5 ones. Students should note that even though the digits are both 5, each represents a different value: 5 thousands and 5 hundreds, respectively, as the digits are in different place-values.
- Students may think that with a number where all the digits are the same, the value of one digit compared to another is achieved by multiplying or dividing by the same digit. For example, in the number 777, ask students how many times as great each 7 is compared to the one immediately to its left or right. Some students may think, for instance, that the value of the 7 in the tens place is 7 times as great as the value of the 7 in the ones place. Students should note that the value the 7 represents in the tens place is 70 and that the value the 7 represents in the ones place is 7. Therefore, the value each digit represents in any place is 10 times as great as the value the same digit represents in the place to the right. Students could also discover this by multiplying the 7 in the ones place by 7. Since the product of these factors is 49, the students can see that their original thought is a misconception.
- Students may confuse and/or associate negative (left) and positive (right) directions, such as on a number line, with the values of consecutive same digits in a number. So when two consecutive equal digits appear in a multi-digit number, students may think the digit further to the right, as on

a number line, is the one that is 10 times as great. Ask students which digit in 8,824 has the greatest value. Students should be reminded that in a standard number, digits increase in value to the left. Therefore, the digit 8 in the thousands place has the greatest value, and it is 10 times the value of the digit 8 in the hundreds place.

► Enrichment

- Ask students to compare the values of equal digits in a whole number that are three or more places away from each other. For example, ask students to find how many times as great the value of the 4 in the thousands place is relative to the value of the 4 in the ones place for the number 4,164. Students should note that the value represented by an equal digit three places to the left is 1,000 (or 10 × 10 × 10) times as great as the value represented by the original digit because each place to the left is 10 times as great as the place to its right. Additionally, students should recognize that the value represented by the same digit three places to the right is $\frac{1}{1,000}$ (or $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$) times the value represented by the original digit since each place is $\frac{1}{10}$ the value of the place to its left.
- Ask students to find values of numbers with the same digits when the numbers are related by powers of 10. For example, ask students to find a number that has an 8 in a place that represents a value that is 10,000 times as great as 800.
- Ask students to find the digit and place-value of a number that is $\frac{1}{500}$ the value of the digit 3 in the ten thousands place of a number. For example, the number 30,000 has the digit 3 in the ten thousands place, and $\frac{1}{500}$ of that value would be the digit 6 in the tens place.

► Key Terms

digit, place-value

A-T.1.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Understand the place-value system.

Demonstrate understanding of place-value of whole numbers and decimals, and compare quantities or magnitudes of numbers.

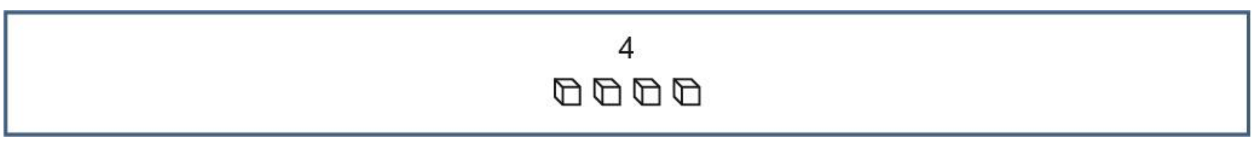
► Eligible Content

A-T.1.1.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

► Instructional Supports

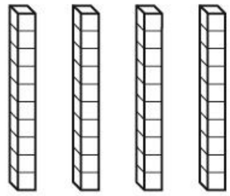
Finding patterns with multiplying by powers of 10

- Ask students to use base-ten blocks or another visual representation to show patterns of a number multiplied by a power of 10.
    - For the first representation, students should represent the number 4 with base-ten blocks.



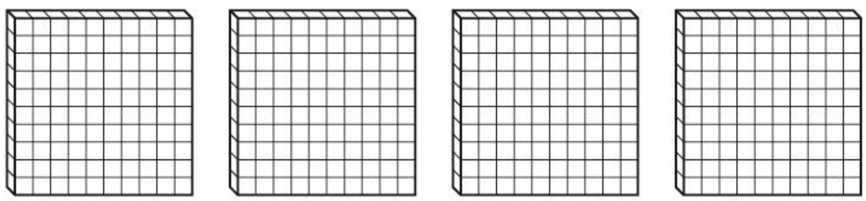
    - Then, students should represent 4 × 10 by repeating their original representation 10 times. Students should note the total number of blocks.

$4 \times 10 = 40$



- Next, students should represent $4 \times 10 \times 10$ by repeating the second representation 10 times. Students should note the total number of blocks.

$4 \times 10 \times 10 = 400$



- Finally, after each number is multiplied by 10, students should identify the pattern for the total number of blocks (i.e., 4, 40, 400, . . .).

- Ask students to simplify expressions and recognize patterns of a number multiplied by a power of 10. Students should write at least three expressions that multiply the same value by different powers of 10. Some examples are shown.
  - 3×10 = 30 (the digit 3 moves left one place)
  - $3 \times 10 \times 10$ = 300 (the digit 3 moves left two places)
  - $3 \times 10 \times 10 \times 10$ = 3,000 (the digit 3 moves left three places)

Using place-value understanding to multiply and divide by powers of 10

- Observe that each time a number is multiplied by 10, the value of each digit becomes 10 times as great and therefore each digit shifts one place to the left, with zeros used as placeholders if necessary. For example, in the multiplication equation $63 \times 10 \times 10 = 6{,}300$, the value of the digit 6 becomes 10 times as great and then 10 times as great again, so the place-value changes from the tens place to the thousands place. Similarly, observe that each time a number is divided by 10, the value of each digit becomes $\frac{1}{10}$ as great and therefore shifts one place to the right. Zeros are used as placeholders between the digits and the decimal point to show when there is no value for that location. For example, $0.8 \times \frac{1}{10}$ will cause the digit 8 to change in value from the tenths to the hundredths place, and there will be no tenths. Therefore, it results in the answer 0.08.

- Ask students to multiply, and then simplify, a number by various powers of 10, such as 1,000, 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1{,}000}$. Some examples are shown.
  - $7.0 \times 1{,}000 = 7{,}000.0$
  - $7.0 \times 100 = 700.0$
  - $7.0 \times 10 = 70.0$
  - $7.0 \times 1 = 7.0$
  - $7.0 \times \frac{1}{10} = 0.7$
  - $7.0 \times \frac{1}{100} = 0.07$
  - $7.0 \times \frac{1}{1{,}000} = 0.007$

  Using the pattern above with different numbers, simplify each product of a number and a power of 10. For example, given the expressions $8 \times \frac{1}{10}$, $9.6 \times 1{,}000$, and $3.1 \times \frac{1}{100}$, students should be able to determine these products as 0.8, 9,600, and 0.031, respectively.

- Ask students to write given numbers or expressions in exponential form. For example, for the numbers and expressions 10,000; 1,000,000; $10 \times 10 \times 10$; and $10 \times 10 \times 10 \times 10 \times 10$, students should write 10^4, 10^6, 10^3, and 10^5, respectively.

- Ask students to write a number with nonzero digits on each side of the decimal point and complete a decimal table. First, students should write each digit of the number in the correct column of the table. Then, to multiply the number by 10, students should multiply the value of each digit by 10. Students should then record the product in the decimal table. Next, students should multiply the original number by 100, shifting each of the digits in the original number two places to the left. Finally, students should multiply the original number by 1,000, shifting each of the digits in the original number three places to the left. Students should explain how each digit shifts according to the specific power of 10 by which it is multiplied. An example table is shown.

	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths
6.23				6	.	2	3
6.23×10			6	2	.	3	0
6.23×10^2		6	2	3	.	0	0
6.23×10^3	6	2	3	0	.	0	0

Have students explain the relationship between the location of the digits, the decimal point, and the power of 10 the number is being multiplied by.

- Given a number pattern with a rule involving multiplying by a power of 10, ask students to recognize the factor that a term is multiplied by to obtain the next term, expressing the answer as a power of 10. For example, given the number pattern 0.0018, 1.8, 1,800, . . ., recognize that each term is multiplied by 10^3 (or 1,000) to determine the next term.

► Identifying and Addressing Common Misconceptions

- Students may think that multiplication always results in a greater number. Ask students why multiplying two fractions that are less than one gives a product that is less than either fraction. Students that have the misconception of the product always being greater may say that a product less than either factor is not possible. Those students should be shown an example that does yield a product less than either factor, such as $\frac{1}{10} \times \frac{1}{10}$.

- When multiplying by powers of 10, students may think that they do not have to follow the order of operations. For example, ask students to simplify 4×10^2. Students may work left to right, ignoring the order of operations, to multiply 4×10 to get 40 and then square 40 to get 1,600.

Remind students that 10^2 needs to be found first since exponents come before multiplication in the order of operations.

- Students may think that when a factor is multiplied by a power of 10, the number of zeros in the power of ten is just “added” onto the end of the product. Ask students to multiply 6.15×10^3. Students that just “add” three zeros to the end of the product will have rewritten the product as 6.15000 or 615,000. Explain to students that just “adding” zeros does not always work, as is the case with a number with a visible decimal point. Multiplying a number by a power of 10 must be thought of differently than simply “adding” zeros to the end of a number. In the case of multiplying a decimal number by a power of 10, all digits are each moved an equal number of places to the left as given by the power of ten.
- When dividing a decimal by a power of 10, students may think that each side of the decimal is divided by the power of 10. Ask students to divide 30.5 ÷ 10. Students that divide 30 by 10 and then 5 by 10 to get $3\frac{1}{2}$ have the misconception that each side of the decimal is treated as a separate division. Ask the students to multiply a number, such as 2.37, by 10. Then ask them what value they would have if they divided their new number by 10. Use this to show them how the position of the digits in relation to the decimal point changes, but the order of the digits remains the same when multiplying and dividing by powers of 10.

► Enrichment

- Ask students to write a given number as a product or quotient of a number and a power of 10 in multiple ways. For example, give students the number 5,600. Possible responses include 56×10^2; $5{,}600{,}000 \div 10^3$; and 5.6×10^3.
- Ask students to explain why it is not possible to have \$0.0017 in the U.S. currency system with real money unless it relates to a situation where we have multiples of that amount, such as 1,000 sheets of paper that cost \$0.0017 each. Discuss with students what other multiples would be possible and what multiples would not be possible.

► Key Terms

base, decimal, decimal point, divide, exponent, expression, factor, multiply, pattern, place-value, power of 10, product, simplify

A-T.1.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Understand the place-value system.

Demonstrate understanding of place-value of whole numbers and decimals, and compare quantities or magnitudes of numbers.

► Eligible Content

A-T.1.1.3 Read and write decimals to thousandths using base-ten numerals, word form, and expanded form.

► Instructional Supports

- Ask students to identify the place-value of the digit at the far right of a given number. Students should then use that specific place-value as a guide to say and write the number. For example, give students the number 53.74. Students should recognize that the 4 is the digit farthest to the right and that it is in the hundredths place. Students should then practice saying and writing the number "fifty-three and seventy-four hundredths."
- Ask students to read numbers that have the same digits in different places. For example, give students the numbers 6.82 and 6.802. Each student should read both numbers aloud to a classmate and explain how they decided to say each number. Students should recognize that 6.82 is read as "six and eighty-two hundredths" since the digit to the far right is in the hundredths place, while 6.802 is read as "six and eight hundred two thousandths" since the digit to the far right is in the thousandths place.
- Given a number written in word form, ask students to write the number in standard form. For example, give students the number "seven and five hundred forty-six thousandths." Ask students to identify and say the place-value of the far right digit. Students should then use that place-value to help them write the number in standard form, 7.546. Students may be given numbers orally, or written in word form, or both.

- Ask students to write a given number in a decimal table and use the table to write the number in expanded form. For example, give students the number 815.732. Students record each digit of the number in the decimal table.

Hundreds $(\times 100)$	**Tens** $(\times 10)$	**Ones** $(\times 1)$	.	**Tenths** $(\times \frac{1}{10})$	**Hundredths** $(\times \frac{1}{100})$	**Thousandths** $(\times \frac{1}{1{,}000})$
8	1	5	.	7	3	2

Next, students use the information from the decimal table to change the number to expanded form. Students should move from left to right, multiplying each digit by its place-value, and then write the number as the sum of the products.

$$8 \times 100 + 1 \times 10 + 5 \times 1 + 7 \times \frac{1}{10} + 3 \times \frac{1}{100} + 2 \times \frac{1}{1{,}000}$$

- Ask students to write a number in base-ten expanded form when given the number in written or standard form. For example, give students the number "sixty-four and thirty-eight hundredths." Students then write the number in base-ten expanded form as $6 \times 10 + 4 \times 1 + 3 \times \frac{1}{10} + 8 \times \frac{1}{100}$. Some students may express the three tenths as $30 \times \frac{1}{100}$ since the written form contains "thirty" and "hundredths" connected by "eight." Students can then divide each number by 10 to simplify that part of the expression to $3 \times \frac{1}{10}$. Some students may also write the number in standard form first to help them with writing the number in expanded form.

▶ Identifying and Addressing Common Misconceptions

- Students may think they can use the word "and" anywhere in a number when saying it orally. For example, some students may express $524.37 as "five hundred **and** twenty-four dollars and thirty-seven cents." If students have more than one "and," as in this case, it may indicate they do not associate the word "and" with the decimal point. Ask students to listen to correctly stated numbers and write them in standard form by including a decimal point when and only when they hear the word "and." Conversely, remind students that the word "and" should only be verbally expressed when the value contains a decimal point; therefore, $524.37 should be stated as "five hundred twenty-four dollars and thirty-seven cents."

- Students may not pay attention to the place-value of the decimal portion when converting from number names to standard form. Ask students to write “two and five hundredths” in standard form. Some students may only notice the “five” and not the “hundredths” and write the number as 2.5. Ask them what the difference is between the 5s in 2.5, 2.05, and 2.005. Discuss the importance of attention to the correct place-value.
- Students may think that a number in base-ten expanded form can be converted to standard form by working from left to right without regard for the order of operations. Ask students to convert the three-digit number in base-ten expanded form $3 \times 100 + 2 \times 10 + 4 \times 1$ to standard form. Students who think the number in standard form is 3,024 may have this misconception. Having these students use parentheses around products, such as $(3 \times 100) + (2 \times 10) + (4 \times 1)$, may help these students successfully complete the conversion from base-ten expanded form to standard form.

► Enrichment

- Ask students to find equivalent values as purely tenths or hundredths for numbers that include values in the ones, tenths, and hundredths places. For example, for the number 6.28, ask students to find the equivalent number of tenths or hundredths.
- Ask students to discuss the advantages and disadvantages of using base-ten expanded form when working with numbers.
- Provide students with a list of numbers in base-ten expanded form to compare to a given number in base-ten expanded form. Have students then choose the number from the list that is the closest in value to the given number and explain how they know it is the closest value.

► Key Terms

base-ten, decimal, expanded form, hundredths, place-value, power of 10, product, standard form, tenths, thousandths

A-T.1.1.4

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Understand the place-value system.

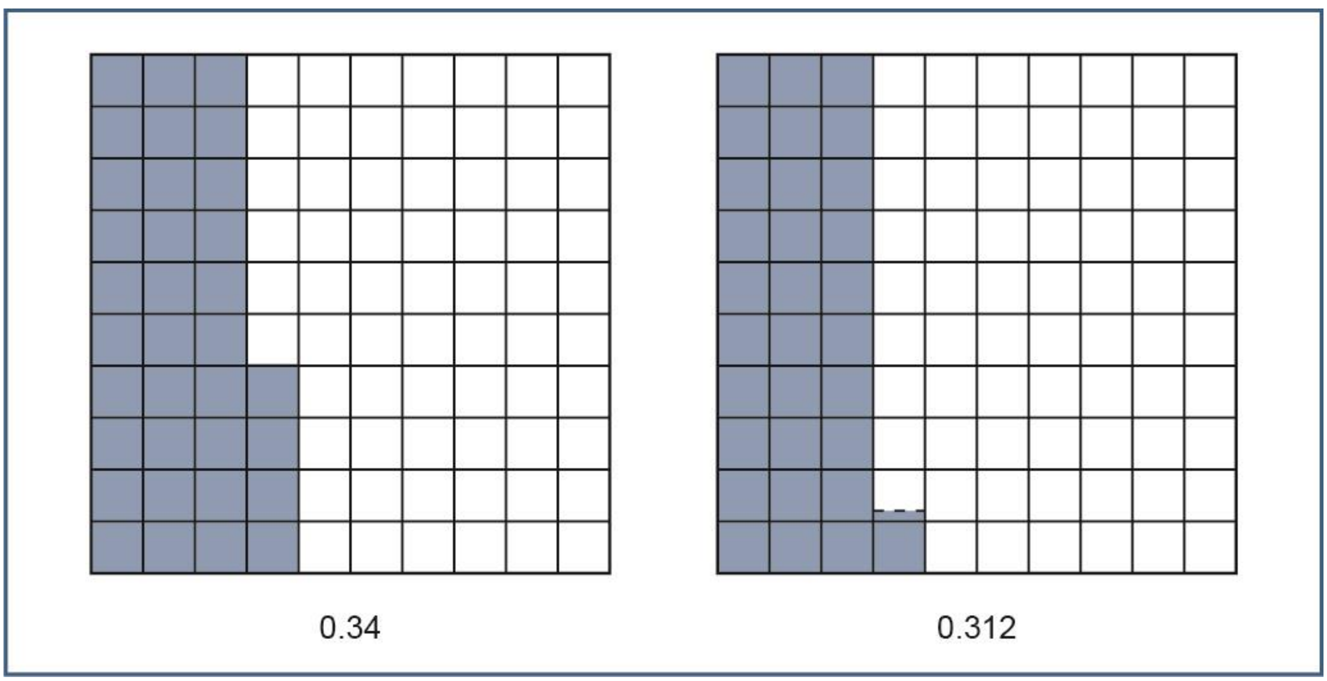
Demonstrate understanding of place-value of whole numbers and decimals, and compare quantities or magnitudes of numbers.

► Eligible Content

A-T.1.1.4 Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols.

► Instructional Supports

- Ask students to create visual models to help them compare two decimal numbers. For example, give students the numbers 0.34 and 0.312. Possible models are shown.



The model for 0.34 has a greater shaded area. Therefore, 0.34 is greater than 0.312.

- Ask students to explain why the leftmost place-values matter more when comparing two numbers. For example, give students the numbers 48.375 and 48.279. Observe that each number contains one digit that is greater than the corresponding digit in the other number. Students may find it helpful to organize the digits in a decimal table as shown.

Tens	Ones	.	Tenths	Hundredths	Thousandths
4	8	.	3	7	5
4	8	.	2	7	9

Students should notice that the digits in most of the place-value columns have the same value, except in the tenths place and the thousandths place. The difference of 1 tenth is greater than the difference of 4 thousandths because tenths are 100 times as great as thousandths. Therefore, 48.375 is greater than 48.279.

- Give students two numbers, with a missing digit in one of the numbers, and ask them to identify possible digits that will make one number greater. For example, give students the two decimal numbers shown.

$\underline{7}\,\underline{2}\,\underline{4}\,.\,\underline{6}\,\underline{5}\,\underline{3} \quad \underline{7}\,\underline{2}\,\underline{4}\,.\,\underline{6}\,\underline{?}\,\underline{3}$

Students determine that when the missing digit is 0, 1, 2, 3, or 4, the second number is less than the first number. Students also note that when the missing digit is 5, the second number is equal to the first number. Finally, when the missing digit is 6, 7, 8, or 9, the second number is greater than the first number.

- Ask students to determine whether the greater than symbol (>), the less than symbol (<), or the equal to symbol (=) is the correct mathematical symbol required to make a true comparison of two decimal numbers. For example, give students the following incomplete comparisons.

7.43 ☐ 7.34

36.281 ☐ 36.278

205.947 ☐ 205.952

The symbols that complete these three comparisons are >, >, and <, respectively. Ask students to pair with a classmate and discuss how they chose the correct symbols.

- Ask students to create two comparisons for a pair of decimal numbers that are not equal, one comparison using greater than and the other using less than. For example, give students the numbers 1.624 and 1.264. Students then identify and record the first comparison using the greater than symbol (>) with the greater number written first: $1.624 > 1.264$. Next, students reverse the order of the numbers and record the second comparison using the less than symbol (<): $1.264 < 1.624$.

► Identifying and Addressing Common Misconceptions

- Students may think that whole number relationships are applied to decimals. Ask students to determine whether 0.34 is greater or less than 0.7 and explain why. Students that say 0.34 is greater may have this misconception. The student might be comparing 34 to 7 instead of being focused on the place-value each number reaches to the right of the decimal. Students should fill in any missing place-values with 0 so the numbers are the same length and then compare the numbers. In this example, the numbers they would compare are 0.34 and 0.70.
- Students may think that one decimal number is greater than another simply due to the number of digits that the decimal number contains, which is a similar point of confusion as in the situation with whole numbers. Ask students to compare the decimal numbers 6.023 and 6.4. Students that consider 6.023 to be greater than 6.4 should line up the decimal points along the same vertical line or place both numbers in a table or on graph paper where the decimal points are in the same column. When students compare digits in the same columns from left to right, they will see that the 4 in the tenths place is greater than the 0 in the tenths place, and so 6.4 is greater than 6.023.
- Students may think that a decimal number is less than another due to extra zeros in places to the right of the other decimal number when the numbers are otherwise the same. Ask students to compare 0.2 and 0.200. Some students may think that 0.2 is less than 0.200 because 2 is less than 200. Students with this misconception may not remember that given a decimal number, zeros may be written on either the far-left or far-right without changing the value of the number. Have students line up the numbers vertically and compare the corresponding place-values. The numbers are visually the same except for the hundredths and thousandths places. Although there

is not a digit written in the hundredths or thousandths places for the 0.2, the digits in the hundredths and thousandths places for 0.200 are zeros, and so the decimal numbers are equal.

► Enrichment

- Ask students to order a list of five decimal numbers up to the ten thousandths place from least to greatest.
- Ask students to write and solve for two mystery decimal numbers when given information about the digits and which number is greater. For example, the first number has a 5 in the hundredths place, an even digit that is a multiple of 3 in the tenths place, and an odd number in the thousandths place that is less than nine but greater than five. The second number has the same digit in the hundredths places as the first number, an even number in the tenths place, and the square of 2 in the thousandths place. Both numbers are less than 1 and the second number is greater than the first number.
- Ask students to find a number between two decimal numbers that appear to be adjacent. For example, ask students to find a number between 0.55 and 0.54.
- Ask students to explain why there is no least positive number.

► Key Terms

compare, decimal, equal, greater than, hundredths, less than, place-value strategies, symbol, tenths, thousandths

A-T.1.1.5

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Understand the place-value system.

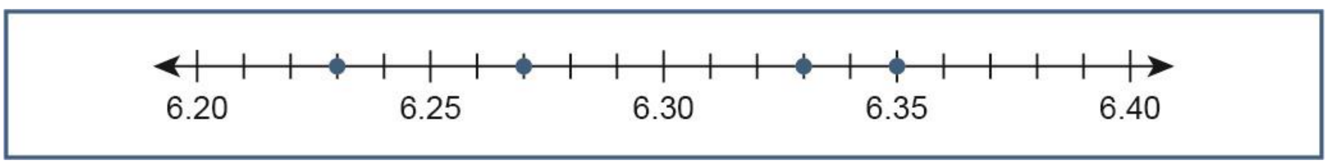
Demonstrate understanding of place-value of whole numbers and decimals, and compare quantities or magnitudes of numbers.

► Eligible Content

A-T.1.1.5 Round decimals to any place (limit rounding to ones, tenths, hundredths, or thousandths place).

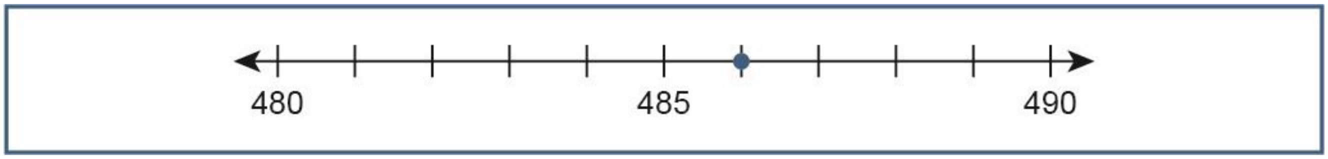
► Instructional Supports

- Ask students to use a number line to round decimal numbers. For example, provide students the numbers 6.23, 6.27, 6.33, and 6.35 and ask them to use a number line to identify which numbers round to 6.30 when rounded to the nearest tenth. Students plot each of the decimal numbers on a number line labeled from 6.20 to 6.40.

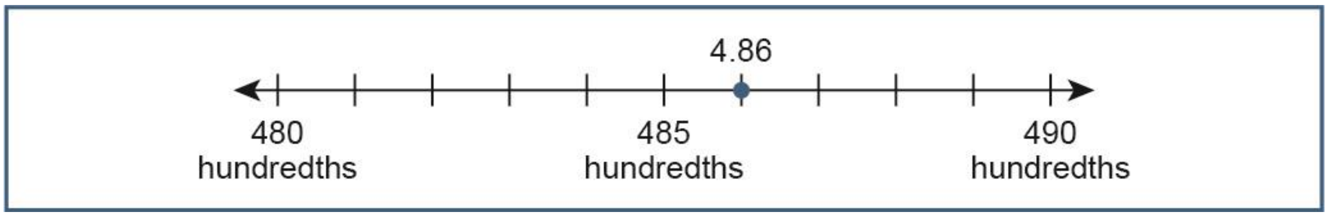


  The numbers 6.27 and 6.33 each round to 6.30 when rounded to the nearest tenth. The other numbers round to 6.20 or 6.40. Ask students to use the number line to explain the rounding of each number by determining the distance from each number to the tenth to the left and the tenth to the right where the hundredths value is 0.

- Ask students to relate the rounding of whole numbers to the rounding of decimal numbers. For example, help students to understand the parallels between rounding 486 to the nearest ten and rounding 4.86 to the nearest tenth. Provide students with a number line labeled from 480 to 490 with 486 plotted. Next, instruct students to round 486 to the nearest ten by using the number line.



Students note that 486 rounds to 490 because 486 is only 4 units away from 490 but 6 units away from 480 on the number line. Then, ask students to equate the number 4.86 to ones, tenths, and hundredths. Students should determine that 4.86 = 4.86 ones = 48.6 tenths = 486 hundredths. Once students recognize that 4.86 is equivalent to 486 hundredths, have them look back at the original number line and compare it to the number line below with 486 hundredths plotted and labeled as 4.86.



Students should note that 486 hundredths rounds to 490 hundredths because 486 hundredths is only 4 hundredths from 490 hundredths but 6 hundredths from 480 hundredths on the number line. Then have students compare the following number line containing 4.80, 4.85, and 4.90 marked and 4.86 plotted with the previous number line showing 486 hundredths plotted.

Ask students to explain how rounding 486 to the nearest ten is similar to rounding 4.86 to the nearest tenth.

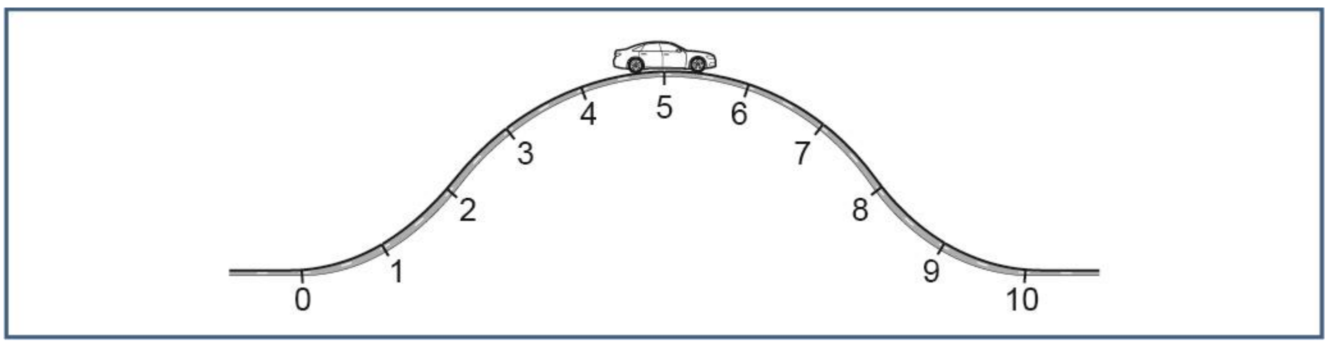
- Ask students to determine the least and greatest numbers, within a certain number of decimal places, that will round to a given number. For example, ask students to determine the least and greatest numbers with digits to the thousandths place that round to 6.3 when rounded to the nearest tenth. Students should recognize 6.250 as the least decimal number (to the thousandths place) and 6.349 as the greatest decimal number (to the thousandths place) that will round to 6.3. Next, ask students to represent their solution processes with a number line or another visual representation.

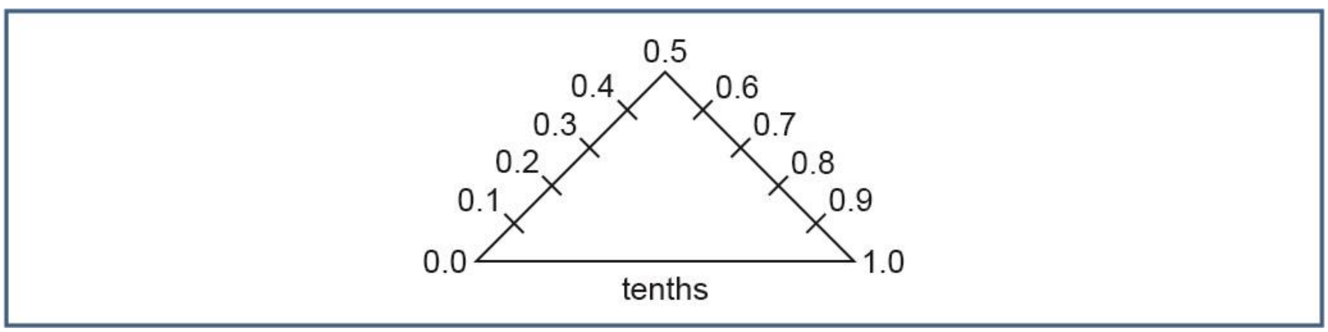
- Ask students to solve real-world problems that involve rounding decimal numbers that contain a deciding digit of 5. For example, give students the following situation:

  > A weather report records the daily low temperatures of six different cities, in degrees Fahrenheit, as shown.
  >
  > 64.5, 65.3, 67.5, 71.5, 75.6, 79.5
  >
  > The temperatures are each rounded to the nearest whole degree for broadcasting. What are the numbers that appear in the broadcast?

  Students should round and record the temperatures as 65, 65, 68, 72, 76, and 80.

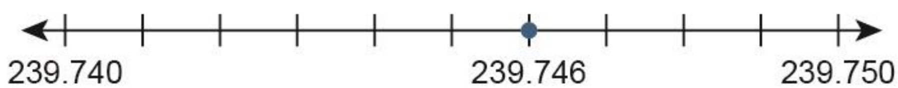
- Ask students to draw a “rounding hill” (pictured) or “rounding triangle” (also pictured) with a certain place-value such as whole numbers or a certain decimal place to help themselves create a visual reminder of when to round up or down. A rounding hill showing rounding to the nearest ten and a rounding triangle showing rounding to the nearest whole number are shown.





► Identifying and Addressing Common Misconceptions

- Students may confuse rounding a decimal number to the nearest ten with rounding to the nearest tenth. Ask students to round 4,975.86 to the nearest tenth. Students who respond with a rounded number of 4,980 or equivalent number may have this misconception. Students could place each digit of the original number into a table that has columns with place-values listed. Students could then reference the tenths place as the place to round to and the hundredths place digit as the deciding digit.
- When looking at a plotted point on a number line, students may round a decimal number based on which interval is greater as opposed to which interval is smaller (or closer to the intended place-value). For example, ask students to use the following number line to round 239.746 to the nearest hundredth.



Some students may round to 239.740 rather than 239.750 because they think they should be looking for the larger interval, not the smaller interval. Discuss with students that the reason for rounding is to make numbers simpler and easier for calculations while still being as accurate as possible. Ask students whether a number that is closer or a number that is farther gives a better estimate. Help them conclude that rounding is done to the closer of the two numbers.

► Enrichment

- Help students investigate and discuss the concept of relative error. For example, ask students to round 870 to the nearest hundred, 8.7 to the nearest whole, and 0.87 to the nearest tenth. Discuss the similarities in the procedures. Then compare the difference between each number and its rounded value (30, 0.3, and 0.03), noting that even though each number is essentially being rounded in the same way, the greater numbers have greater differences and, therefore, would have greater errors in performing calculations.
- Ask students to round a decimal number to intervals other than powers of 10. For example, ask students to round 32.83 to the nearest 0.25.

- Ask students to investigate how operations affect rounding. For example, note to students that rounding 3.2 to the nearest whole number results in 3. Then ask students to add 4 to both 3.2 and 3 and examine whether the first sum (3.2 + 4 = 7.2) rounds to the second sum (3 + 4 = 7). Then ask students to multiply both 3.2 and 3 by 4 and examine whether the first product (3.2 × 4 = 12.8) rounds to the second product (3 × 4 = 12). Discuss why addition and multiplication give different rounding results.

► Key Terms

decimal, hundredths, number line, place-value, round, tenths, thousandths

A-T.2.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Use whole numbers and decimals to compute accurately (straight computation or word problems).

► Eligible Content

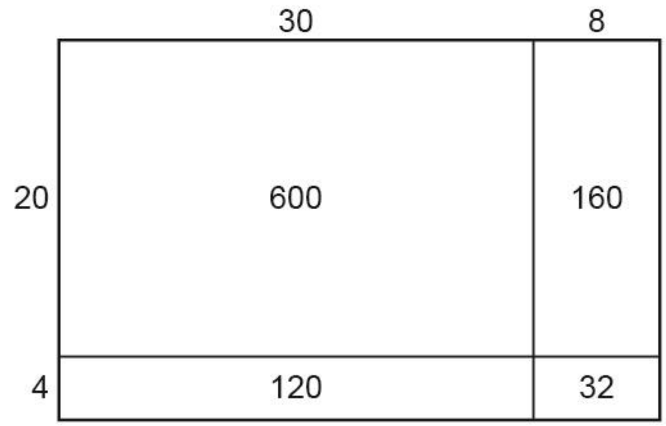
A-T.2.1.1 Multiply multi-digit whole numbers (not to exceed 3-digit by 3-digit).

► Instructional Supports

- Using the standard algorithm, ask students to find the product of two given numbers. Then ask students to relate the standard algorithm to the process of using an area model. For example, ask students to multiply 24 and 38. The students may perform either of the calculations below.

$$\begin{array}{r} 1 \\ \cancel{3} \\ 24 \\ \times\ 38 \\ \hline 192 \\ +\ 720 \\ \hline 912 \end{array} \qquad\qquad \begin{array}{r} 1 \\ \cancel{3} \\ 38 \\ \times\ 24 \\ \hline 152 \\ +\ 760 \\ \hline 912 \end{array}$$

Students then make an area model to find the product of the given numbers.

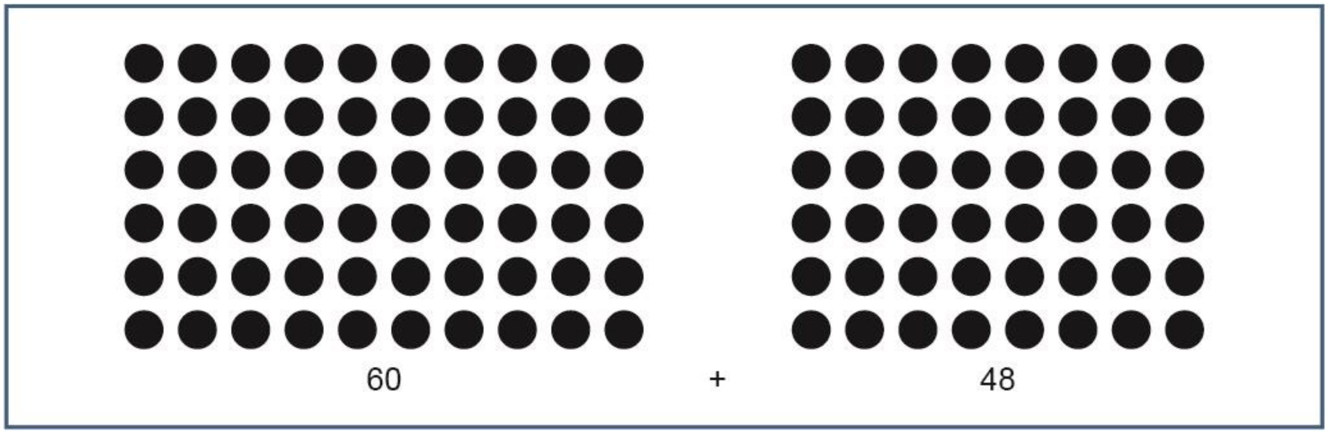


Students match the multiplication steps in their work from the standard algorithm with their area model. Ask students to discuss how the two methods relate. Guide students to see that the sum of 192 and 720 is the same as the sum of (160 + 32) and (600 + 120) from the vertical addition of the boxes. Also, guide students to see that the sum of 152 and 760 is the same as the sum of (120 + 32) and (600 + 160) from the horizontal addition of the boxes. Students should realize that each method requires finding the sum of the partial products 600, 160, 120, and 32 to find the overall product of 24 and 38.

- Ask students to use the distributive property and the standard algorithm to find the product of two given numbers. Students then model the partial products using an array. For example, give students 6 × 18, which can be decomposed to 6 × (10 + 8). Students then apply the distributive property to create the equation 60 + 48 = 108. Next, students find the product of 6 and 18 using the standard algorithm, as shown.

$$\begin{array}{r} {}^{4} \\ 18 \\ \times \quad 6 \\ \hline 108 \end{array}$$

Then, ask students to represent 6 × 18 by creating an array that represents the expression 6 × (10 + 8).



Discuss the similarities between the three methods: the distributive property, the standard algorithm, and the array. Guide students to recognize that all three methods involve finding the sum of 60 and 48.

► Identifying and Addressing Common Misconceptions

- Students may treat each digit as a number of ones in the standard algorithm for multiplication. Ask students to multiply 36 by 45 using the standard algorithm. Students that have partial products of 30, 15, 24, and 12 with an overall product of 81 may have this misconception. Help students recognize that the digit 3 in 36 and the digit 4 in 45 are both in the tens place of each number and have values of 30 and 40, respectively. Then discuss how this changes students' work by shifting the places of the digits so their values represent the true values of the partial products. Ask students to try more multiplication problems and look for patterns in which partial products need to have place-values shifted. Once a pattern is found, help students connect the pattern to the zero placed in the ones place of the partial product in the standard algorithm.
- Students may not know the reason for the zero in the second line of partial products and may omit the zero in the ones place of the second partial product. Ask students to calculate 20 × 56 using the standard algorithm. Students that have partial products of 120 and 100 may have this misconception. Ask students whether 20 × 5 or 20 × 50 is a more reasonable estimate of the product. Help students recognize that the latter is the better estimate because it has factors closer to the original factors. Then ask if 20 × 56 is greater or less than 20 × 50. Because the product of 20 × 50 has four digits, 20 × 56 will have at least four digits in its product as well. Show the students that when 20 × 50 is multiplied using the standard algorithm, the digit 5 in the second

factor represents 50. A zero is placed in the ones place of the second partial product since there is no value for the ones place of the second partial product. Have students start with a zero in the ones place of the second partial product before they start multiplying and explain why this is necessary.

- When using the standard algorithm for multiplication, students may start correctly but introduce an error when multiplying numbers with the tens digit of the second factor. For example, they may not regroup correctly from the ones place to the tens place because there is already a number present from the first multiplication performed. Students may incorrectly use the same regrouped value twice. Ask students to calculate 76 × 48. Students that have an 8 in the hundreds place of their product may have this misconception. A sample student process with this misconception is shown.

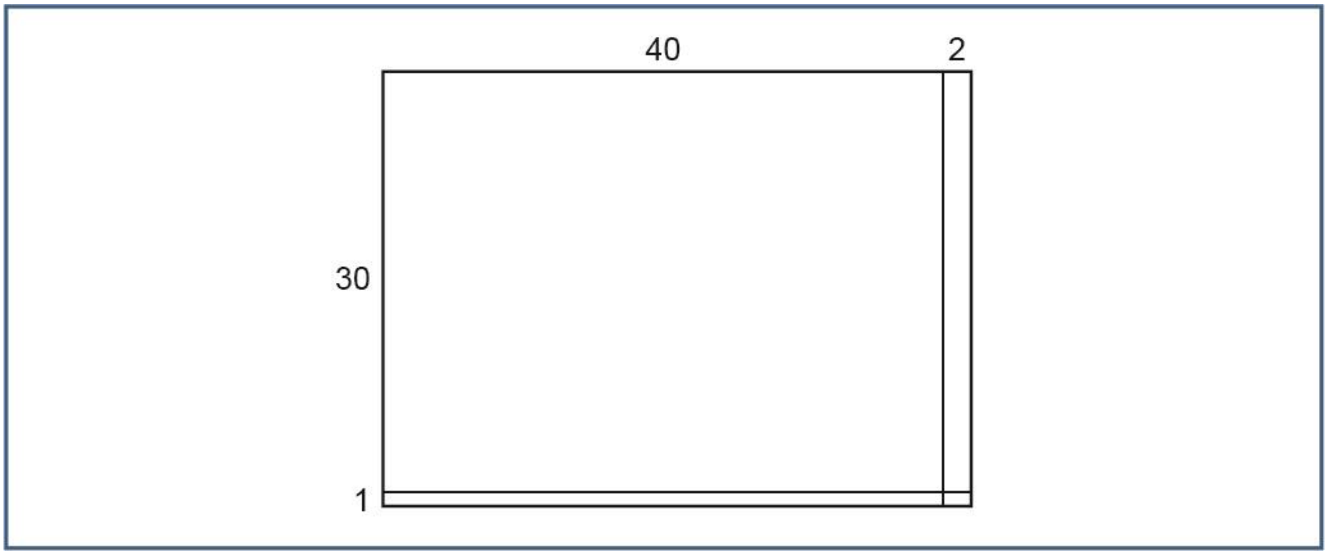
$$\begin{array}{r} {}^{4} \\ 76 \\ \times\ 48 \\ \hline 608 \\ +\ 3{,}240 \\ \hline 3{,}848 \end{array}$$

Have students with this misconception cross off any carried values after using those values. In the example shown, the 4 gets crossed out after it is used in the first partial product and a 2 would be placed in that location for the second partial product.

- Students may use the standard algorithm for multiplying by using each column as a separate single-digit multiplication. Ask students to calculate 42 × 31. An example of this misconception is shown.

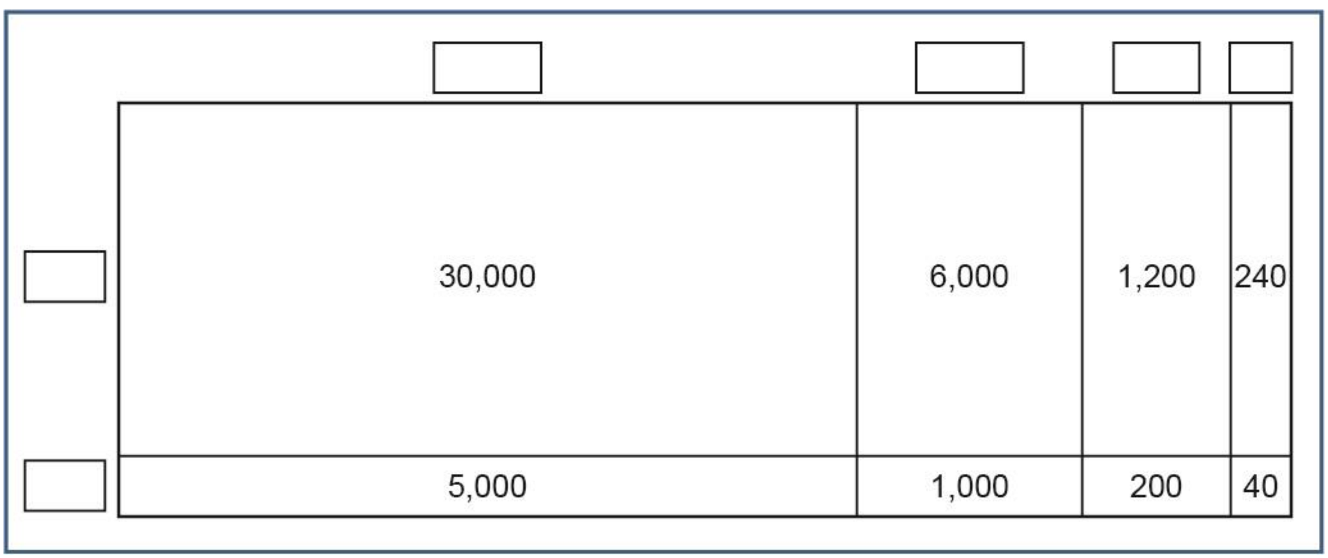
$$\begin{array}{r} 42 \\ \times\ 31 \\ \hline 122 \end{array}$$

Students with this misconception may only multiply 4 × 3 and 2 × 1, resulting in 12 and 2 (or 122). Have students with this misconception use an area model so that they see all partial products of the overall product.



► Enrichment

- Ask students to find the product of two three-digit numbers using multiple methods for multiplication and explain differences between the methods. For example, ask students to solve 246 × 123 using the standard algorithm for multiplication and using an area model. Ask students to explain similarities and differences of the two methods.
- Given an area model with partial products, ask students to find the two-digit and four-digit factors and connect the partial products in the area model with the partial products found using the standard algorithm. For example, give students the following area model.



Students can use various methods to find that the figure models the product 1,248 × 35.

- Ask students to find the missing digits in a multiplication problem involving the standard algorithm. For example, give students the following partial multiplication problem.

$$\begin{array}{r} \square\square\;\; \\ 3\square 7 \\ \times \qquad 6 \\ \hline 2{,}08\square \end{array}$$

► Key Terms

area model, array, distributive property, multi-digit, multiply, partial products, standard algorithm

A-T.2.1.2

► Standard

Numbers and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Use whole numbers and decimals to compute accurately (straight computation or word problems).

► Eligible Content

A-T.2.1.2 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.

► Instructional Supports

- Give students a four-digit dividend and a two-digit divisor and ask them to decompose the dividend into expanded form to help find partial quotients. For example, give students a dividend of 6,690 and a divisor of 30 and then decompose 6,690 into 6,000 + 600 + 90. Next, students divide each term of the decomposition by the divisor, 30.

  $$(6{,}000 \div 30) + (600 \div 30) + (90 \div 30)$$

  $$200 + 20 + 3$$

  Students then find the sum of the partial quotients: 200 + 20 + 3 = 223. Therefore, 6,690 ÷ 30 = 223. Ask students to represent the division with an area model and then explain their models to a classmate.

- Give students a division problem with a four-digit dividend and a two-digit divisor and ask them to rewrite the division problem as a related multiplication problem. For example, give students the division problem 1,350 ÷ 30 = ?. Students then write the related multiplication problem: ? × 30 = 1,350. Students may use skip-counting, as shown, or other like strategies to aid in solving the multiplication problem.

Skip-Count	30	60	90	120	150	. . .
Groups of 30	1	2	3	4	5	. . .

Due to the large dividend, students may find it helpful to skip-count by greater intervals.

Skip-Count	150	300	450	600	750	900	1,050	1,200	1,350
Groups of 30	5	10	15	20	25	30	35	40	45

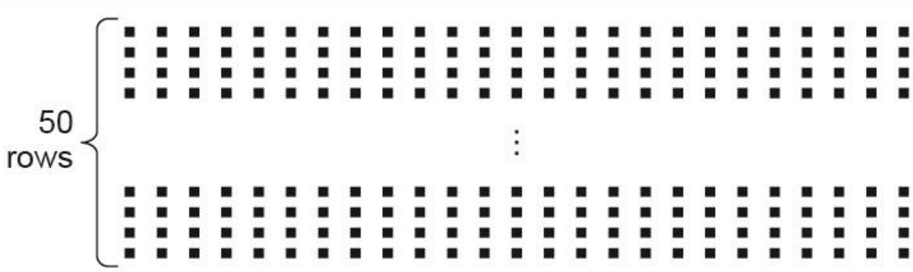
Students should determine that there are 45 groups of 30 in 1,350; therefore, 1,340 ÷ 30 = 45.

- Ask students to describe a given division area model and create the corresponding division equation. For example, give students the following model.

	100	50	2
	6,384	2,184	84
42	– 4,200	– 2,100	– 84
	2,184	84	0

The dividend of 6,384 can be divided into 100 groups of 42 with 2,184 remaining. The remaining 2,184 can be divided into 50 groups of 42 with 84 remaining. The last 84 can be divided into 2 groups of 42 with no remainder. The corresponding division equation for the division area model is 6,384 ÷ 42 = 152.

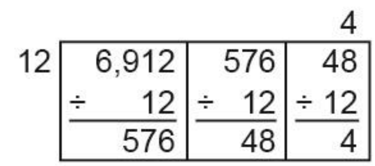
- Ask students to explain how an array can model a given real-world division problem. For example, give students this problem: “There are 1,250 seats in an auditorium. The seating is arranged in rows of 25 seats each for a concert. How many rows of seats are available for the concert?” A partial array to represent this example is shown.



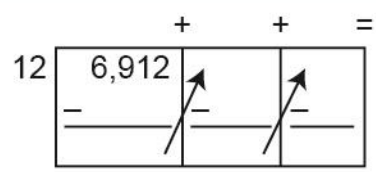
Explain how each part of the problem is modeled in the array. For example, the array includes 1,250 squares to represent all the seats in the auditorium. There are 25 squares that go from left to right in each row to represent the number of seats in each row in the auditorium, and there are 50 rows in the auditorium.

▶ Identifying and Addressing Common Misconceptions

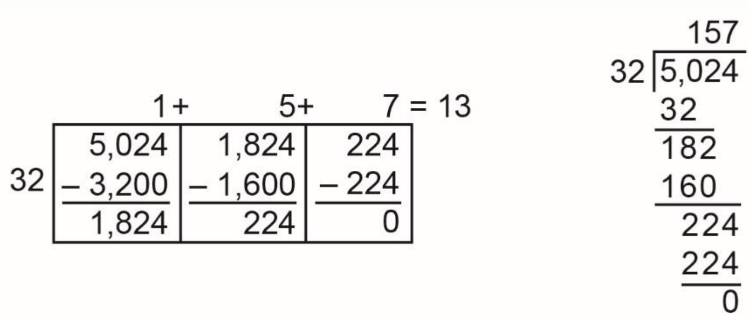
- When using a division area model, some students may think that division is the only operation used. Ask students to divide 6,912 by 12 using an area model. Students who give an answer of 4 may have this misconception and may make a model like the one shown.



Before starting the problem, have students set up the interior of their area model for subtraction, the portion above the boxes for addition of the partial quotients, and the portion to the outer right for any potential remainder of the quotient. Remind students that another way to think of division is as repeated subtraction.



- When using an area model, some students may write the partial quotients as single digits. Ask students to divide 5,024 by 32 using an area model. Students that give an answer of 13 may be confusing area models with the standard algorithm and write single digits instead of the entire value as shown in the example.



Ask students to explain how the values 3,200, 1,600, and 224 in the model were found. Remind students that to calculate those values, the partial quotients need to include the place-values, not just the digits. The place-values are not needed in the standard algorithm because the place-value is reflected in where the digits are written.

► Enrichment

- Give students a start time and ask them to identify the time after a given amount of time has passed. For example, ask students to find what time and date it was exactly 2,000 minutes after the start of the new year in the year 2000 (January 1, 2000, at 12:00 A.M.).
- Ask students to divide a five-digit number by a three-digit number using an area model.
- Give students a division problem and ask them to find other division problems with the same quotient. Encourage students to either divide both the dividend and the divisor by a common factor or multiply both the dividend and the divisor by the same number.
- Ask students to create division problems in which the quotient meets given requirements. For example, ask students to find a four-digit even number and a two-digit odd number that will have a quotient with a tens digit of zero and a remainder of five.

► Key Terms

area model, decompose, divide, dividend, divisor, equation, multiple, multiply, partial quotients, quotient, remainder

A-T.2.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

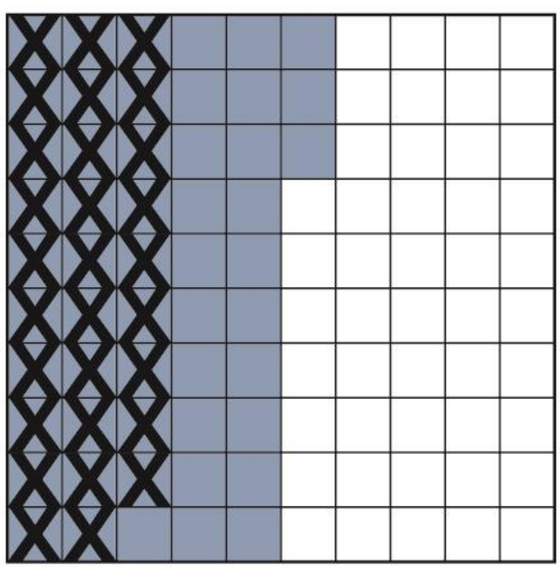
Use whole numbers and decimals to compute accurately (straight computation or word problems).

► Eligible Content

A-T.2.1.3 Add, subtract, multiply, and divide decimals to hundredths (no divisors with decimals).

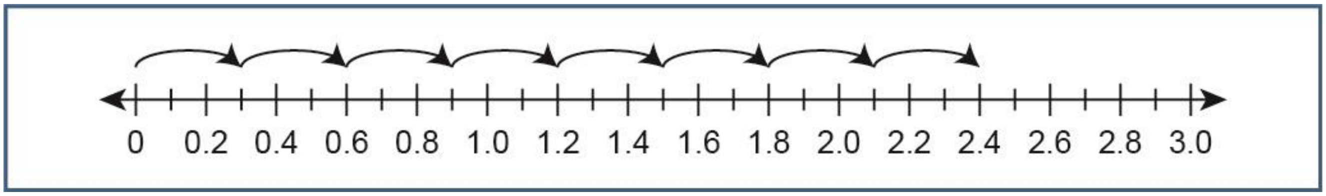
► Instructional Supports

- Ask students to compare the process of adding or subtracting two-digit whole numbers to the process of performing the same operation with two-digit decimal numbers. For example, ask students to compare methods of finding the difference between 53 and 29 with methods of finding the difference between 0.53 and 0.29. Students can calculate the first difference of 24 using any method. Next, have students use a hundreds grid to find the difference between 0.53 and 0.29 by shading 53 squares (to represent 53 hundredths) and then crossing out 29 (to represent subtracting 29 hundredths).

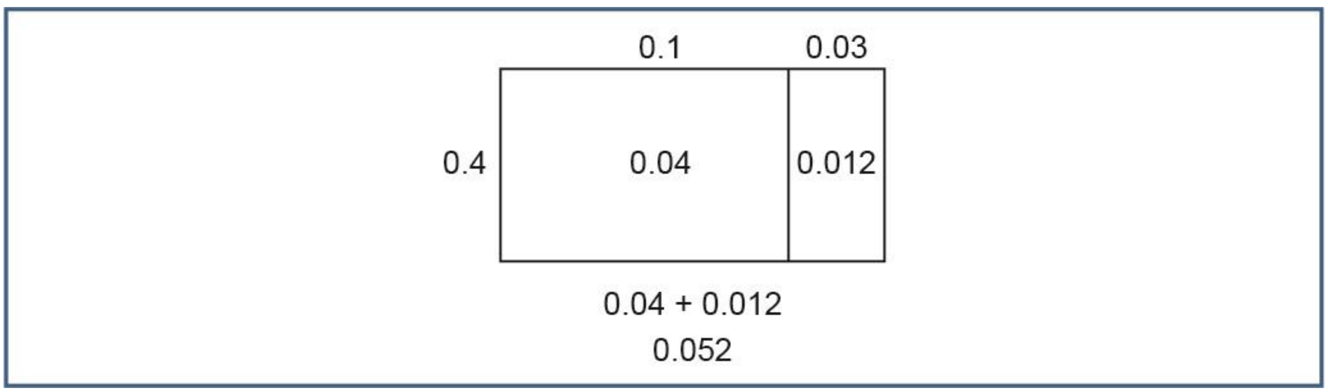


Students should notice the similarity of 53 – 29 = 24 and 0.53 – 0.29 = 0.24. Ask students to repeat with other numbers and other strategies.

- Ask students to find the product of a decimal number and a whole number using repeated addition. For example, give students the multiplication problem 0.3 × 8. Ask students to create a number line with tick marks representing every tenth. Next, ask students to skip-count 3 tick marks eight times and note the location of the product 2.4 on the number line.



- Ask students to find the product of two decimal numbers using two or more methods. For example, give students the product 0.13 × 0.4 and ask them to find the product by using an area model and by using properties of multiplication. Two possible solutions are shown.



$$0.13 \times 0.4$$

$$(13 \times \frac{1}{100}) \times (4 \times \frac{1}{10})$$

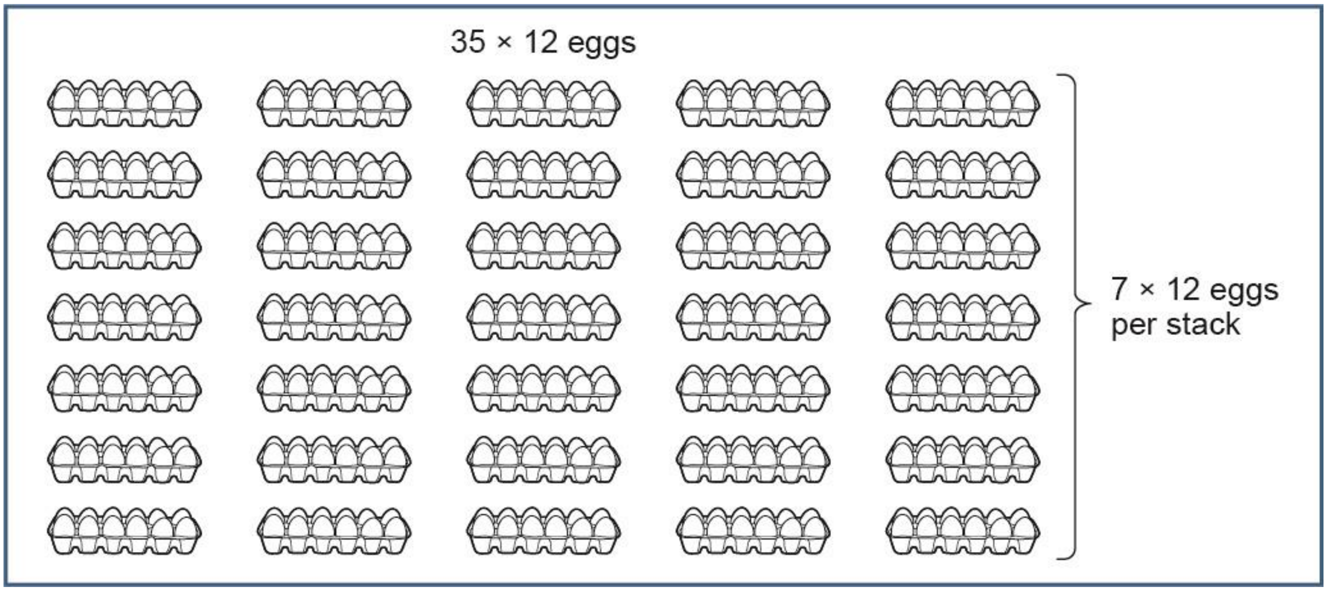
$$(13 \times 4) \times (\frac{1}{100} \times \frac{1}{10})$$

$$52 \times \frac{1}{1{,}000}$$

$$0.052$$

Students should note that both methods result in the same product of 0.052.

- Ask students to reason about the meaning of division to conclude that multiplying both a dividend and a divisor by the same number yields the same quotient. For example, give students the quotient 35 ÷ 7 and ask them to explain why multiplying both 35 and 7 by 12 results in the same quotient. The problem 35 ÷ 7 can be modeled using the situation of a student putting 35 cardboard cartons into stacks of 7 cartons each. This can be done with 5 stacks of 7 cartons. Multiplying each number by 12 is equivalent to imagining that each cardboard carton is a carton of a dozen eggs. The problem is now modeled by putting 35 × 12 eggs into stacks of 7 × 12 eggs each.



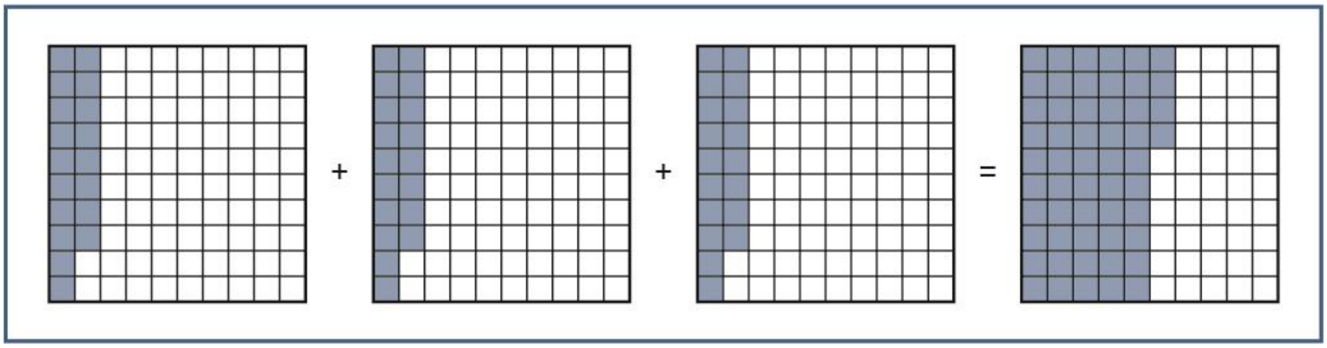
  Since this is the exact amount of eggs that are in the original stacks of cartons, this also can be done with 5 stacks. Therefore, the quotient is still 5.

- Ask students to connect decimal number division with whole number division. For example, give students the quotient 26.4 ÷ 0.12 and ask them to multiply both the dividend and divisor by a multiple of 10 to create a division problem involving only whole numbers. Some possible attempts are shown.

Multiply dividend and divisor by	Resulting equation
10	264 ÷ 1.2
100	2,640 ÷ 12

Because multiplying both the dividend and the divisor of a division problem does not change the quotient, the quotient of 26.4 ÷ 0.12 is the same as the quotient of 2,640 ÷ 12.

- Give students a model of an operation on decimal numbers and ask them to create an equation for the model. For example, give students the following figure.



Students should write 0.18 + 0.18 + 0.18 = 0.54 or 3 × 0.18 = 0.54 to represent the model.

► Identifying and Addressing Common Misconceptions

- Some students may add or subtract numbers without paying attention to the place-values of the digits. Ask students to solve 5.26 + 0.3. Some students may respond with a sum of 5.29. Ask students to place each digit of the original numbers into a table that has columns with place-values listed. Students then add the numbers in each column.

- Some students may align the decimal in their product with the decimals in the factors when multiplying, just as they do with addition. Ask students to solve 1.1 × 3.4. Some students may notice that both factors have digits in the tenths place and conclude that the product should also be a decimal number that ends in the tenths place, resulting in an answer of 37.4. Ask students to compare the product of 1.1 × 3.4 to the product of 2 × 4 and note that the product of 2 × 4 should be greater. Remind students that multiplying 3 tens by 2 tens does not result in a product of 6 tens but rather 6 hundreds. Similarly, multiplying two numbers with tenths does not result in a product with tenths but rather with hundredths. For example, 9 tenths times 9 tenths results in 81 hundredths.

- Some students may think that they can multiply each side of a decimal by a whole number separately. For example, ask students to solve 3.2 × 8. Some students may solve 3 × 8 and 2 × 8 separately to arrive at a product of 24.16. Ask students to multiply the numbers as 32×8 using the standard algorithm and then adjust the product for 3.2×8 by recognizing that the value of 3.2×8 is $\frac{1}{10}$ as great as the value of 32×8.

► Enrichment

- Ask students to add, subtract, multiply, or divide two decimals that each extend to the thousandths place in a real-world context.
- Ask students to add two numbers using expanded form. For example, give students the numbers 1.67 and 3.85. The first number can be written as $1 \times 1 + 6 \times \frac{1}{10} + 7 \times \frac{1}{100}$, and the second number can be written as $3 \times 1 + 8 \times \frac{1}{10} + 5 \times \frac{1}{100}$. Following the order of operations, and then rearranging the products, the sum of the two numbers can be written as $1 + 3 + \frac{6}{10} + \frac{8}{10} + \frac{7}{100} + \frac{5}{100}$. Adding like terms leads to the regrouping of hundredths and tenths for a final sum of $5 + \frac{5}{10} + \frac{2}{100}$ or 5.52.

► Key Terms

add, decimal, divide, hundredths, multiply, operation, subtract, tenths

B-O.1.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Write and interpret numerical expressions.

Analyze and complete calculations by applying the order of operations.

▶ Eligible Content

B-O.1.1.1 Use multiple grouping symbols (parentheses, brackets, or braces) in numerical expressions, and evaluate expressions containing these symbols.

▶ Instructional Supports

- Ask students to determine the order in which the operations of a given expression should be performed. For example, give students the expression $8 \times (6 - 4) \div 2$. The parentheses indicate that subtraction should be done first. Because there are no other grouping symbols to indicate whether the multiplication or the division should be done next, the order of operations dictates that they be done from left to right. Therefore, the order is: subtraction, multiplication, division.
- Ask students to show the steps used to evaluate expressions by using the order of operations. For example, give students the expression $[6 + (8 - 5) \times 2] \times 3$. A possible student response is shown.

$$[6 + (3) \times 2] \times 3$$

$$[6 + 6] \times 3$$

$$12 \times 3$$

$$36$$

- Ask students to evaluate an expression multiple times with different sets of grouping symbols. For example, give students the expression $16 + 8 \div 4 \times 6 - 1$ and have them evaluate the expression with different groupings to show how the parentheses change the value. Some possible responses are shown.

$16 + 8 \div 4 \times 6 - 1$ $16 + 2 \times 6 - 1$ $16 + 12 - 1$ $28 - 1$ 27	$(16 + 8) \div 4 \times 6 - 1$ $24 \div 4 \times 6 - 1$ $6 \times 6 - 1$ $36 - 1$ 35
$(16 + 8) \div (4 \times 6) - 1$ $24 \div (4 \times 6) - 1$ $24 \div 24 - 1$ $1 - 1$ 0	$(16 + 8 \div 4) \times 6 - 1$ $(16 + 2) \times 6 - 1$ $18 \times 6 - 1$ $108 - 1$ 107

- Give students an expression and ask them to insert grouping symbols to create a target value. For example, give students the expression $7 \times 4 + 8 \div 2$ and ask them to insert grouping symbols so that the expression has a value of 56. A possible student response is shown.

$$7 \times (4 + 8 \div 2)$$

$$7 \times (4 + 4)$$

$$7 \times 8$$

$$56$$

- Give students an expression and ask them to insert grouping symbols so that the operations are executed in a specified order. For example, give students the expression $8 + 3 \times 2 \div 4 - 1$ and ask them to insert grouping symbols so that the multiplication and subtraction are performed first, followed by the division and then the addition. One possible solution is $8 + (3 \times 2) \div (4 - 1)$. Note that in this example, parentheses are not actually necessary to make multiplication the first step.

► Identifying and Addressing Common Misconceptions

- Some students may interpret the order of operations as indicating that multiplication must come before division and addition must come before subtraction. Ask students to evaluate the expressions $8 \div 4 \times 2$ and $9 - 3 + 1$. Some students may give the answers 1 and 5. Help students understand that multiplication and division are a pair and are performed together from left to right with neither taking priority over the other. Similarly, addition and subtraction are also paired with neither taking priority over the other.
- Some students may not recognize that the order of operations also applies when evaluating expressions within grouping symbols. Ask students to evaluate the expression $3 + (9 - 2 \times 2)$. Some students may subtract before multiplying and arrive at the value of 17. Ask students to evaluate $9 - 2 \times 2$. Help them recognize that the order of operations dictates that the multiplication be performed before the subtraction and that this order still applies when the expression occurs inside the grouping symbols of the original expression.

► Enrichment

- Give students expressions without parentheses and ask students to determine which combinations of parentheses would and would not change the value of the expression.
- Ask students to use grouping symbols to find the greatest and the least possible values for an expression. For example, give students the expression $10 \times 9 + 8 - 7$ and have them determine where to insert parentheses in order to create the greatest and least possible values. Note that in this example, parentheses are not actually necessary to create the least possible value.

  - $10 \times (9 + 8) - 7 = 163$

  - $10 \times 9 + (8 - 7) = 91$

- Ask students to use parentheses to indicate multiplication while also using them as grouping symbols. For example, ask students to rewrite $3 \times (5 + 8)$ as $3(5 + 8)$. The parentheses indicate multiplication by the lack of an operation symbol. However, in the expression $(10 + 2) \div 3$, the parentheses indicate the order of operations. Students can practice writing expressions using parentheses instead of a multiplication symbol, i.e., students could rewrite $4 \times (7 + 2)$ as $4(7 + 2)$.

► Key Terms

braces, brackets, equations, evaluate, expressions, grouping symbols, order of operations, parentheses

B-O.1.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Write and interpret numerical expressions.

Analyze and complete calculations by applying the order of operations.

► Eligible Content

B-O.1.1.2 Write simple expressions that model calculations with numbers, and interpret numerical expressions without evaluating them.

► Instructional Supports

- Ask students to write mathematical expressions from verbal descriptions. For example, give students the following descriptions.
    - nine less than the product of two and seven

      $(2 \times 7) - 9$

    - the sum of 8 and 4 divided by the sum of 1 and 2

      $(8 + 4) \div (1 + 2)$

- Ask students to write verbal descriptions of mathematical expressions. For example, give students the expression (33 – 12) ÷ 5. Students can describe this as "the difference of thirty-three and twelve, divided by 5" or "the quotient of the difference of 33 and 12, and 5."
- Ask students to identify key words and phrases in verbal descriptions or contexts, identify the mathematical operations that are indicated, and write expressions to represent the situations. For example, give students the description "A 5-ounce glass of water and an 11-ounce glass of water are combined and then poured evenly into 2 new glasses." Students should recognize that, in this

context, “combined” indicates addition and “poured evenly” indicates division. The resulting mathematical expression is $(5 + 11) \div 2$.

- Ask students to compare two similar verbal descriptions and explain the difference in the mathematical expressions they describe. For example, give students these descriptions: “the sum of forty-five and one, times five” and “the sum of forty-five and the quantity, one times five.” Just as parentheses can be used to indicate groups of operations, commas and phrases such as “the quantity” can be used to indicate groups of operations. The first description contains the “sum of forty-five and one” together on one side of the comma, indicating that the addition should be grouped together as in the expression $(45 + 1) \times 5$. The second description contains “one times five” separated by a comma, indicating that the multiplication should be grouped together as in the expression $45 + (1 \times 5)$.
- Ask students to make comparisons between the values of expressions without using calculations. For example, give students the expressions $3 + (4 \times 1)$ and $5 + (4 \times 1)$. Students may reason that the value of the second expression is greater than the value of the first because adding 5 results in a greater sum than adding 3 to the same quantity. As an additional example, the expressions $(12 - 8) \times \frac{1}{4}$ and $(12 - 8) \div 4$ are equivalent because multiplying by $\frac{1}{4}$ is equivalent to division by 4.
- Ask students to describe the difference in the values of two expressions. For example, give students the expressions $(8 + 12) \times 2$ and $(8 + 12)$. The expression $(8 + 12) \times 2$ has a value that is twice as great as the value of $(8 + 12)$. As an additional example, the value of the expression $(18 - 7) + 9$ is five more than the value of $(18 - 7) + 4$.

► Identifying and Addressing Common Misconceptions

- Some students may not be able to compare the values of expressions without calculating. Ask students to determine whether (4 + __) or 2 × (4 + __) is greater when __ represents the same unknown number in each expression. (If students have already learned negative numbers, specify that the missing value is positive.) Some students may think that it is not possible to determine which expression has a greater value. Ask students to identify whether multiplying a (positive) number by 2 increases or decreases the value. Then help them recognize that because the

(4 + __) is identical in both expressions, the two expressions are comparing a number with that same number doubled.

- Some students may not recognize how the use of punctuation translates into mathematical expressions. Ask students to write a mathematical statement for both “the sum of ten and six divided by two” and “the sum of ten, and six divided by two.” Some students may write the same expression for both descriptions. Remind them that commas are often used to separate different parts or concepts.

► Enrichment

- Ask students to create multiple mathematical representations of verbal descriptions. For example, give students the expression “the sum of 13 and 7, divided by 2.” Some possible mathematical expressions include $(13 + 7) \div 2$, $\frac{13+7}{2}$, $\frac{1}{2}(13 + 7)$, $(13 + 7) \times \frac{1}{2}$, $0.5(13 + 7)$, and $(13 + 7) \times 0.5$.
- Ask students to write expressions requiring nested grouping symbols when given a verbal description.
- Give students an expression and ask them to write a new expression with a value that is twice as great and another expression with a value that is greater by two. Ask students to explain why one expression may require adding a new set of parentheses and the other does not.

► Key Terms

calculate, compare, evaluate, expression, interpret, reason

B-O.2.1.1 & B-O.2.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Analyze patterns and relationships.

Create, extend, and analyze patterns.

► Eligible Content

B-O.2.1.1 Generate numerical patterns using given rules.

B-O.2.1.2 Identify apparent relationships between corresponding terms of two patterns with the same starting numbers that follow different rules.

► Instructional Supports

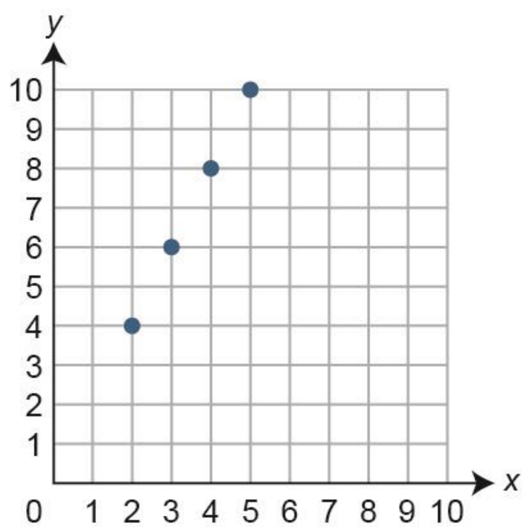
- Ask students to generate a numeric pattern from a given starting number and a rule. For example, give students the starting number of 2 and the rule "add 5." Students write the sequence 2, 7, 12, 17, . . . and describe the pattern in words as "the first number is 2, and each number is 5 more than the one before it."
- Give students descriptions of two numeric sequences and ask them to create ordered pairs using the corresponding terms of the sequences. For example, give students a sequence that starts at 0 and follows the rule "add 1" and a sequence that starts at 3 and follows the rule "add 3." Students determine the sequences to be 0, 1, 2, 3, . . . and 3, 6, 9, 12, ... Students can then use these sequences to create a table.

x	y
0	3
1	6
2	9
3	12

Students should recognize that both the table and the sequences represent the ordered pairs (0, 3), (1, 6), (2, 9), (3, 12), ... Note that a different set of ordered pairs could also be created by reversing which sequence is used for the *x*-coordinates and which is used for the *y*-coordinates.

- Give students two sequences and ask them to graph the sequences by creating ordered pairs from corresponding terms of the sequences. For example, give students the following descriptions of sequences.
    - starts at 2 and follows the rule "add 1"
    - starts at 4 and follows the rule "add 2"

  Ask students to write the sequences as 2, 3, 4, 5, . . . and 4, 6, 8, 10, ... Students using the numbers from the first sequence as the *x*-values create the following graph.



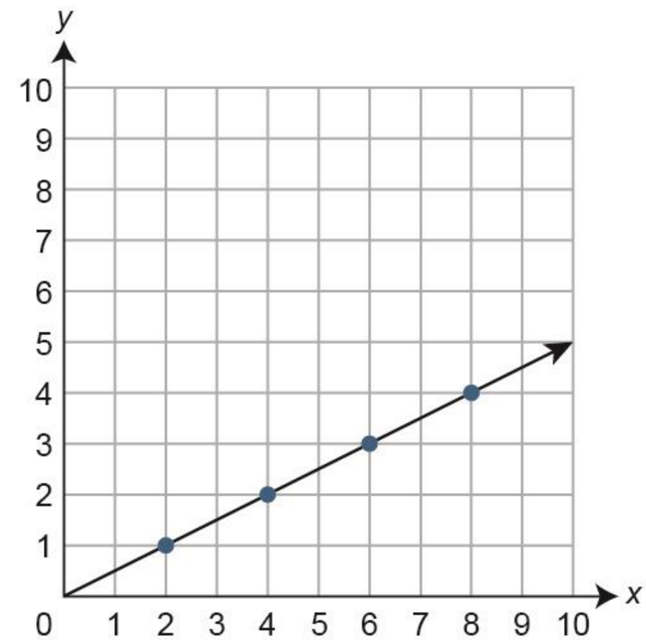
- Give students two sequences and ask them to find relationships between the corresponding terms of the sequences. For example, give students a sequence that starts with 2 and follows the rule "add 1" and a sequence that starts with 3 and follows the rule "add 2." Ask students to write the sequences as 2, 3, 4, 5, . . . and 3, 5, 7, 9, ... Help students recognize that each number in the second sequence is one less than twice the corresponding number in the first sequence.

▶ Identifying and Addressing Common Misconceptions

- Some students may confuse x- and y-values when plotting points on the coordinate grid. Ask students to plot the point (2, 3). Some students may plot (3, 2). Discuss ways for students to recall which coordinate is first, such as, "x is before y in the alphabet, so x is first in an ordered pair" or "y goes to the sky."
- Some students may assume that sequences must start with 0 or 1. Give students two rules and ask them to write the corresponding sequences and plot the points. Some students may either ignore the starting values and start with (0, 0) or (1, 1), while others may append (0, 0) or (1, 1) before different given starting values. Show students a sequence that does not start with 0 or 1 and ask them to describe the sequence. Then help them apply the same reasoning with the two given rules.

▶ Enrichment

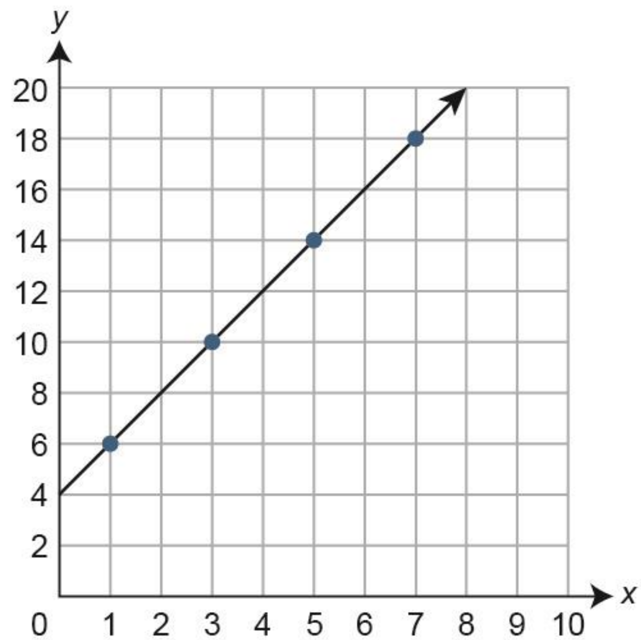
- Give students two sequences, ask them to plot the first few pairs of corresponding points on a graph, and ask them to use the graph to answer questions about the sequences. For example, give students the sequences 2, 4, 6, 8, . . . and 1, 2, 3, 4, . . . and ask them to make the following graph.



Ask them to observe patterns in the graph and answer the following questions.

- By extending the line created by connecting the points, what other pairs of numbers should occur in the sequences?
- Is the point (10, 10) on the line? How can you tell from the sequences?
- Would the numbers 12 and 8 occur as corresponding terms in the sequences? How can you tell from the graph?

- Give students two sequences and a graph created from those sequences, then ask students to use the graph to find another pair of sequences, with different numbers, that would have the same graph. For example, give students the following graph.



The plotted points suggest sequences of 1, 3, 5, 7, . . . and 6, 10, 14, 18, ... However, the sequences of 0, 2, 4, 6, . . . and 4, 8, 12, 16, . . . would also have points that lie on the same line.

- Ask students to investigate how operations on terms of the sequences affect the relationships between the corresponding terms of the sequences. For example, when given two sequences in which the terms of one sequence are always 3 times the corresponding terms of the other sequence, then multiplying all terms of both sequences by 2 results in two sequences with the same relationship as the original pair (the terms of one sequence are 3 times the corresponding terms of the other). However, adding 2 to all terms of both sequences does not preserve the relationship.

► Key Terms

coordinate plane, generate, graph, number pattern, ordered pairs, origin, sequence, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

B-O.2.1.1 & B-O.2.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Analyze patterns and relationships.

Create, extend, and analyze patterns.

► Eligible Content

B-O.2.1.1 Generate numerical patterns using given rules.

B-O.2.1.2 Identify apparent relationships between corresponding terms of two patterns with the same starting numbers that follow different rules.

► Instructional Supports

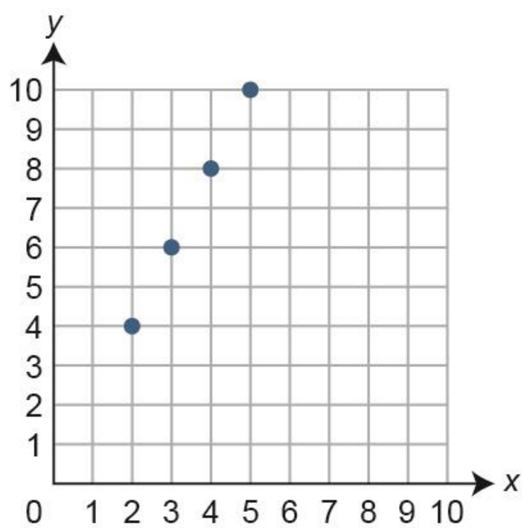
- Ask students to generate a numeric pattern from a given starting number and a rule. For example, give students the starting number of 2 and the rule “add 5.” Students write the sequence 2, 7, 12, 17, . . . and describe the pattern in words as “the first number is 2, and each number is 5 more than the one before it.”
- Give students descriptions of two numeric sequences and ask them to create ordered pairs using the corresponding terms of the sequences. For example, give students a sequence that starts at 0 and follows the rule “add 1” and a sequence that starts at 3 and follows the rule “add 3.” Students determine the sequences to be 0, 1, 2, 3, . . . and 3, 6, 9, 12, ... Students can then use these sequences to create a table.

x	y
0	3
1	6
2	9
3	12

Students should recognize that both the table and the sequences represent the ordered pairs (0, 3), (1, 6), (2, 9), (3, 12), ... Note that a different set of ordered pairs could also be created by reversing which sequence is used for the *x*-coordinates and which is used for the *y*-coordinates.

- Give students two sequences and ask them to graph the sequences by creating ordered pairs from corresponding terms of the sequences. For example, give students the following descriptions of sequences.
  - starts at 2 and follows the rule "add 1"
  - starts at 4 and follows the rule "add 2"

  Ask students to write the sequences as 2, 3, 4, 5, . . . and 4, 6, 8, 10, ... Students using the numbers from the first sequence as the *x*-values create the following graph.



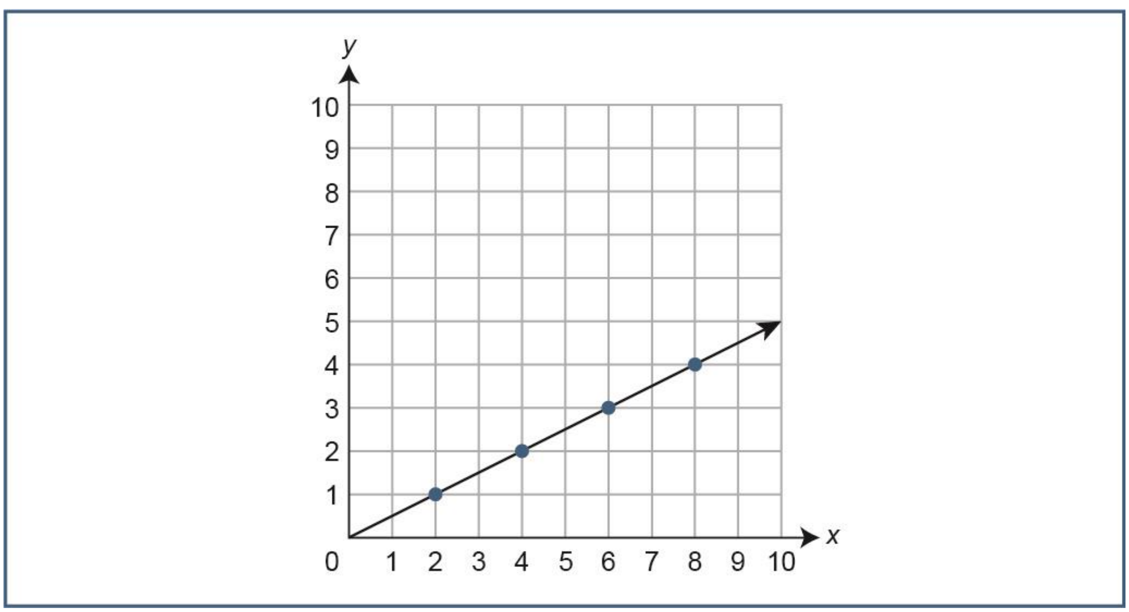
- Give students two sequences and ask them to find relationships between the corresponding terms of the sequences. For example, give students a sequence that starts with 2 and follows the rule "add 1" and a sequence that starts with 3 and follows the rule "add 2." Ask students to write the sequences as 2, 3, 4, 5, . . . and 3, 5, 7, 9, ... Help students recognize that each number in the second sequence is one less than twice the corresponding number in the first sequence.

▶ Identifying and Addressing Common Misconceptions

- Some students may confuse x- and y-values when plotting points on the coordinate grid. Ask students to plot the point (2, 3). Some students may plot (3, 2). Discuss ways for students to recall which coordinate is first, such as, "x is before y in the alphabet, so x is first in an ordered pair" or "y goes to the sky."
- Some students may assume that sequences must start with 0 or 1. Give students two rules and ask them to write the corresponding sequences and plot the points. Some students may either ignore the starting values and start with (0, 0) or (1, 1), while others may append (0, 0) or (1, 1) before different given starting values. Show students a sequence that does not start with 0 or 1 and ask them to describe the sequence. Then help them apply the same reasoning with the two given rules.

▶ Enrichment

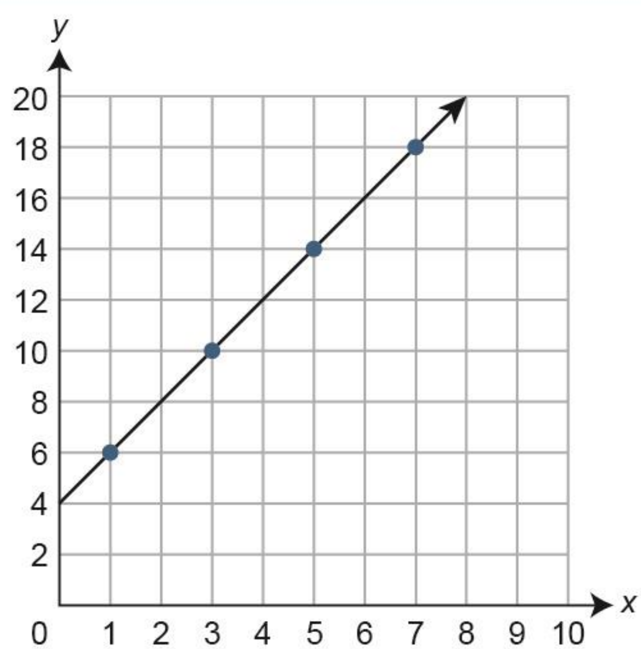
- Give students two sequences, ask them to plot the first few pairs of corresponding points on a graph, and ask them to use the graph to answer questions about the sequences. For example, give students the sequences 2, 4, 6, 8, . . . and 1, 2, 3, 4, . . . and ask them to make the following graph.



Ask them to observe patterns in the graph and answer the following questions.

- By extending the line created by connecting the points, what other pairs of numbers should occur in the sequences?
- Is the point (10, 10) on the line? How can you tell from the sequences?
- Would the numbers 12 and 8 occur as corresponding terms in the sequences? How can you tell from the graph?

- Give students two sequences and a graph created from those sequences, then ask students to use the graph to find another pair of sequences, with different numbers, that would have the same graph. For example, give students the following graph.



The plotted points suggest sequences of 1, 3, 5, 7, . . . and 6, 10, 14, 18, ... However, the sequences of 0, 2, 4, 6, . . . and 4, 8, 12, 16, . . . would also have points that lie on the same line.

- Ask students to investigate how operations on terms of the sequences affect the relationships between the corresponding terms of the sequences. For example, when given two sequences in which the terms of one sequence are always 3 times the corresponding terms of the other sequence, then multiplying all terms of both sequences by 2 results in two sequences with the same relationship as the original pair (the terms of one sequence are 3 times the corresponding terms of the other). However, adding 2 to all terms of both sequences does not preserve the relationship.

► Key Terms

- coordinate plane, generate, graph, number pattern, ordered pairs, origin, sequence, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

C-G.1.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

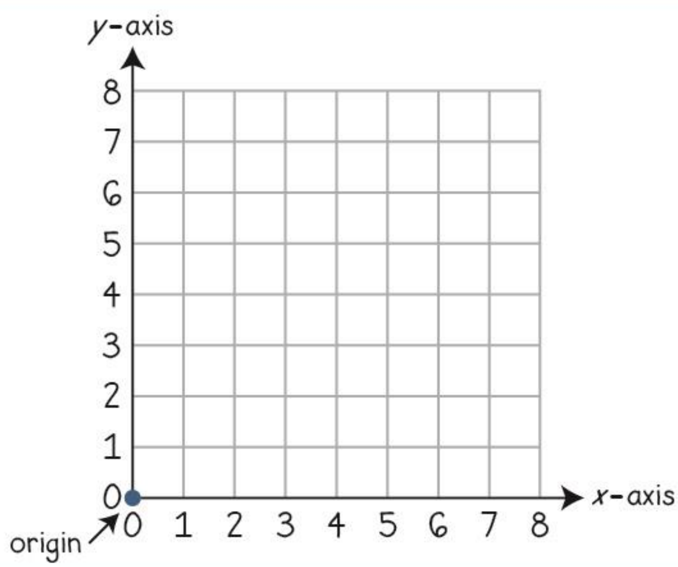
Identify parts of a coordinate grid, and describe or interpret points given an ordered pair.

▶ Eligible Content

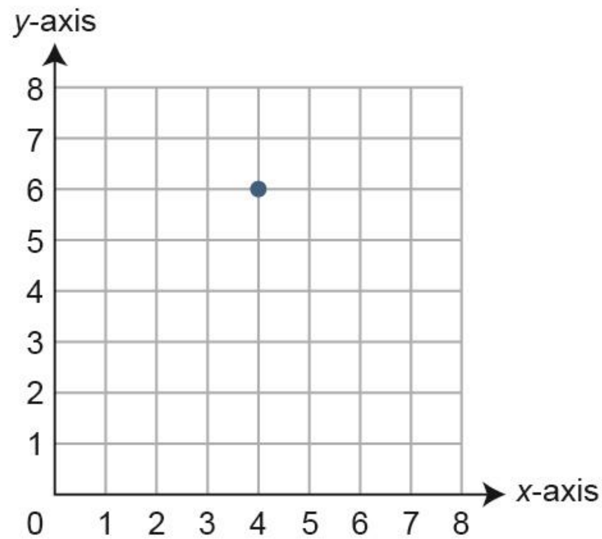
C-G.1.1.1 Identify parts of the coordinate plane (x-axis, y-axis, and the origin) and the ordered pair (x-coordinate and y-coordinate). Limit the coordinate plane to quadrant I.

▶ Instructional Supports

- Give students a blank copy of the first quadrant of a coordinate system and ask them to identify and label the *x*-axis, the *y*-axis, numbers along both axes, and the origin. A possible student response is shown.

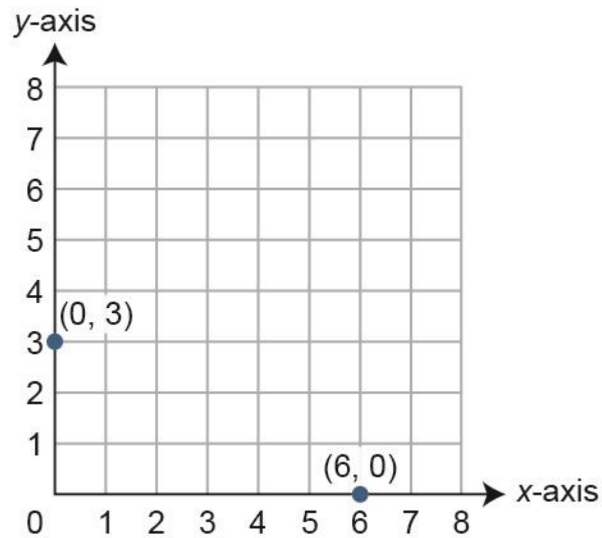


- Give students a point on the coordinate plane and ask them to identify the ordered pair for the point. For example, give students the following graph.



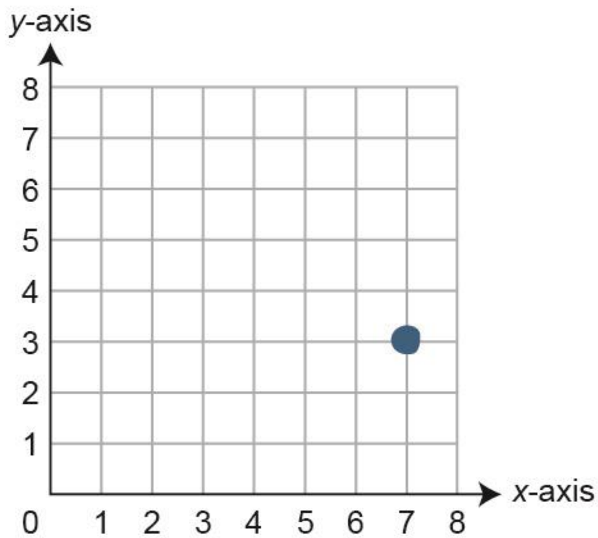
Students start at the origin and move along the *x*-axis until they are directly under the given point. The corresponding label on the *x*-axis, 4, gives the first number in the ordered pair. Then, students move up from this location on the *x*-axis until they reach the given point. Students read the corresponding label on the *y*-axis, 6, to determine the second number in the ordered pair. The coordinates of the point are (4, 6).

- Ask students to plot points on the coordinate plane that are located on either the *x*-axis or the *y*-axis when given instructions for the location or an ordered pair. For example, ask students to plot a point at a location that is six units from the origin along the *x*-axis and label it with its coordinates. Then, ask students to plot a point given the ordered pair (0, 3) and label it with its coordinates. A possible student response is shown.



Ask students to compare the coordinates and discuss similarities and differences between the two ordered pairs and the role that the number 0 plays in each ordered pair.

- Ask students to explain how to graph an ordered pair. For example, give students the point (2, 5). Students may respond by saying that an ordered pair gives "right" and "up" instructions. The two numbers in the ordered pair (2, 5) provide directions on how to reach that point: first by moving "right" 2 units from the origin and then by moving "up" 5 units. If the order of the movement is reversed, the resulting point will not correctly match the ordered pair unless both coordinates are the same.
- Ask students to plot a point on the coordinate plane when given an ordered pair. For example, ask students to plot the ordered pair (7, 3). Students start at the origin and move right seven units along the *x*-axis, then move up three units, and then plot a point to show the location (7, 3).



► Identifying and Addressing Common Misconceptions

- Some students may think that the first number in an ordered pair indicates the number of units to move up (along the y-axis) and the second number in an ordered pair indicates the number of units to move right (along the x-axis). Ask students to plot the point (2, 5). Some students may plot the point (5, 2). Point out that, in the alphabet, the letter x comes before the letter y, and so in an ordered pair, the number corresponding to movement along the x-axis comes before the number corresponding to movement along the y-axis. Additionally, point out that the English language is read from left to right first and from top to bottom second. Likewise, ordered pairs list coordinates horizontally first and vertically second.

- Some students, when graphing more than one point at a time, may start "counting" the location of the second point based on the location of the first rather than the origin. Ask students to plot the point (4, 5) and then the point (1, 2). Some students may plot (4, 5) and (5, 7). Remind students that, when graphing each point, they should begin counting spaces from the origin, not from any other point on the graph.

- Some students may think that a point on the x-axis should have an x-coordinate of 0 and a point on the y-axis should have a y-coordinate of 0. Ask students to identify the coordinates of a point that lies 4 units above the origin on the y-axis. Some students may say the coordinates are (4, 0). Show students a graph with points plotted at (5, 1) and (1, 5). Then ask them to explain how to identify the coordinates of both points. Help them apply the same procedure to help identify points on the axes.

► Enrichment

- Ask students to shade in a square on the coordinate plane that contains a point with a given ordered pair in which both coordinates of the ordered pair are fractions or mixed numbers. Likewise, ask students to shade in a line segment of length 1 on the coordinate plane that contains a point with a given ordered pair in which one coordinate is a whole number and the other is a fraction.
- Ask students to find patterns in horizontal and vertical lines. For example, ask students to write coordinates of points on a vertical line passing through the 3 on the *x*-axis. Students should notice that all points on this line have an *x*-coordinate of 3. Help them arrive at informal descriptions like "*x*-coordinate is 3," or "$x = 3$."
- Ask students to graph a point given an ordered pair and then reverse the order of the coordinates and graph the other point. For example, ask students to graph (3, 6) and (6, 3). Then, ask them to connect the two points and repeat the activity with a few pairs of points. Ask the students to make observations about the corresponding line segments and where they would cross the *x*- and *y*-axes if extended into lines.
- Ask students to graph a series of points given ordered pairs in which each *x*-coordinate is greater than its corresponding *y*-coordinate. Then, ask students to graph a series of points given ordered pairs in which each *x*-coordinate is less than its corresponding *y*-coordinate. Ask students to make observations about the groupings of points. Finally, ask students to graph a series of points given ordered pairs in which each *x*-coordinate is equal to its corresponding *y*-coordinate.

► Key Terms

coordinate, coordinate system, horizontal, ordered pair, origin, plane, vertical, *x*-axis, *y*-axis

C-G.1.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

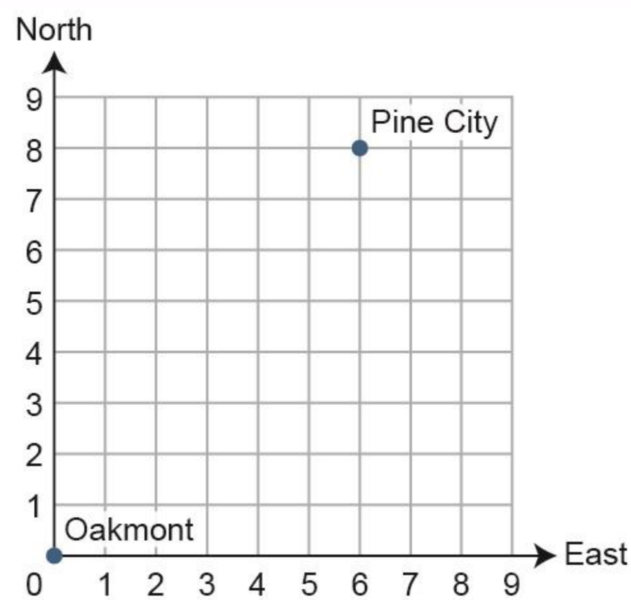
Identify parts of a coordinate grid, and describe or interpret points given an ordered pair.

► Eligible Content

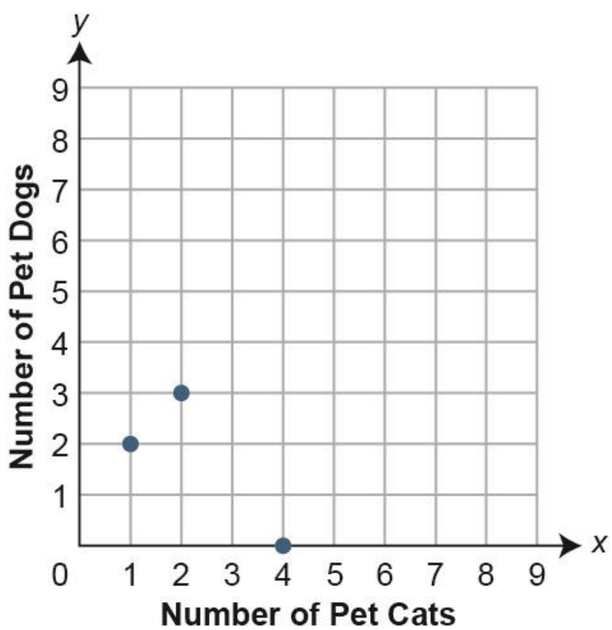
C-G.1.1.2 Represent real world and mathematical problems by plotting points in quadrant I of the coordinate plane, and interpret coordinate values of points in the context of the situation.

► Instructional Supports

- Ask students to interpret the location of points based on a given context. For example, give students this problem: "A coordinate grid is placed on a map containing the towns of Pine City and Oakmont as shown. Each unit on the grid represents 1 mile. Describe how to travel from Oakmont to Pine City based on this coordinate grid." Students determine that the location of Pine City is (6, 8), which would mean traveling 6 miles east and 8 miles north from Oakmont.

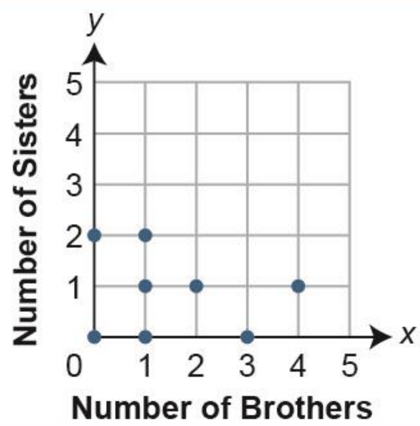


- Ask students to use points on the coordinate plane in contexts that use points to show data. For example, the points on the following graph show the numbers of pet cats and pet dogs that people have.



Ask students to explain the meaning of each point on the graph. Then ask students to plot a point on the graph that shows the number of pet cats and dogs they have themselves.

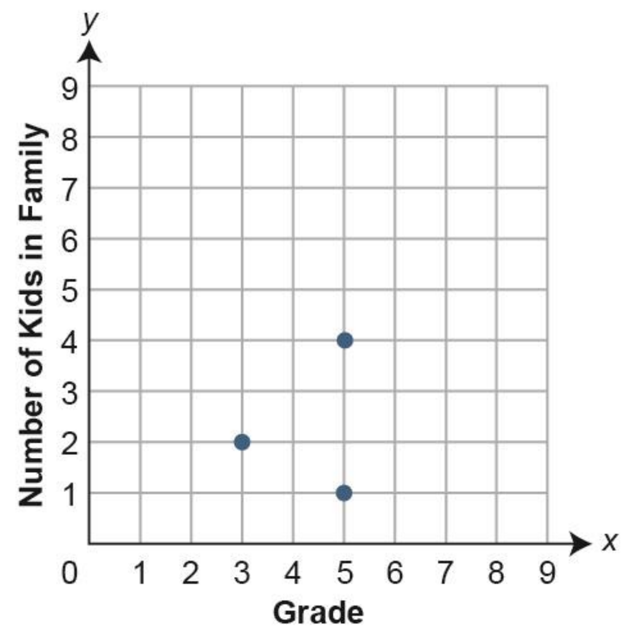
- Ask students to answer questions based on the coordinate values of points in a data display. For example, give students the following data display showing the numbers of brothers and sisters students have. Ask students to answer questions such as “How many students have more sisters than brothers?”



Students should be able to see there are two points with greater y-coordinates than x-coordinates and conclude there are two students who have more sisters than brothers.

► Identifying and Addressing Common Misconceptions

- Some students may think that reversing the numbers in an ordered pair does not change the location of the point represented by the ordered pair. Describe a situation in which an ordered pair represents the number of wins (x) and losses (y) a student has in a chess tournament. Ask what record is represented by (4, 6) and what record is represented by (6, 4). Some students may give the same interpretation for both ordered pairs. If students say that both ordered pairs represent a record of 4 wins and 6 losses, ask them to identify what point would indicate a record of 6 wins and 4 losses.
- Some students may interpret the x- and y-axes as having a value of 1, especially in contexts that do not allow for values of 0. Give students the following graph and ask them to interpret the coordinates of each point.



Some students may count along each axis starting at 1 and think the graph shows a fourth grader and two sixth graders. Help students understand that even though 0 is not a reasonable value for either coordinate in this context, graphs are always assumed to start at 0 unless indicated otherwise.

► Enrichment

- Give students a graph that represents locations on a map and ask them to use a ruler to estimate the straight-line or "airplane" distance between two points and compare it to the orthogonal or "taxicab" distance between two points.
- Ask students to describe, in the context of a city map, all the points that lie on a given horizontal or vertical gridline.

► Key Terms

coordinate plane, coordinates, first quadrant, ordered pair, *x*-axis, *y*-axis

C-G.2.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Classify two-dimensional figures into categories based on their properties.

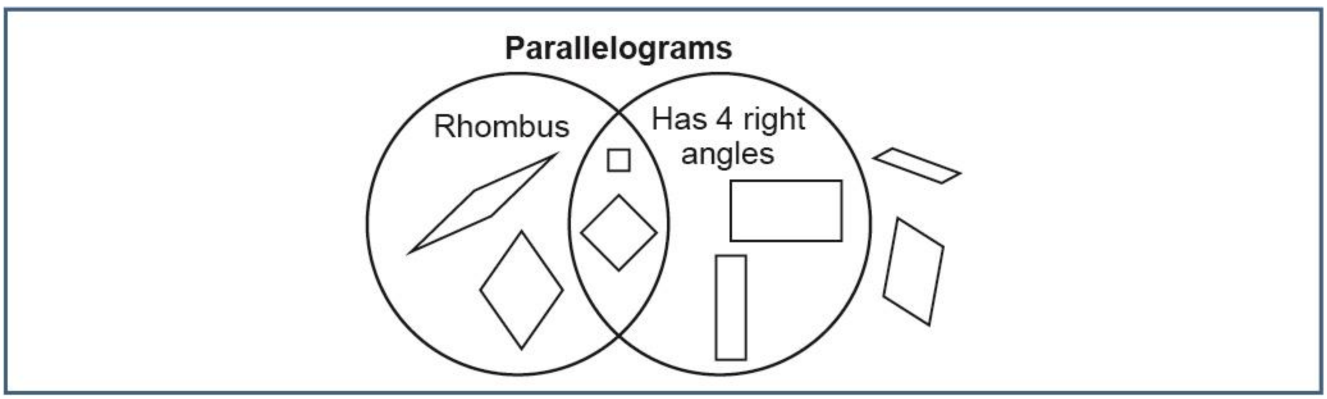
Use basic properties to classify two-dimensional figures.

► Eligible Content

C-G.2.1.1 Classify two-dimensional figures in a hierarchy based on properties.

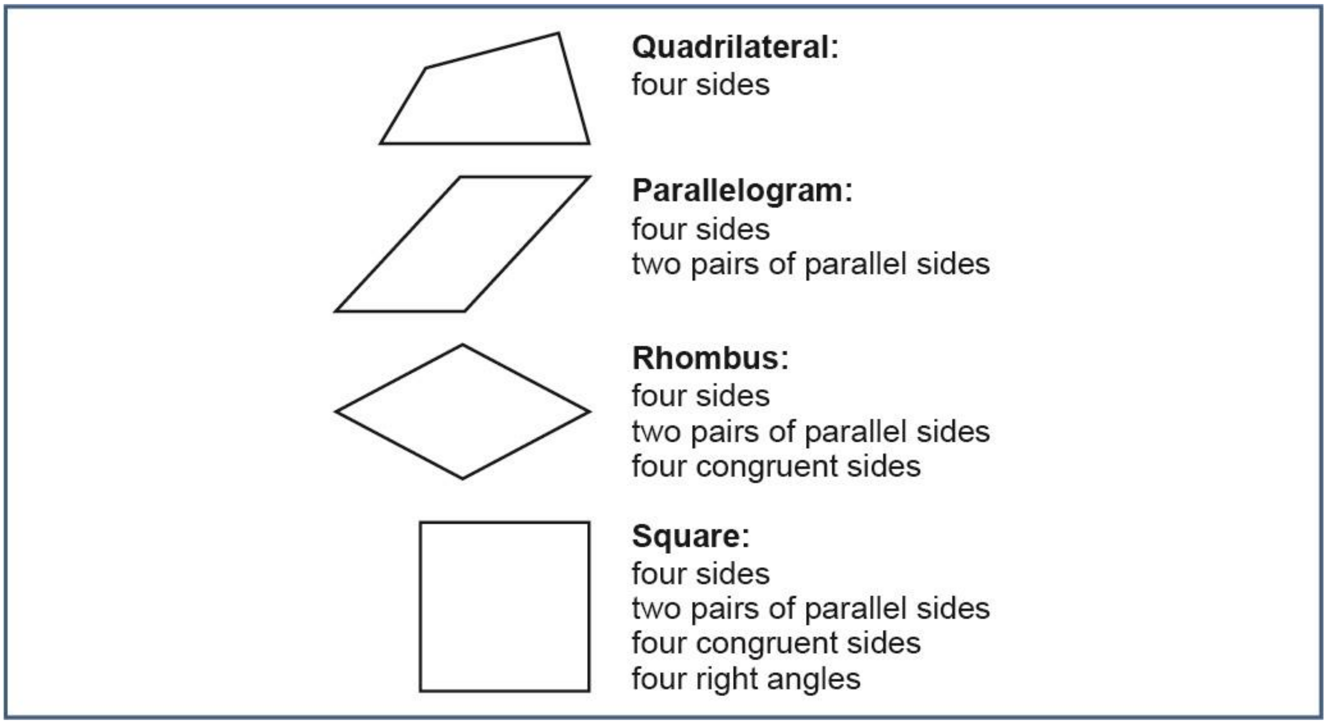
► Instructional Supports

- Ask students to draw various shapes so there are example shapes in all possible regions of a given Venn diagram. For example, the following Venn diagram shows two possible categories with shapes placed in all the regions.



  Discuss with students what would change if the rightmost circle was designated as "squares" instead of shapes that "have 4 right angles." There would be no shapes in the rightmost crescent because every square is a rhombus.

- Ask students to classify figures in a hierarchy based on the properties of the figures. For example, ask students to identify the properties of a quadrilateral, a parallelogram, a rhombus, and a square. Then, ask them to arrange the figures in a hierarchy such that each figure has all the attributes of all the figures above it.



► Identifying and Addressing Common Misconceptions

- Some students may think that two-dimensional figures are arranged in a linear hierarchy where each category either completely contains or is contained in each other category. Give students the hierarchy of "polygon → triangle → right triangle" and ask them where isosceles triangles fit. Some students may try to create a hierarchy shaped like a chain rather than a branching tree. Ask students to draw a right triangle that is not isosceles and an isosceles triangle that is not right. Help them use those drawings to recognize that both right triangles and isosceles triangles are subcategories of triangles, but neither is a subcategory of the other. Then help them draw the correct hierarchy.

► Enrichment

- Ask students to make a hierarchy including types of triangles and types of quadrilaterals. As an additional challenge, introduce students to regular polygons, convex polygons, and concave polygons and ask students to include these shapes in the hierarchy.

- Ask students to create categorical hierarchies using other contexts, such as animals, warm-blooded, vertebrates, mammals, fish, and reptiles.

- Ask students to make a Venn diagram with three circles and populate as many regions as possible based on the three chosen categories.
- Ask students to make Venn diagrams in which one circle is contained entirely within another or where the intersection of both circles is empty.

► Key Terms

properties, two-dimensional, hierarchy

D-M.1.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Convert like measurement units within a given measurement system.

Solve problems using simple conversions (may include multi-step, real-world problems).

► Eligible Content

D-M.1.1.1 Convert among different-sized measurement units within a given measurement system. A table of equivalencies will be provided.

► Instructional Supports

- Ask students to convert measurements within the metric system.
  - For example, ask students to convert 3.5 meters to centimeters. To convert from a larger unit to a smaller unit, students use the conversion factor combined with multiplication. Because there are 100 centimeters in each meter, and because 3.5 meters can be thought of as 3.5 groups of size 100 centimeters, students multiply 3.5 by 100 and get 350. Therefore, 350 centimeters is equivalent to 3.5 meters. Similarly, to convert 6.2 kilometers to meters, students multiply 6.2 by 1,000 because there are 1,000 meters in each kilometer and 6.2 kilometers can be thought of as 6.2 groups of size 1,000 meters. Therefore, 6,200 meters is equivalent to 6.2 kilometers.

  - For example, ask students to convert 86 millimeters to centimeters. To convert from a smaller unit to a larger unit, students use the conversion factor combined with division. Students divide 86 by 10 because there are 10 millimeters in each centimeter and the division determines the number of groups of size 10 that are in 86. Therefore, 86 millimeters is equivalent to 8.6 centimeters. Similarly, to convert 4.9 grams to kilograms, students divide 4.9 by 1,000 since there are 1,000 grams in each kilogram. Therefore, 4.9 grams is equivalent to 0.0049 kilograms.

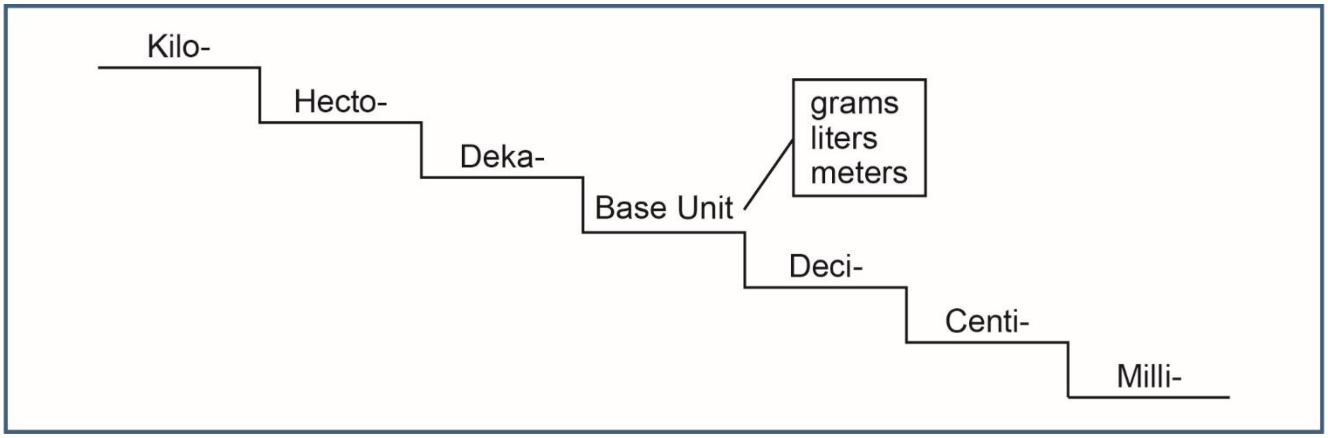
- Ask students to convert measurements within the U.S. Customary system.
  - For example, ask students to convert 3 pounds to ounces. To convert from a larger unit to a smaller unit, students use the conversion factor combined with multiplication. Because 3 pounds can be thought of as 3 groups of size 16 ounces, students multiply 3 by 16 to get 48 and conclude that 3 pounds is equivalent to 48 ounces. Similarly, to convert 7 gallons to quarts, students multiply 7 by 4 (because there are 4 quarts in each gallon) and conclude that 7 gallons is equivalent to 28 quarts.
  - For example, ask students to convert 48 inches to feet. To convert from a smaller unit to a larger unit, students use the conversion factor combined with division. There are 12 inches in each foot, and division determines the number of groups of size 12 inches that are in 48 inches. Therefore, students divide 48 by 12 to get 4 and conclude that 48 inches is equivalent to 4 feet. Similarly, to convert 20 cups to pints, students divide 20 by 2 because there are 2 cups in each pint. Therefore, 20 cups are equivalent to 10 pints.

- Ask students to solve a multistep conversion problem with the help of a conversion table. For example, ask students to use the table shown to help solve this problem: "Jamie buys 2 gallons of laundry soap. Each load of laundry needs 1 cup of laundry soap. How many loads of laundry can Jamie do with 2 gallons of laundry soap?"

Unit	Equivalent Unit
1 gallon	4 quarts
1 quart	2 pints
1 pint	2 cups

First, students use the conversion table to convert 2 gallons to quarts. Since there are 4 quarts in 1 gallon, students solve 4×2 and conclude that there are 8 quarts in 2 gallons. Then, students convert 8 quarts to pints. Since there are 2 pints in 1 quart, there are 16 pints in 8 quarts (because $8 \times 2 = 16$). Finally, students convert 16 pints to cups. Since there are 2 cups in 1 pint, there are 32 cups in 16 pints (because $16 \times 2 = 32$). Therefore, Jamie can do 32 loads of laundry with the 2 gallons of laundry soap.

- Ask students to use a metric stairstep to solve a multistep problem. For example, give students this problem: "Ahmad measures the length of a peapod. The first time he measured it, the peapod was 3.8 centimeters long. When he measured it 2 weeks later, it was 4.2 centimeters long. If the peapod grew the same length each day, how many millimeters did it grow in the first week after Ahmad measured it?"



First, students calculate 4.2 – 3.8 = 0.4 to determine that the peapod grew a total of 0.4 centimeters in 2 weeks. Therefore, it grew half that amount, or 0.4 ÷ 2 = 0.2 centimeters, in the first week. Then ask students to use the metric stairstep to convert 0.2 centimeters to millimeters. On the metric stairstep, each step down requires multiplication by 10. Since millimeters is only 1 step down from centimeters, students need to multiply by 10 just once. Therefore, students multiply 0.2 centimeters by 10 to determine that the peapod grew 2 millimeters in the first week after Ahmad measured it.

- Ask students to create graphic organizers to model and help solve multistep problems. For example, to convert between units in the U.S. Customary system for length, students can create a graphic organizer like the one shown.

Then, give students this problem: “Kayla kicked a soccer ball 26 yards. Mika kicked a soccer ball 28 yards. How many more inches did Mika kick the soccer ball than Kayla?” Students determine that Mika kicked the soccer ball 2 more yards than Kayla since 28 – 26 = 2. Then, students use the graphic organizer to determine that each yard contains 3 feet and each foot contains 12 inches, so each single yard contains 36 inches. Since Mika kicked the soccer ball 2 more yards, she kicked it 72 more inches than Kayla (because 2 × 36 = 72).

► Identifying and Addressing Common Misconceptions

- Some students may perform conversions using the wrong operation. For example, students may use multiplication instead of division. Ask students to convert 80 centimeters to millimeters. Some students may respond with 8 millimeters because 80 ÷ 10 = 8. Ask students whether a centimeter or a millimeter is smaller. Explain that since a millimeter is smaller, there must be more millimeters than centimeters. If students have trouble conceptualizing why, have them imagine filling a jar with large marbles and then dumping those out and filling the jar with small marbles. Ask students whether they would need more large marbles or more small marbles to fill the jar. Students should recognize that they would need more small marbles to fill the jar, and so they need more millimeters than centimeters to “fill” or “cover” the original 80 centimeters. Therefore, the conversion requires multiplication, and there are 800 millimeters in 80 centimeters (because 80 × 10 = 800).

- Some students may struggle with converting units in problems that require more than one conversion step. Ask students to convert 4 yards to inches. Some students may respond with 12 inches or 48 inches, having multiplied by 3 (the conversion for yards to feet) or 12 (the conversion for feet to inches). Have students write out the U.S. Customary units for length, which should include yards, feet, and inches. Ask students what units they can convert using their conversion rates. Since they do not have a conversion rate from yards to inches but do have a conversion rate from yards to feet, ask them to first convert yards to feet. Then ask them to convert feet to inches. Remind students that each step requires a conversion, and since there are two steps "between" yards and inches, there must be two conversions. If they do not have a rate for a conversion, they should start by using whatever conversion rates they do have that go in the correct direction (larger or smaller).
- Some students may not know the metric prefixes. For example, ask students what the prefix "centi-" means in "centimeter." Some students may respond that it means a factor of 10 rather than 100. Help students develop mnemonics by connecting the meanings of the prefixes to other words they are familiar with. For example, "centi-" connects to words such as "century" and "cents" (relating to money), and they are all connected to 100.

► Enrichment

- Have students solve conversion problems in which some measurements are given in two different units. For example, give students this problem: "Gerry is 5 feet 8 inches tall. Amanda is 6 feet 1 inch tall. How many inches taller is Amanda than Gerry?"
- Have students make conversion charts that combine multiple steps into one. For example, have students make a conversion chart that shows the conversions from gallons into quarts, pints, and cups, as shown.

Unit	Equivalent Unit
1 gallon	4 quarts
1 gallon	8 pints
1 gallon	16 cups

► Key Terms

conversion, division, equivalent measurements, measurement system, metric system, multiplication, unit, U.S. Customary system

D-M.2.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Represent and interpret data.

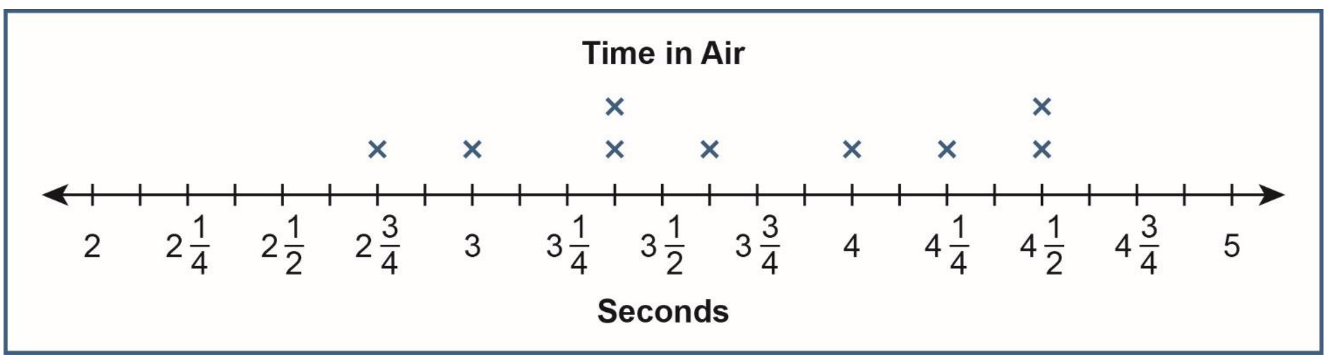
Organize, display, and answer questions based on data.

► Eligible Content

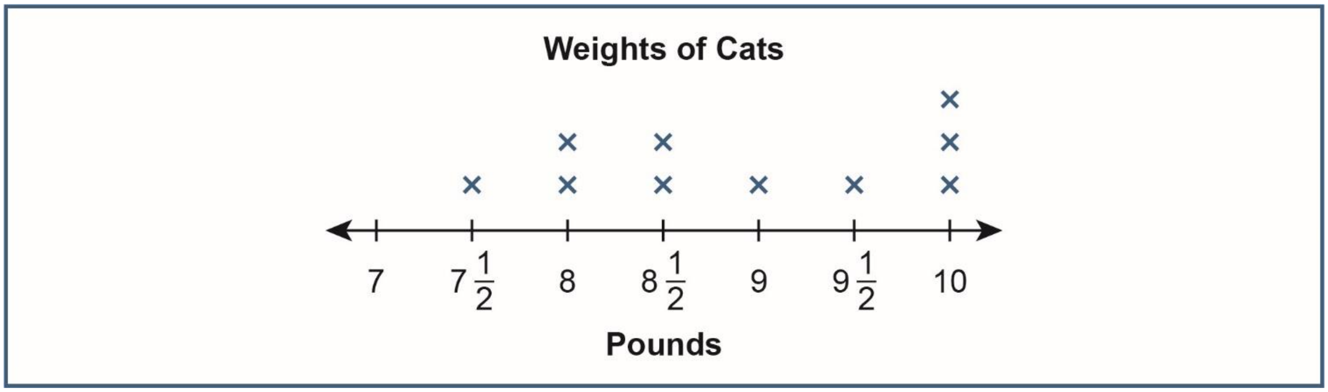
D-M.2.1.1 Solve problems involving computation of fractions by using information presented in line plots.

► Instructional Supports

- Ask students to make a line plot to represent a given data set. For example, give students the following data set showing the number of seconds 9 footballs stay in the air after being kicked, each rounded to the nearest $\frac{1}{8}$ second: $3\frac{3}{8}$, $3\frac{3}{8}$, $4\frac{1}{4}$, $4\frac{1}{2}$, $2\frac{3}{4}$, 3, 4, $3\frac{5}{8}$, $4\frac{1}{2}$.

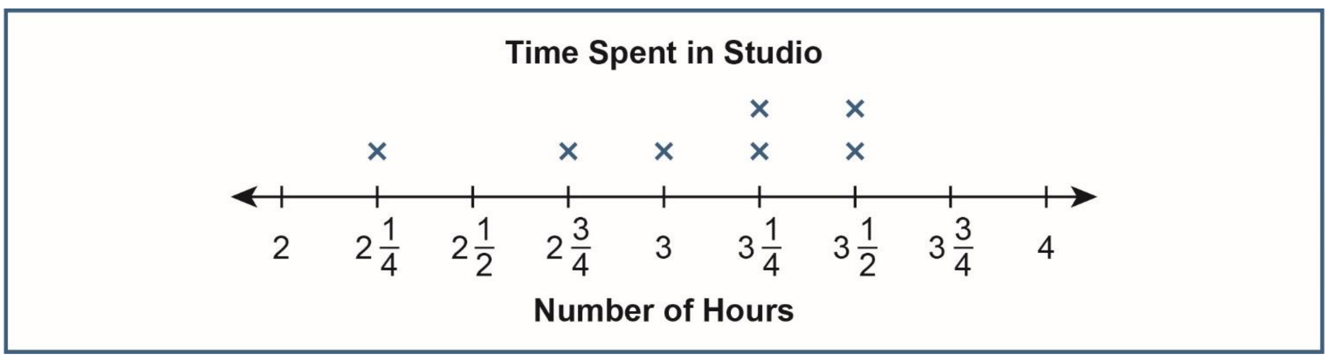


- Ask students to solve problems using information from a line plot. For example, give students the following line plot showing the weights of 10 adult cats at a cat groomer's shop, with each weight rounded to the nearest $\frac{1}{2}$ pound, and ask them to determine the difference between the heaviest cat and the lightest cat.



The lightest weight in the line plot is $7\frac{1}{2}$ pounds, and the heaviest weight in the line plot is 10 pounds. The fact that 10 occurs multiple times does not matter regarding the difference in weights. The difference is $2\frac{1}{2}$ pounds because $10 - 7\frac{1}{2} = 2\frac{1}{2}$.

- Ask students to identify the missing data value in a data set given an incomplete line plot and the sum of all the measurements, including the missing one. For example, give students this prompt: "A group of 8 artists recorded the number of hours they each spent in their studios yesterday. The times for 7 of the artists are shown in the line plot. All 8 artists combined spent a total of 25 hours in their studios yesterday. Determine how many hours the eighth artist spent in the studio yesterday."

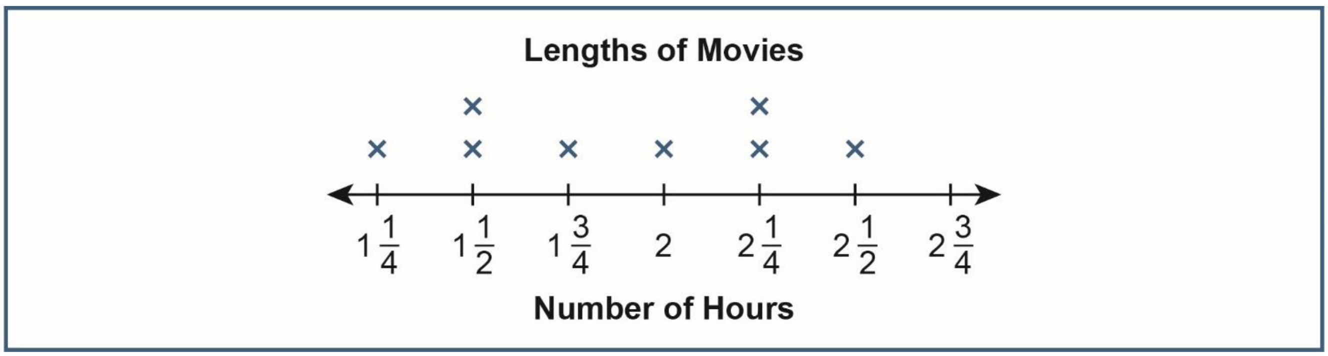


The total of the 7 values shown is $21\frac{1}{2}$ hours because adding together the fractional values $(\frac{1}{4} + \frac{3}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} + \frac{1}{2})$ gives $2\frac{1}{2}$, adding together the whole number parts of the values $(2 + 2 + 3 + 3 + 3 + 3 + 3)$ gives 19, and $19 + 2\frac{1}{2} = 21\frac{1}{2}$. To determine the missing value, subtract

$21\frac{1}{2}$ hours from the given total of 25 hours. Therefore, the missing data value is $3\frac{1}{2}$, and a complete line plot would have an additional X above $3\frac{1}{2}$.

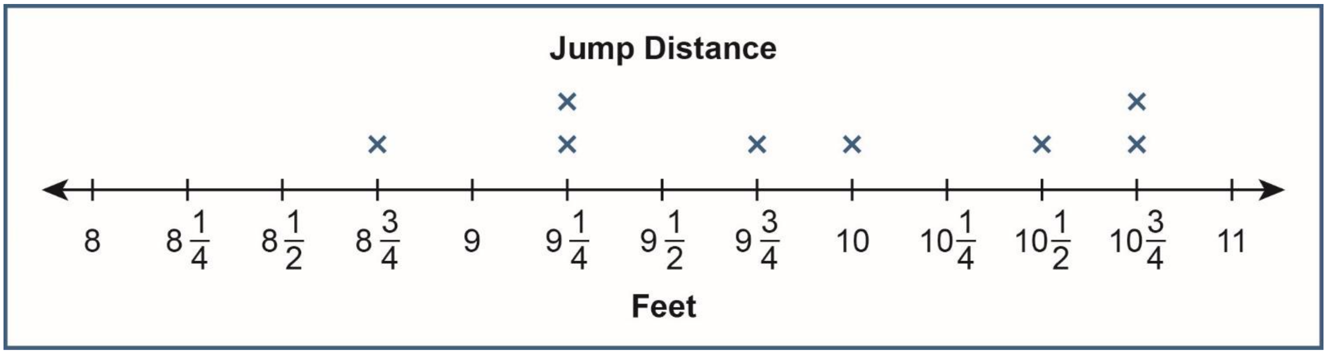
▶ Identifying and Addressing Common Misconceptions

- Some students may accidentally omit data points when creating a line plot. Ask students to make a line plot representing a data set with 12 data points. Some students may include only 11 points. Ask students to count the number of X's they marked and make sure they plotted the correct number of values. Point out that while this does not verify their marks are plotted in the correct locations, it at least verifies they have plotted the correct number of values.
- Some students may answer questions about a line plot using the numbers on the number line rather than using the data. Give students the following line plot showing the lengths, in hours, of 8 movies, and ask students to calculate the difference between the length of the longest movie and the length of the shortest movie.



Some students may identify the greatest and least values on the number line as $2\frac{3}{4}$ and $1\frac{1}{4}$ and give a response of $1\frac{1}{2}$ hours. Ask students to explain the meaning of each X on the line plot and make a list of the movie lengths as they go through each X. Then ask them to calculate the difference using their list of numbers.

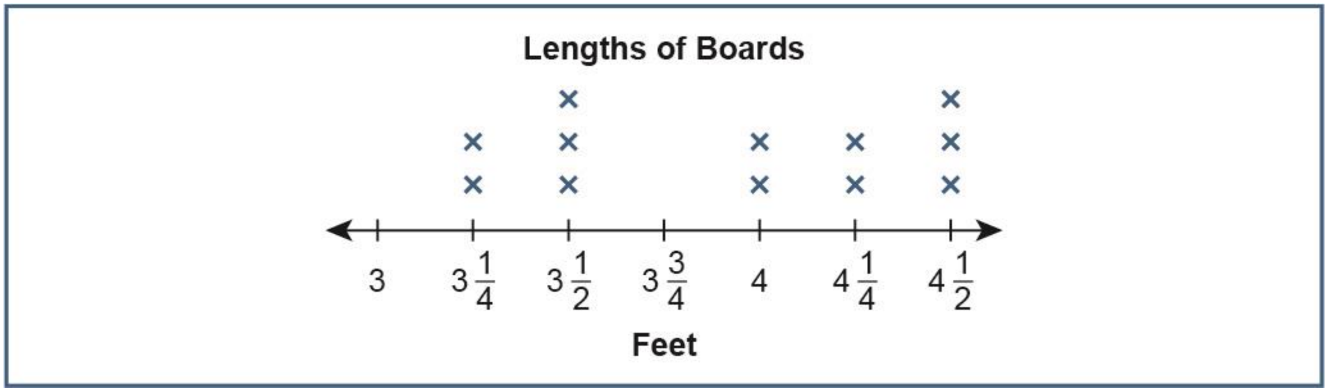
- Some students notice only whether the numbers have X's over them and do not pay attention to how many X's there are. Give students the following line plot showing the distances, in feet, jumped in a long jump contest, and ask them to determine how many jumpers had a jump length of 10 feet or more.



Some students may look only at the lengths of the jumps and answer with 3 jumpers because of the three values of 10, $10\frac{1}{2}$, and $10\frac{3}{4}$. These students have not considered the number of jumpers who jumped each of those lengths. Point to the two X's above $10\frac{3}{4}$ and ask students to explain the meaning of the two X's.

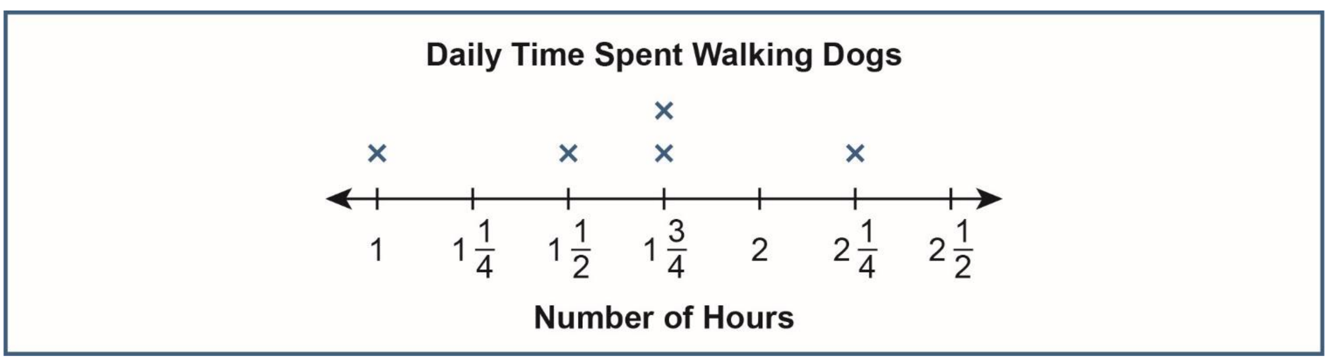
► Enrichment

- Give students a line plot and ask them to create a new line plot using additional information that affects all the values on the original line plot. For example, give students a line plot that represents the length of time each of 8 students has been reading and have students create a new line plot assuming that all 8 students read for an additional $\frac{1}{2}$ hour. Students can notice that the shape of the line plot doesn't change—just the values change. This can lead to discussions of variability, shape, and spread.
- Ask students to write expressions to represent the total of the values shown in a line plot. For example, give students the line plot shown, which represents the lengths of 12 boards in Geraldine's garage with each length rounded to the nearest $\frac{1}{4}$ foot.



Ask students to write expressions that can be used to determine the total of all 12 lengths, in feet. Students might write the straightforward sum of all 12 values. Students might also multiply each length by its frequency and add the products, as in $(2 \times 3\frac{1}{4}) + (3 \times 3\frac{1}{2}) + \ldots$, etc. Or, students might note that because every value has at least two X's, the sum could be written as $2 \times (3\frac{1}{4} + 3\frac{1}{2} + 4 + 4\frac{1}{4} + 4\frac{1}{2}) + (3\frac{1}{2} + 4\frac{1}{2})$.

- Give students a line plot and ask them to write questions involving missing data values for a classmate. Each question should involve more than one missing data value but have enough information to have only one correct response. For example, give students the line plot shown, which represents the number of hours Melinda spent walking dogs on Monday through Friday of last week.



Some possible student questions are shown.

  - Melinda spent a total of 9 hours walking dogs Monday through Sunday. She spent 1 hour walking dogs on Saturday. How many hours did she spend walking dogs on Sunday?

- Melinda spent a total of $10\frac{1}{4}$ hours walking dogs Monday through Sunday. She spent the same number of hours walking dogs on Saturday as she did on Sunday. How many hours did she spend walking dogs on Sunday?
- The difference in the number of hours Melinda walked dogs on the day she did the least walking and the number of hours she walked on Saturday is $1\frac{1}{2}$ hours. The difference in the number of hours she walked on the day she did the least walking and the number of hours she walked on Sunday is $1\frac{3}{4}$ hours. How many hours, in total, did Melinda spend walking dogs Monday through Sunday?

▶ Key Terms

data, fraction, fractional intervals, frequency, line plot, number line, operation

D-M.3.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

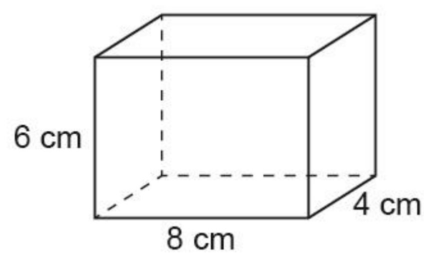
Use, describe, and develop procedures to solve problems involving volume.

► Eligible Content

D-M.3.1.1 Apply the formulas V = l × w × h and V = B × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. Formulas will be provided.

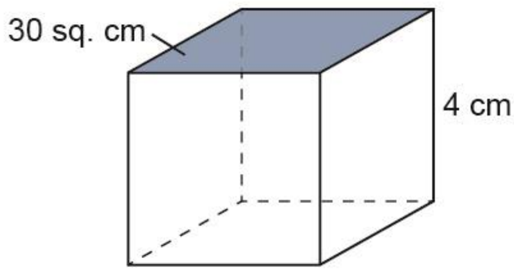
► Instructional Supports

- Ask students to calculate the volume of a prism given the length, width, and height. For example, a rectangular prism with a length of 8 centimeters, a width of 4 centimeters, and a height of 6 centimeters is shown.



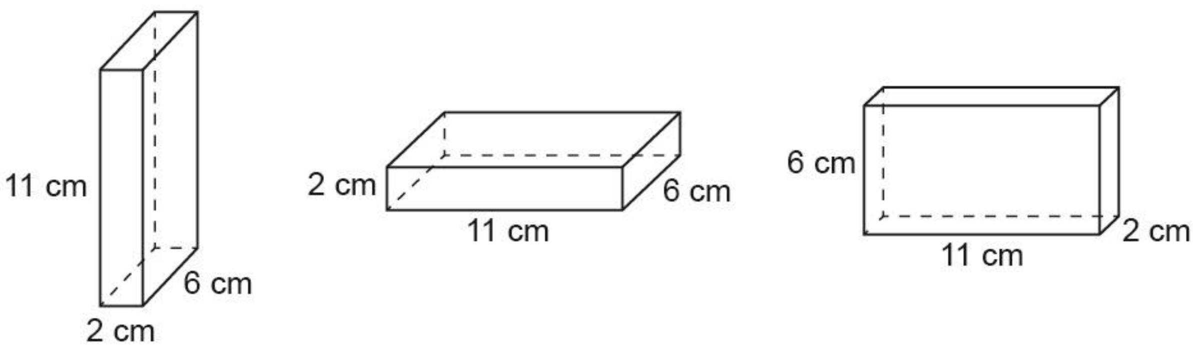
  Students use the formula $V = l \times w \times h$ to calculate the volume (V) of a rectangular prism with a length of l units, a width of w units, and a height of h units. The volume is 192 cubic centimeters because 8 × 4 × 6 = 192.

- Ask students to calculate the volume of a prism when given the area of its base and its height. For example, a right rectangular prism with a height of 4 centimeters and a base that has an area of 30 square centimeters is shown.



Students use the formula $V = B \times h$ to calculate the volume (V) of a rectangular prism with a base area of B square units and a height of h units. The volume is 120 cubic centimeters because $30 \times 4 = 120$.

- Ask students to calculate the volume of a rectangular prism in three different ways, each time treating a different face of the prism as the base. For example, give students a rectangular prism with edge lengths of 2 centimeters, 6 centimeters, and 11 centimeters. The same prism is shown in three different orientations.



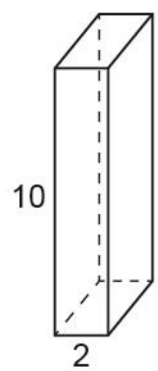
  - Using the 2×6 rectangle as the base, students determine that the base area is 12 square centimeters. Then, students recognize that if the 2×6 rectangle is the base, the height is 11 centimeters. Students use the formula $V = B \times h$ to calculate $12 \times 11 = 132$ cubic centimeters.
  - Using the 6×11 rectangle as the base, students determine the base area is 66 square centimeters. Because the 6×11 rectangle is the base, the height is now

considered to be 2 centimeters. Students use the formula $V = B \times h$ to calculate $66 \times 2 = 132$ cubic centimeters.

- Using the 2×11 rectangle as the base, students determine a base area of 22 square centimeters. Recognizing that the height is now 6 centimeters, students use the formula $V = B \times h$ to calculate $22 \times 6 = 132$ cubic centimeters.

Students should note that all three orientations of the prism result in the same volume of 132 cubic centimeters.

- Ask students to determine the missing dimension of a right rectangular prism when given the prism's volume and two different edge lengths. For example, given that a right rectangular prism has a volume of 100 cubic inches, a length of 2 inches, and a height of 10 inches, ask students to determine the width of the rectangular prism.



Volume = 100 cubic inches

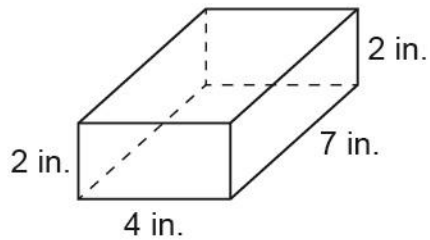
Students determine the missing dimension by using the formula $V = l \times w \times h$. When students substitute the volume of 100 for V, the length of 2 for l, and the height of 10 for h, the equation becomes $100 = 2 \times w \times 10$, which simplifies to $100 = 20 \times w$. That means the product of the width and 20 is 100. Therefore, the width is 5 inches because $20 \times 5 = 100$ or, alternatively, because $100 \div 20 = 5$.

- Ask students to determine possible pairs of values for two missing dimensions of a right rectangular prism given the prism's volume and one edge length. For example, give students a rectangular prism with a volume of 60 cubic inches and an edge length of 4 inches. Since

$60 \div 4 = 15$, the two unknown dimensions have a product of 15. Therefore, possible whole-number answers are the factor pairs of 15, such as 1×15 or 3×5.

► Identifying and Addressing Common Misconceptions

- Some students may multiply all dimensions shown when calculating the volume of a rectangular prism. Ask students to calculate the volume of the prism shown.



Some students may calculate the volume as 112 cubic inches by calculating $2 \times 4 \times 7 \times 2$. Ask students to highlight all the edges in the figure that represent the height in the same color. Then ask students to use a second color to highlight all the edges in the figure representing the width and a third color to highlight all the edges representing the length. Each of these dimensions should be used only once in the product $l \times w \times h$.

- Some students may confuse the height of the base and the height of the prism when calculating volume with the formula $V = B \times h$. Ask students to use that formula to determine the volume of a rectangular prism with a length of 12 units, a width of 4 units, and a height of 5 units. Some students may give answers such as 100 cubic units by using 5 as the height of the base and 4×5 as the area of the base, but then multiplying by the height of 5 again. Once students have determined which two dimensions they are going to use to determine the area of the base, have them sketch the prism and shade the corresponding face. Students can then identify the edge that is perpendicular to the shaded face as the "height" of the prism in order to complete the calculation.

- Some students may not recognize that, in the formula $V = B \times h$, the B is an area that may have to be derived from a figure first, and that it does **not** represent an edge length. Ask students to use the formula $V = B \times h$ to determine the volume of a rectangular prism with a length of 5 units, a width of 4 units, and a height of 15 units. Some students may respond that the volume is either 60 cubic units or 75 cubic units, having multiplied the height by one side of the base instead of the area of the base. Ask students to identify the units of the numbers they multiplied, then help them to understand that multiplying the length by the height would only result in a product with a dimension of square units, not cubic units. Once students recognize that the dimensions of their volume calculation are incorrect, ask what dimensions are represented by B in the formula. Help them connect the fact that B must have dimensions of square units because it represents an area, not a linear measure.

► Enrichment

- Give students six random whole numbers between 1 and 10 and ask students to arrange them into two groups of three such that if the three numbers in each group represent the length, width, and height of a rectangular prism, the two prisms have volumes that are as close as possible to one another.
- Give students a volume measure and ask them to determine the dimensions of at least two right rectangular prisms that have that same volume.
- Ask students to extend the $V = B \times h$ formula for use in any right prism, not just right rectangular prisms. For example, given that a right pentagonal prism has a base with an area of 60 square inches and a height of 8 inches, have students determine the volume of the prism.

► Key Terms

base, edge, formula, height, length, rectangular prism, volume, width

D-M.3.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

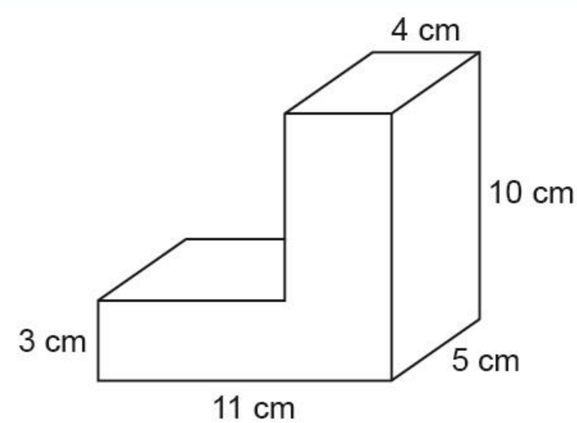
Use, describe, and develop procedures to solve problems involving volume.

► Eligible Content

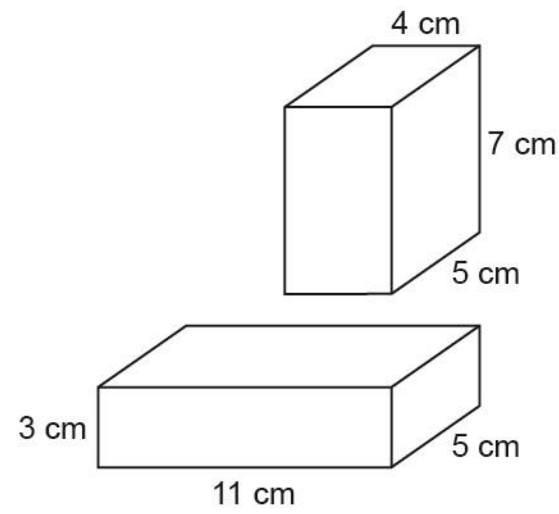
D-M.3.1.2 Find volumes of solid figures composed of two non-overlapping right rectangular prisms.

► Instructional Supports

- Ask students to calculate the volume of a three-dimensional solid figure by decomposing it into two rectangular prisms. For example, ask students to determine the volume of the figure shown.

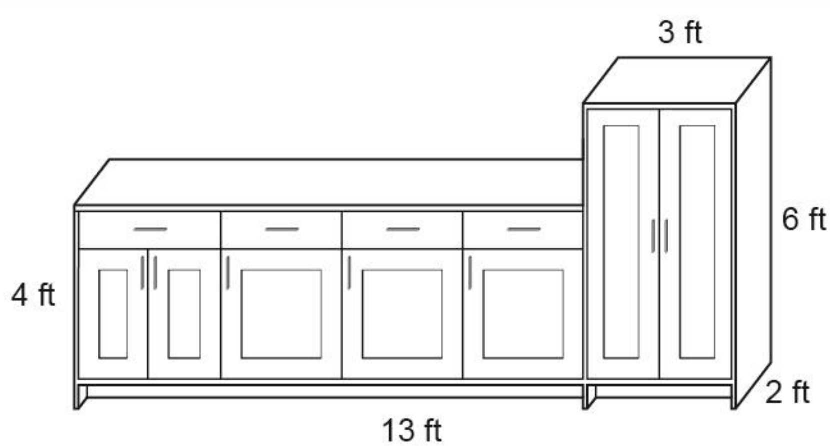


The figure can be decomposed into the two rectangular prisms as shown.



The volume of the bottom prism is 165 cubic centimeters because 3 × 11 × 5 = 165. The volume of the top prism is 140 cubic centimeters because 5 × 7 × 4 = 140. Therefore, the total volume of the solid is equal to 165 + 140 or 305 cubic centimeters.

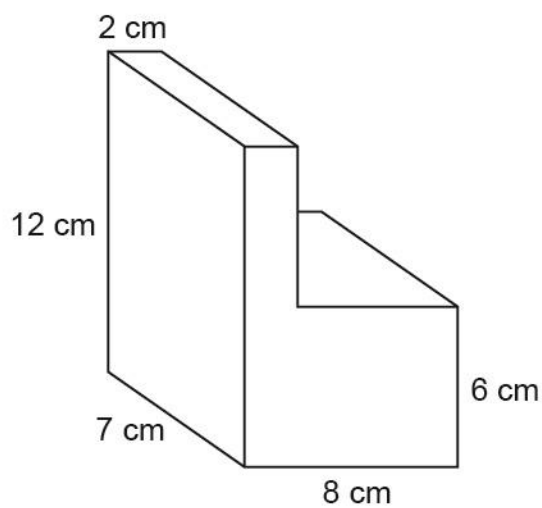
- Ask students to calculate the volume of a figure made up of multiple rectangular prisms in a real-world context. For example, give students this problem: "Alexia has a set of rectangular storage cabinets as shown. What is the total volume of the set of storage cabinets?"



Students can split the set of cabinets into two separate rectangular prisms. One possible way is to split the cabinets into a left prism with dimensions of 10 feet by 2 feet by 4 feet and a right prism with dimensions of 3 feet by 2 feet by 6 feet. To calculate the total volume of the cabinets, students must determine the volume of each of the two prisms that make up the set of storage cabinets and then add those volumes together. The equations for the volumes of the prisms are $10 \times 2 \times 4 = 80$ and $3 \times 2 \times 6 = 36$. The total volume for the set of storage cabinets is 116 cubic feet because $80 + 36 = 116$.

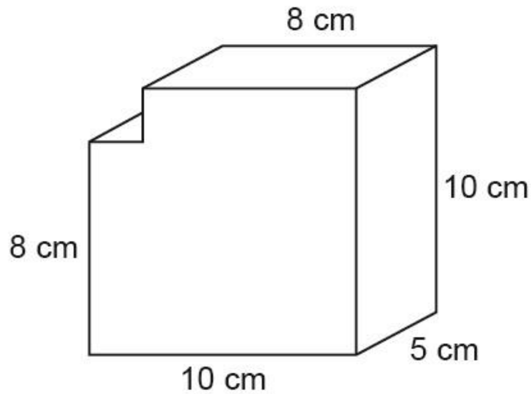
► Identifying and Addressing Common Misconceptions

- Some students may multiply the longest length, width, and height of a given solid composed of two rectangular prisms in order to calculate its volume. Ask students to determine the volume of the figure shown.

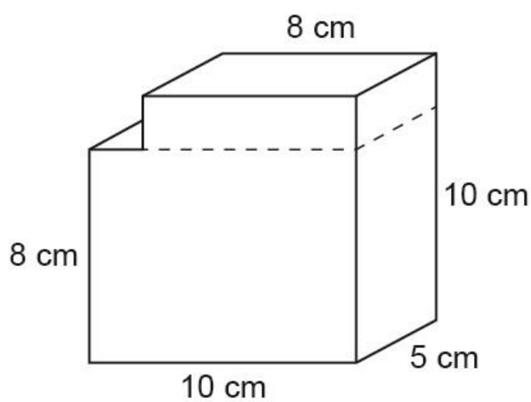


Some students may respond that the volume is 672 cubic centimeters since $8 \times 7 \times 12 = 672$. Ask students to draw a rectangular prism with a length of 8 centimeters, a width of 7 centimeters, and a height of 12 centimeters and calculate the volume of that solid. Students should respond that the volume is 672 cubic centimeters. Help students recognize that the original solid must have a volume that is less than the volume of the entire rectangular prism by shading the part of the prism that would have to be removed in order to create the original solid.

- Some students may decompose a solid figure into overlapping prisms. Ask students to decompose the figure shown into two rectangular prisms and give the dimensions of each prism.

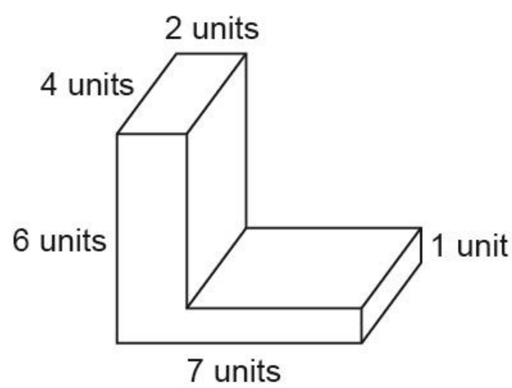


Some students may decompose the figure into two rectangular prisms with dimensions of $8 \times 10 \times 5$, having divided the figure into two rectangular prisms but included the lower right portion of the solid in the volume of both prisms. Help students construct both prisms using blocks and then put them together to show that the combined shape is not the same as the original. Then have students divide the figure by drawing a horizontal line to show one way to divide the composite solid, as shown.



Help students recognize that for at least one of the two prisms, one of its dimensions is always going to be shortened from that dimension of the original figure.

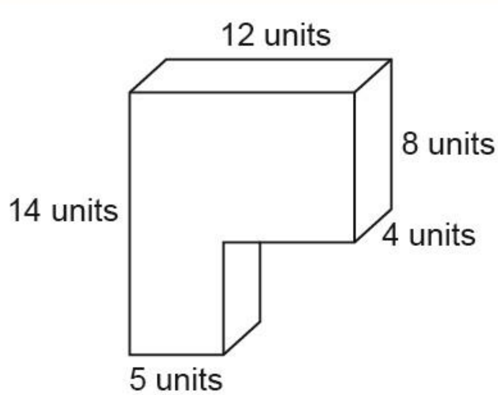
- Some students may determine the volume of a solid composed of two rectangular prisms by multiplying the volumes of the two prisms instead of adding them. Ask students to calculate the volume of the solid shown.



Some students may respond that the volume is 960 cubic units since the volume of the "upper" prism is 48 (2 × 4 × 6), the volume of the "lower" prism is 20 (5 × 4 × 1), and 48 × 20 = 960. Remind students that multiplication is used to calculate the volume of a single prism because the volume of the base layer is repeatedly added until the prism is full of unit cubes. However, when decomposing solids into two separate prisms, the decomposed parts are not being repeatedly added. With the strategy of decomposing, addition is used because the two volumes are being put together.

► Enrichment

- Have students divide composite figures into two rectangular prisms in two different ways and show that the volume is the same, regardless of how the composite figure is divided.
- Have students determine the volume of a figure by subtracting instead of adding. For example, give students the figure shown.



The volume can be determined by calculating the volume of a whole rectangular prism with dimensions of 12 units by 4 units by 14 units and subtracting the volume of a missing rectangular prism with dimensions of 7 units by 4 units by 6 units. The whole prism has a volume of 672 cubic units because $12 \times 4 \times 14 = 672$. The missing prism has a volume of 168 cubic units because $7 \times 4 \times 6 = 168$. Therefore, the equation for the volume of the figure shown is $672 - 168 = 504$ cubic units.

- Have students determine the volume of a figure by decomposing the figure into more than two rectangular prisms.

► Key Terms

additive, composition, decomposition, equivalent, rectangular prism, sum, three-dimensional, volume