A-F.1.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Extend understanding of fraction equivalence and ordering.

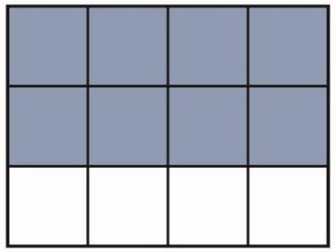
Find equivalencies and compare fractions.

▶ Eligible Content

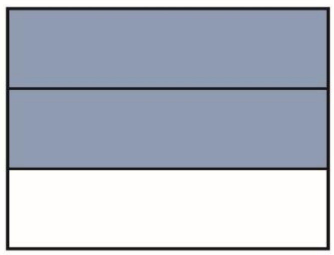
A-F.1.1.1 Recognize and generate equivalent fractions.

▶ Instructional Supports

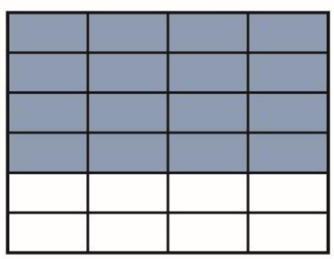
- Ask students to use physical or visual models to determine whether two fractions are equivalent. For example, give students fraction strips divided into fourths and twelfths and ask them whether $\frac{1}{4}$ is equivalent to $\frac{3}{12}$. Students can align the fraction strips and recognize that the fraction $\frac{3}{12}$ has the same area as $\frac{1}{4}$. Therefore, both fractions are equivalent.
- Give students a fraction and ask them to determine an equivalent fraction with a given denominator by comparing physical models. For example, give students pieces of circles that are cut into fifths and tenths and ask students to match up fractions that are equivalent to $\frac{3}{5}$. Students can use the fifths to show the fraction $\frac{3}{5}$. Students can then create the same shape using the tenths and count how many tenths are needed. Because 6 tenth pieces are required, students conclude that $\frac{3}{5} = \frac{6}{10}$.
- Ask students to create equivalent fractions by adding partitions to or removing partitions from a fraction model. For example, give students the fraction $\frac{8}{12}$. Students can draw the rectangle shown.



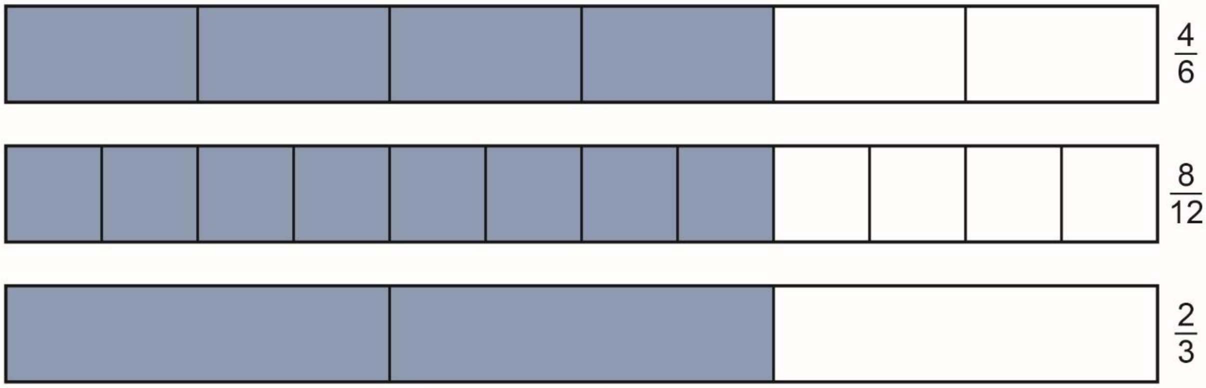
- Students can redraw the rectangle, removing some of the partitions as shown, to discover the equivalent fraction $\frac{2}{3}$.



- Students can redraw the rectangle, dividing each square into 2 parts as shown, to discover the equivalent fraction $\frac{16}{24}$.



- Ask students to explain why equivalent fractions can be created by multiplying or dividing both the numerator and denominator by the same number. For example, students can explore the fraction $\frac{4}{6}$ by using paper strips to discover the following equivalent fractions.



$$\frac{4 \times 2}{6 \times 2} = \frac{8}{12}$$

$$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

Students should recognize that when each section of $\frac{4}{6}$ is split into 2 parts, the number of shaded regions (the numerator) is doubled and the number of total regions (the denominator) is also doubled. When sections of $\frac{4}{6}$ are merged together, the number of shaded regions (the numerator) is divided by 2 and the number of total regions (the denominator) is also divided by 2.

- Ask students to numerically determine whether two fractions are equivalent. For example, give students the fractions $\frac{3}{8}$ and $\frac{6}{10}$. The numerator of $\frac{3}{8}$ can be multiplied by 2 to get 6, the numerator of $\frac{6}{10}$. However, multiplying the first denominator by 2 does not also give the same denominator ($8 \times 2 \neq 10$). Therefore, the fractions are not equivalent.

▶ Identifying and Addressing Common Misconceptions

- Some students may think that equivalent fractions can be created by adding the same number to both the numerator and denominator. Ask students whether $\frac{5}{8}$ and $\frac{7}{10}$ are equivalent and explain why. Some students may think the fractions are equivalent because $5 + 2 = 7$ and $8 + 2 = 10$. Ask students how many eighths are "missing" from a whole in the first fraction and how many tenths are "missing" from a whole in the second fraction. Then ask students whether one eighth is the same size as one tenth. Help students recognize that the gap between $\frac{5}{8}$ and 1 is not the same as the gap between $\frac{7}{10}$ and 1.

▶ Enrichment

- Ask students to explain why any fraction in which the numerator and denominator are equal is equivalent to 1.
- Show students how to multiply fractions, then ask students to explain why multiplying the numerator and denominator of a fraction by the same number results in an equivalent fraction. For example, ask students to multiply both the numerator and denominator of $\frac{2}{3}$ by 5.

  $$\frac{2 \times 5}{3 \times 5} = \frac{2}{3} \times \frac{5}{5} = \frac{2}{3} \times 1$$

  Because $\frac{2}{3} \times 1 = \frac{2}{3}$, $\frac{2 \times 5}{3 \times 5}$ is equivalent to $\frac{2}{3}$. Ask students to determine equivalent fractions that do not have whole number numerators or denominators. For example, ask students to determine a fraction with a denominator of 3 that is equivalent to $\frac{1}{2}$. Students can use visual models to create the fraction $\frac{1.5}{3}$.
- Ask students to recognize equivalent fractions in which the numerators and denominators are not multiples of each other. For example, ask students whether $\frac{4}{6}$ is equivalent to $\frac{6}{9}$. Though neither numerator is a multiple of the other, students can show that both fractions are equivalent to $\frac{2}{3}$ or $\frac{12}{18}$.

▶ Key Terms

area model, denominator, equivalent, fraction, fraction models, multiples, multiply, numerator, tape diagram

A-F.1.1.2

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Extend understanding of fraction equivalence and ordering.

Find equivalencies and compare fractions.

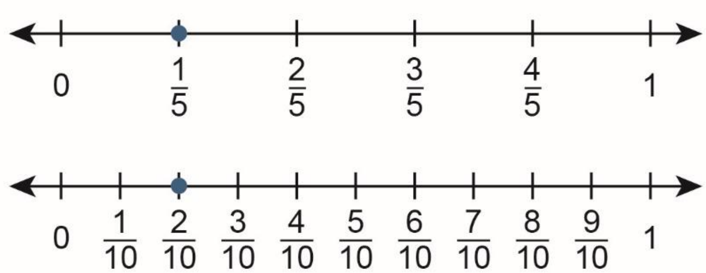
▶ Eligible Content

A-F.1.1.2 Compare two fractions with different numerators and different denominators (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100) using the symbols >, =, or <, and justify the conclusions.

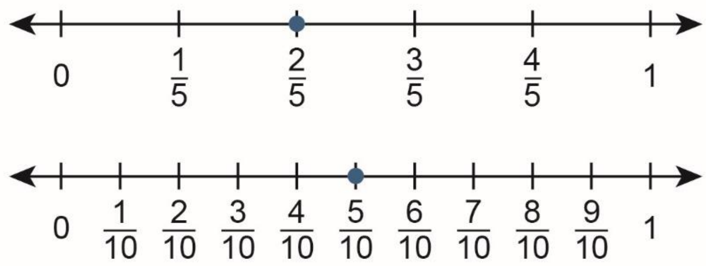
▶ Instructional Supports

Comparing fractions

- Ask students to use visual models to show fraction equivalence. For example, ask students to show that the fraction $\frac{1}{5}$ is equivalent to $\frac{2}{10}$. A possible response is shown.



- Ask students to demonstrate the knowledge that less than (or <) means "to the left of" on a number line, while greater than (or >) means "to the right of" on a number line. For example, ask students to write a comparison statement for the fractions shown.

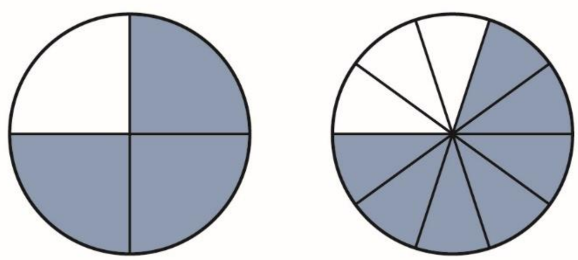


Because $\frac{2}{5}$ is to the left of $\frac{5}{10}$, students write the comparison $\frac{2}{5} < \frac{5}{10}$.

- Ask students to use the symbols <, >, and = to compare fractions. For example, ask students to compare $\frac{5}{12}$ and $\frac{1}{4}$. Students determine, using visual models or numeric methods, that $\frac{5}{12}$ is greater and write the comparison $\frac{5}{12} > \frac{1}{4}$.

Strategies for comparisons

- Ask students to use visual models to compare fractions. For example, ask students to create visual models to compare $\frac{3}{4}$ and $\frac{7}{10}$. Students might create the following models.



The model for $\frac{3}{4}$ has a greater shaded area than the model for $\frac{7}{10}$. Therefore, $\frac{3}{4}$ is greater than $\frac{7}{10}$.

- Ask students to use benchmark fractions to compare fraction size. For example, ask students to compare $\frac{3}{8}$ and $\frac{7}{12}$. Students can compare both fractions to the benchmark fraction of $\frac{1}{2}$. Students recognize that $\frac{3}{8}$ is less than $\frac{1}{2}$ because $\frac{3}{8}$ is less than $\frac{4}{8}$, which is equivalent to $\frac{1}{2}$. Likewise, students recognize that $\frac{7}{12}$ is greater than $\frac{1}{2}$ because $\frac{7}{12}$ is greater than $\frac{6}{12}$, which is equivalent to $\frac{1}{2}$. Using this, students determine that $\frac{3}{8} < \frac{7}{12}$.

- Ask students to compare fractions by using lists of multiples to determine common denominators. For example, give students the fractions $\frac{4}{5}$ and $\frac{5}{6}$. Because the denominators are different, lists of multiples may be used to determine a potential common denominator.

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, . . .

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, . . .

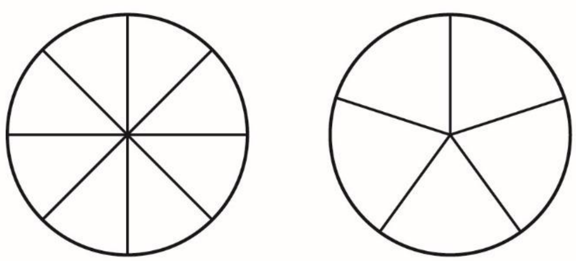
A multiple discovered in both lists is 30. Therefore, both fractions may be written as equivalent fractions with a denominator of 30.

$$\frac{4\times 6}{5\times 6} = \frac{24}{30}$$

$$\frac{5\times 5}{6\times 5} = \frac{25}{30}$$

Since 25 is greater than 24, $\frac{5}{6}$ is greater than $\frac{4}{5}$.

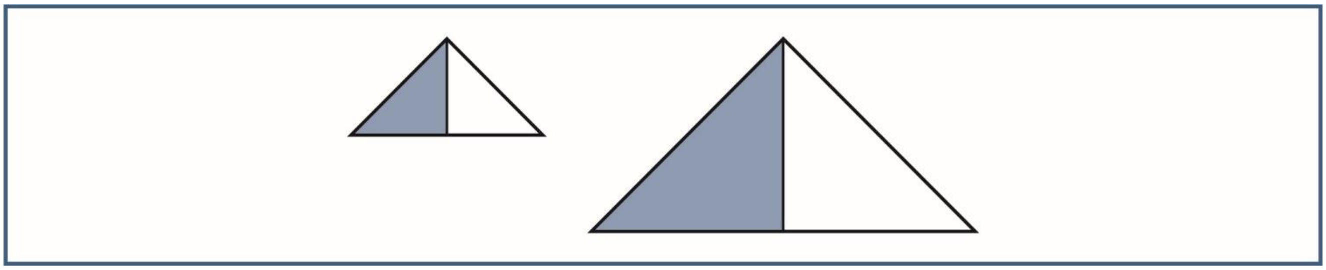
- Ask students to compare fractions by reasoning about the sizes of the pieces in the fraction. For example, ask students to compare $\frac{2}{5}$ with $\frac{2}{8}$ and to compare $\frac{4}{5}$ with $\frac{7}{8}$. Students may recognize, through either reasoning or visual models, that fifths are larger than eighths.



Therefore, $\frac{2}{5}$ is greater than $\frac{2}{8}$. On the other hand, $\frac{4}{5}$ is less than $\frac{7}{8}$ because the first fraction is one fifth away from being a complete whole and the second fraction is one eighth away from being a complete whole. Since eighths are smaller than fifths, $\frac{7}{8}$ is closer to a complete whole than $\frac{4}{5}$. So, $\frac{7}{8}$ is greater than $\frac{4}{5}$.

Comparisons in practice

- Ask students to recognize that fractions may only be compared when both wholes are the same size. For example, give students the model shown.



The shapes are similar triangles, and both have $\frac{1}{2}$ shaded; however, since they are of differing sizes, the fractions cannot be directly compared. The $\frac{1}{2}$ in the case of the smaller triangle does not represent the same area as $\frac{1}{2}$ of the larger triangle.

- Ask students to make real-world comparisons of fractional parts. Some examples are shown.
  - Ask students to compare the amount of pizza in $\frac{1}{8}$-slices of pizza from two different restaurants. The amount of pizza cannot be compared unless it is known whether the pizzas are the same size.
  - Ask students to compare running $\frac{1}{3}$ of a lap and running $\frac{2}{5}$ of a lap on the same track. Students can compare the fractions by creating equivalent fractions with a common denominator of 15. Because $\frac{1}{3} = \frac{5}{15}$ and $\frac{2}{5} = \frac{6}{15}$, students conclude that $\frac{1}{3}$ of a lap is a shorter distance than $\frac{2}{5}$ of a lap.
  - Ask students to compare the amount of liquid in $\frac{5}{6}$ carton of milk and $\frac{2}{3}$ bottle of water. The amount of liquid cannot be compared unless it is known whether the carton and bottle are the same size.
  - Ask students to compare the weights of $\frac{1}{3}$ pound of grapes and $\frac{1}{3}$ pound of cantaloupe. Though the fruit being compared are different, the whole involved with both fractions is a pound. Therefore, the weights can be compared and are equal.

► Identifying and Addressing Common Misconceptions

- Some students may think that fractions with greater denominators have greater values. Ask students which is greater, $\frac{1}{5}$ or $\frac{1}{6}$, and ask them to explain their thinking. Some students may respond that $\frac{1}{6}$ is greater than $\frac{1}{5}$ because $\frac{1}{6}$ has a greater denominator than $\frac{1}{5}$. Ask students to create a visual fraction model for fifths and sixths. Help students understand that $\frac{1}{5}$ is greater than $\frac{1}{6}$ because dividing a whole into 5 parts results in larger pieces than a whole divided into 6 parts.
- Some students may only compare numerators when comparing fractions. Ask students to compare $\frac{3}{5}$ with $\frac{4}{8}$. Some students may claim $\frac{4}{8}$ is greater than $\frac{3}{5}$ because the numerator in $\frac{4}{8}$ is greater than the numerator in $\frac{3}{5}$. With visual fraction models, help students see that $\frac{3}{5}$ is greater than $\frac{4}{8}$ and understand that it is important to compare the size of the units, not just the quantities of each unit, when comparing fractions.
- Some students may confuse the "less than" and "greater than" symbols. Ask students to use < or > to compare $\frac{1}{2}$ and $\frac{1}{8}$. Some students may say $\frac{1}{2}$ is greater than $\frac{1}{8}$ but write $\frac{1}{2} < \frac{1}{8}$ or $\frac{1}{8} > \frac{1}{2}$. Encourage students to come up with a mnemonic to help them remember the meaning of each symbol. For example, they could remember that the "mouth" or opening of the inequality symbol opens to the greater value, that the "point" of the symbol points to the lesser number, or that the "less than" symbol looks like the letter "L."

► Enrichment

- Ask students to discover two new fractions between two given fractions. For example, ask students to write two fractions that are between $\frac{3}{5}$ and $\frac{5}{6}$.
- Ask students to write a word problem that involves comparing two fractions.
- Place 9 cards labeled with one of each of the digits 1–9 inside an empty box. Ask students to pick a card without replacement and use it for one of the values of *a*, *b*, *c*, or *d* for the fractions $\frac{a}{b}$ and $\frac{c}{d}$. The student continues to pick cards without replacement with the goal of creating two fractions that are as far apart as possible on a number line. Students can play with each other to

locate their fractions on the number line and determine which pairs are farthest apart.

► Key Terms

benchmark fraction, common denominator, common numerator, compare, denominator, equal to (=), equation, equivalent fractions, fraction circle, greater than (>), less than (<), model, number line, numerator, tape diagram, whole

A-F.2.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

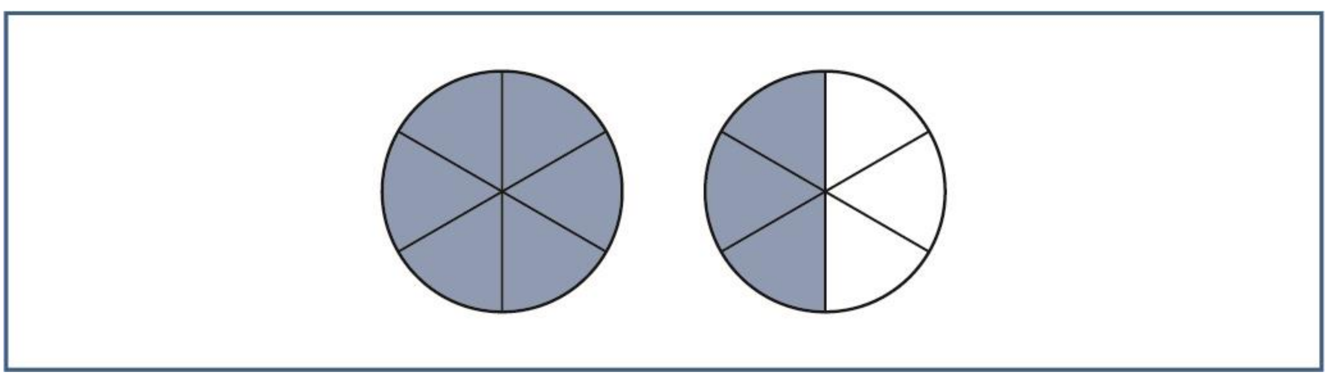
Solve problems involving fractions and whole numbers (straight computation or word problems).

► Eligible Content

A-F.2.1.1 Add and subtract fractions with a common denominator (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100; answers do not need to be reduced; and no improper fractions as the final answer).

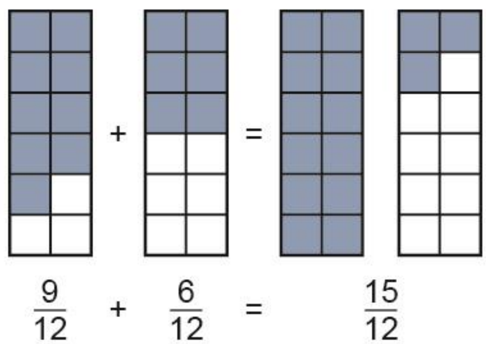
► Instructional Supports

- Ask students to decompose a given fraction into unit fractions that can be added to determine the value of the initial fraction. For example, provide the fraction $\frac{4}{5}$ and ask students to show that the fraction can be decomposed into $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$. Similarly, $\frac{7}{4}$ can be decomposed into $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$.
- Ask students to interpret a model (i.e., a rectangle, a tape diagram, a number line, or a fraction circle) as both a single fraction and as a sum of unit fractions. For example, give students the model shown.

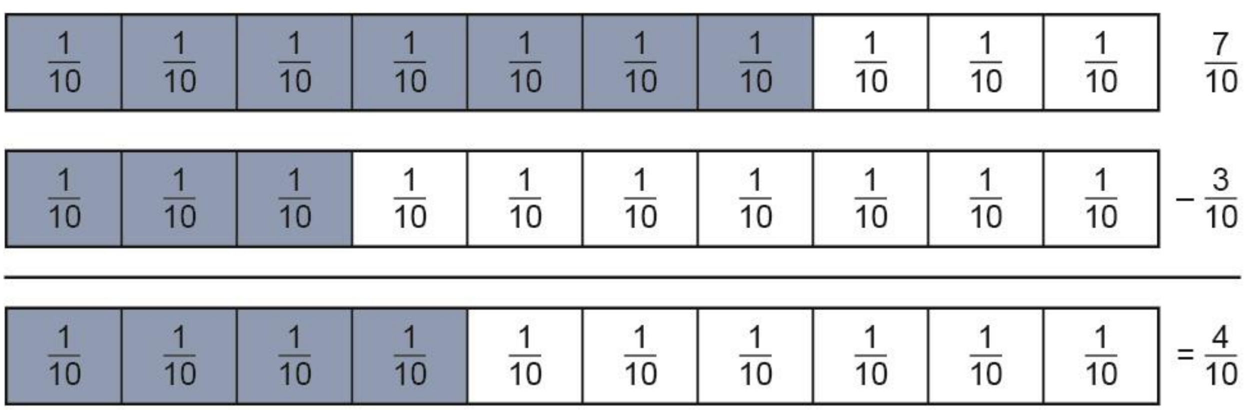


The circles are divided into sixths, and 9 of the sixths are shaded. Therefore, the model shows $\frac{9}{6}$, which can also be represented as $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$.

- Ask students to use models to show the sum or difference of fractions with like denominators as joining or separating parts.
    - Students may show $\frac{9}{12}+\frac{6}{12}$ as follows:



    - Students may show $\frac{7}{10}-\frac{3}{10}$ as follows:



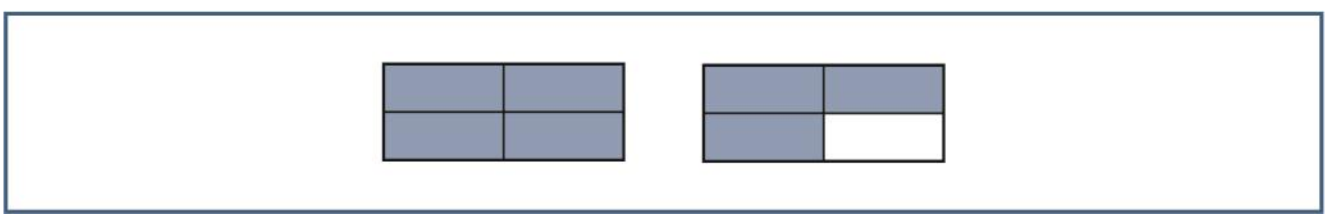
Students should observe that the models in each example have the same size partitions of the same size whole.

- Ask students to recognize when addition and subtraction are required in word problems and identify the key words or phrases that indicate which operation to use. For example, give students this word problem: “Josh eats $\frac{1}{2}$ of a pizza, and Jane eats $\frac{1}{3}$ of the same pizza. What fraction of the pizza did Josh and Jane eat altogether?” Addition is needed to solve the problem because the word “altogether” is an indicator for joining parts together. Note that solving problems with fractions that have unlike denominators is above grade level and is not used within this standard. Its purpose here is solely to determine the operation.

- Ask students to note similarities between addition and subtraction with whole numbers and addition and subtraction with fractions. For example, the expressions 9 + 7 and $\frac{9}{10} + \frac{7}{10}$ both indicate parts being joined together. Similarly, the expressions 9 – 7 and $\frac{9}{10} - \frac{7}{10}$ both indicate parts being removed. Students should note that in addition (joining), the order of the addition is not important, while in subtraction (removing), the order is important.
- Ask students to interpret word problems in order to understand whether the fractions refer to the same whole. For example, give students this word problem: “Charlie has $\frac{7}{8}$ of an apple. Malia has $\frac{2}{3}$ of a banana. How much more fruit does Charlie have than Malia?” In this example, subtracting the fractions does not make sense within the context because the wholes that each fraction refers to are different. Bananas may not be subtracted from apples.

► Identifying and Addressing Common Misconceptions

- Some students may misinterpret models for fractions greater than 1. Ask students to determine the fraction shown in the following model.



Some students may respond with $\frac{7}{8}$. Ask students to identify the whole that is shown in the model. Then cover up one half of the model at a time and ask them to evaluate each part separately.

- Some students may not be aware of whether fractions in a word problem have the same whole.

Ask students to explain how to solve this problem: “Jalen ate $\frac{3}{4}$ of a candy bar. Liana ate $\frac{1}{2}$ cup of ice cream. How much did Jalen and Liana eat altogether?” Some students may simply add the fractions. Ask students to identify the whole that is associated with each fraction. Then ask them whether the wholes can be directly compared.

- Some students may add both the numerator and denominator when adding fractions. Ask students to calculate the sum of $\frac{4}{10} + \frac{3}{10}$. Some students may give an answer of $\frac{7}{20}$. Ask students to show an area model of $\frac{4}{10}$. Then ask them to show an area model of $\frac{3}{10}$ that has the same whole. Help students understand that when the two models are combined, the whole is still the same whole that was used for each individual fraction. Even though there are 20 parts in the combined model, the whole is still only 10 parts.

► Enrichment

- Ask students to write word problems that utilize addition or subtraction of fractions.

► Key Terms

addition, compose, denominator, difference, equations, fractions, numerator, subtraction, sum, unit fraction, whole, whole numbers

A-F.2.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Solve problems involving fractions and whole numbers (straight computation or word problems).

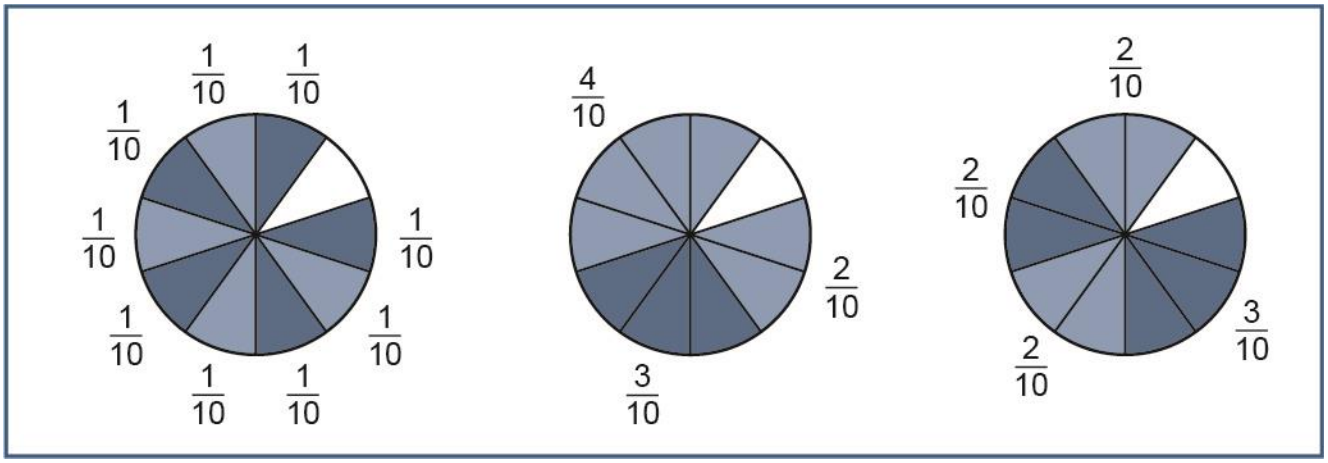
► Eligible Content

A-F.2.1.2 Decompose a fraction or a mixed number into a sum of fractions with the same denominator (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100), recording the decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model).

► Instructional Supports

Decomposing fractions

- Ask students to decompose both whole numbers and fractions in multiple ways. For example, ask students to decompose both 11 and $\frac{11}{12}$. The number 11 can be decomposed as 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1, as 10 + 1, or in many other ways. The fraction $\frac{11}{12}$ can be decomposed as $\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$, as $\frac{10}{12}+\frac{1}{12}$, or in many other ways. Students should note that in whole numbers, the entire number is decomposed, and in fractions, only the numerator is decomposed.
- Ask students to use models and/or manipulatives to decompose a fraction in multiple ways. For example, give students a model of $\frac{9}{10}$ and ask them to create different decompositions. Some possible responses are shown.



Further, have students express each decomposed model algebraically. For the examples shown, students may write $\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}$, or $\frac{4}{10}+\frac{3}{10}+\frac{2}{10}$, or $\frac{2}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}$.

Decomposing mixed numbers

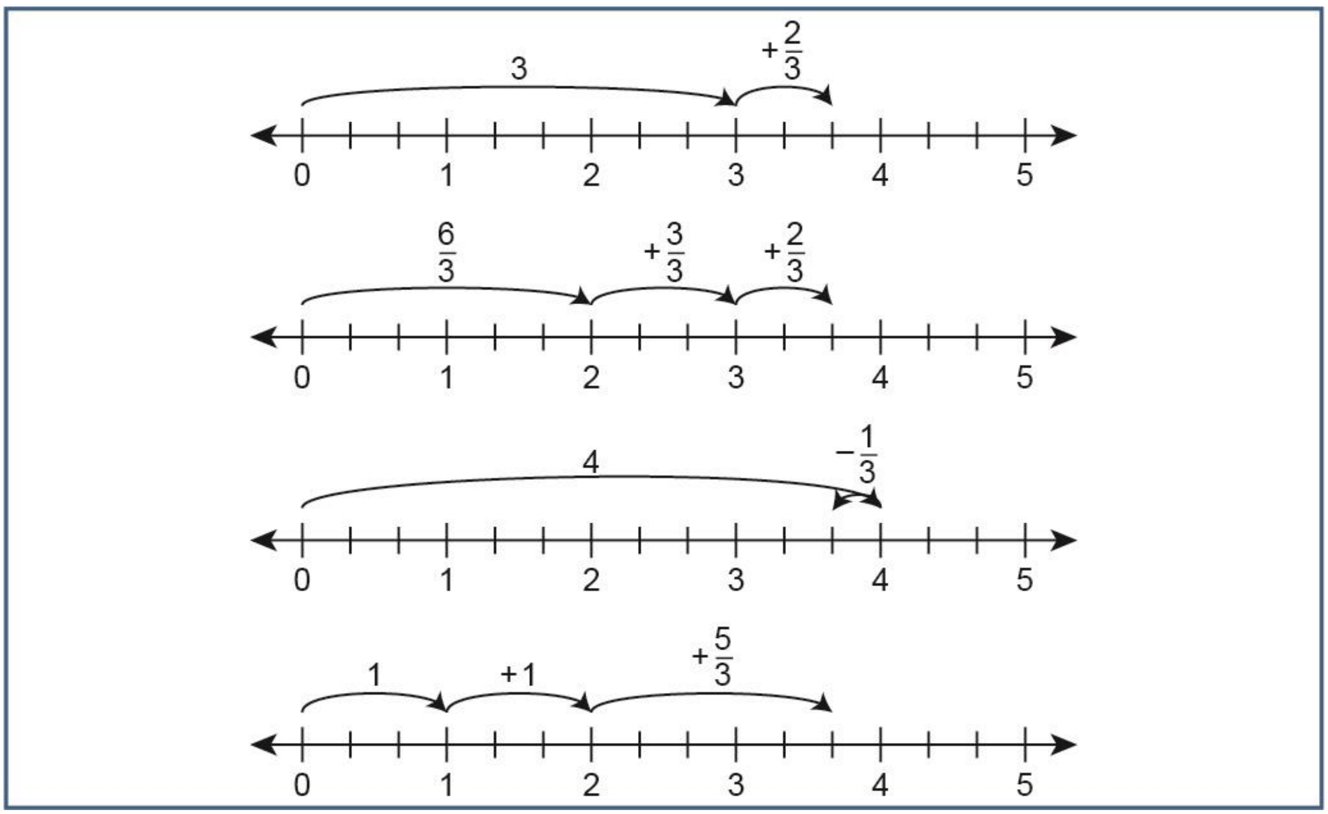
- Ask students to decompose mixed numbers using addition of fractions with like denominators. For example, give students the mixed number $6\frac{3}{8}$. Students may decompose it as $6+\frac{3}{8}$, as $1+1+4+\frac{3}{8}$, as $6+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$, or in many other ways.
- Ask students to interpret the connection between composed and decomposed mixed numbers. For example, students should note that $1\frac{11}{12}$ is the same as $1+\frac{11}{12}$, and they should be able to describe the relationship as "one **and** eleven-twelfths is the same as one **plus** eleven-twelfths."
- Ask students to decompose each of the following numbers in a variety of ways: 8, $\frac{4}{5}$, and $4\frac{8}{10}$. Then, with their list of decomposed numbers, ask the following questions:
  - What does it mean to decompose a number?
  - What are the similarities and differences between decomposing fractions and whole numbers?

Decomposing with subtraction

- Ask students to decompose fractions using addition and subtraction. For example, provide the mixed number $2\frac{7}{10}$. Student responses may include the following expressions:

- $1 + 1 + \frac{7}{10}$
- $2 + \frac{3}{10} + \frac{4}{10}$
- $\frac{20}{10} + \frac{7}{10}$
- $3 - \frac{3}{10}$
- $\frac{30}{10} - \frac{3}{10}$

- Ask students to use number lines to model decompositions of fractions and mixed numbers. For example, give students the value $3\frac{2}{3}$. Some possible student responses are shown.



► Identifying and Addressing Common Misconceptions

- Some students may think of decomposing a fraction as decomposing the numbers in the fraction rather than decomposing the quantity represented by the fraction. Ask students to decompose $\frac{3}{8}$. Some students may decompose the numerator and denominator separately and give a response like $\frac{1}{5}+\frac{2}{3}$. Ask students to decompose a whole number and create a visual model that shows why this decomposition is correct. Help students recognize that the decomposed parts can be recombined to form the original whole. Then ask them to create visual models for their decomposition of $\frac{3}{8}$ and help them show that their parts cannot be recombined to make $\frac{3}{8}$.

► Enrichment

- Ask students to decompose numbers using fractions as numerators instead of using whole number numerators. For example, students can decompose $3\frac{9}{12}$ as $1 + 2 + \frac{4\frac{1}{2}}{12} + \frac{4\frac{1}{2}}{12}$.
- Ask students to discover patterns in the number of decompositions different fractions have.
- Ask students to decompose fractions greater than 1 into mixed numbers. For example, ask students to decompose $\frac{19}{5}$ and then recompose the parts into a mixed number. A possible response is shown.

$$
\begin{aligned}
\frac{19}{5} &= \boxed{\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}} + \boxed{\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}} + \boxed{\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}} + \boxed{\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\,\tfrac{1}{5}\quad} \\
&= \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{4}{5} \\
&= 1 + 1 + 1 + \frac{4}{5} \\
&= 3 + \frac{4}{5} \\
&= 3\frac{4}{5}
\end{aligned}
$$

► Key Terms

compose, decompose, denominator, difference, equation, equivalence, fraction, like denominators, mixed number, numerator, sum, whole number

A-F.2.1.3

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

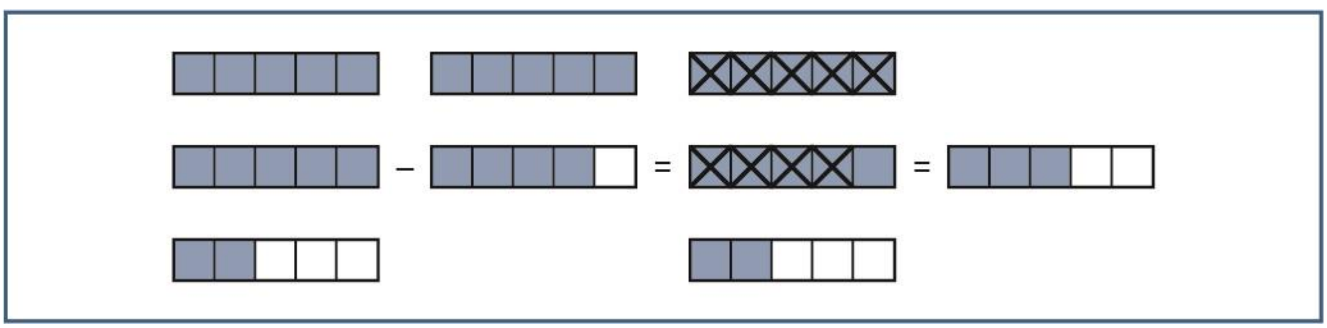
Solve problems involving fractions and whole numbers (straight computation or word problems).

▶ Eligible Content

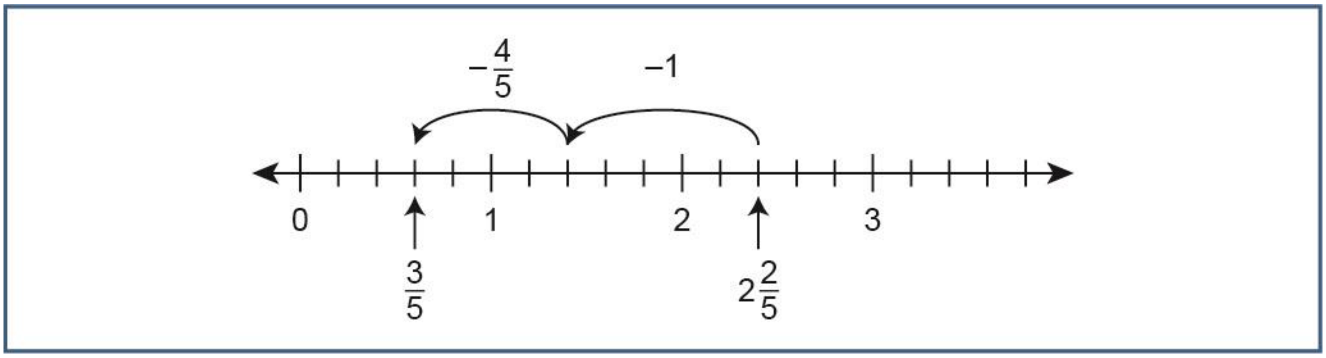
A-F.2.1.3 Add and subtract mixed numbers with a common denominator (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100; no regrouping with subtraction; fractions do not need to be simplified; and no improper fractions as the final answers).

▶ Instructional Supports

- Ask students to use visual models to solve problems involving addition and subtraction of fractions with like denominators. For example, give students the problem $2\frac{2}{5} - 1\frac{4}{5}$. Two possible solutions are shown.
    - Some students may use a bar model.



- Some students may use a number line.



In the number line example, the mixed numbers are decomposed into a whole number and a fraction and then those parts are subtracted separately. In both examples, the difference is $\frac{3}{5}$.

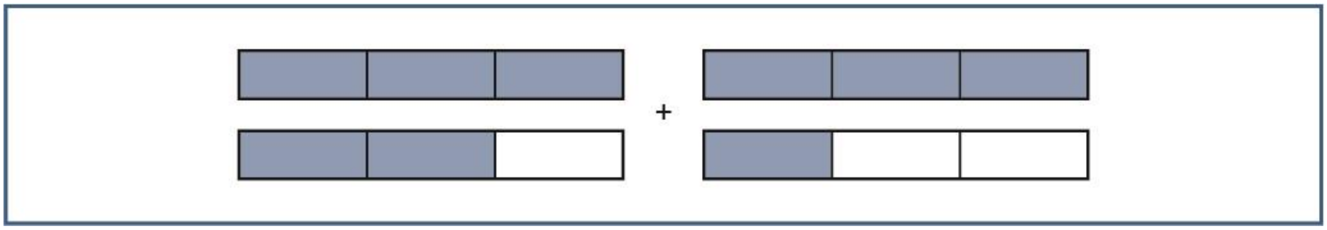
- Ask students to add or subtract fractions by decomposing them into parts and applying properties of addition and subtraction. For example, ask students to solve $8\frac{7}{8} + 3\frac{3}{8}$. A possible explanation is shown.

$8 + \frac{7}{8} + 3 + \frac{3}{8}$	decompose into whole and fraction parts
$8 + 3 + \frac{7}{8} + \frac{3}{8}$	use commutative property
$8 + 3 + \frac{10}{8}$	add fractions
$11 + \frac{10}{8}$	add whole numbers
$11 + \frac{8}{8} + \frac{2}{8}$	decompose the fraction
$11 + 1 + \frac{2}{8}$	rewrite as an equivalent value
$12 + \frac{2}{8}$	add whole numbers
$12\frac{2}{8}$	compose into a mixed number

- Ask students to add or subtract mixed numbers by replacing them with equivalent fractions. For example, ask students to solve $1\frac{3}{5} + 2\frac{4}{5}$. Because 1 is equivalent to $\frac{5}{5}$, the first addend can be rewritten as $\frac{8}{5}$. Because 2 is equivalent to $\frac{10}{5}$, the second addend can be rewritten as $\frac{14}{5}$. Students calculate the sum as $\frac{8}{5} + \frac{14}{5} = \frac{22}{5}$.

▶ Identifying and Addressing Common Misconceptions

- Some students may have inconsistent interpretations of the whole in visual models. Ask students to use the following figure to find the sum of $1\frac{2}{3} + 1\frac{1}{3}$.



  Some students may recognize that there are two wholes but still combine the other fractional models to get a sum of $2\frac{3}{6}$. Ask students to point out the parts of the model that indicate the 2 and ask them to describe the whole in that situation. Then ask them to explain the parts and whole that result in $\frac{3}{6}$. Point out to students that their whole in the two situations is inconsistent and help them understand that the whole needs to remain consistent through the entire sum.

- Some students may rewrite mixed numbers as improper fractions incorrectly. Ask students to write $5\frac{1}{4}$ as a single fraction. Some students may add or multiply the 5 and 1 to get an answer of either $\frac{6}{4}$ or $\frac{5}{4}$. Ask students how many fourths make up 1 whole and ask them to create a visual model for 5 wholes. Show how 5 wholes can be divided into 20 fourths. Help students extend this model to show why $5\frac{1}{4}$ is equivalent to $\frac{21}{4}$.

▶ Enrichment

- Ask students to draw connections between adding mixed numbers and adding decimals.
- Give students six random digits and ask them to arrange them into the expression $a\frac{b}{c} + d\frac{e}{f}$ in a way that makes the sum as great as possible.

▶ Key Terms

addition, decompose, denominator, equivalent fraction, fraction, like denominators, mixed number, numerator, subtraction, whole number

A-F.2.1.4

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

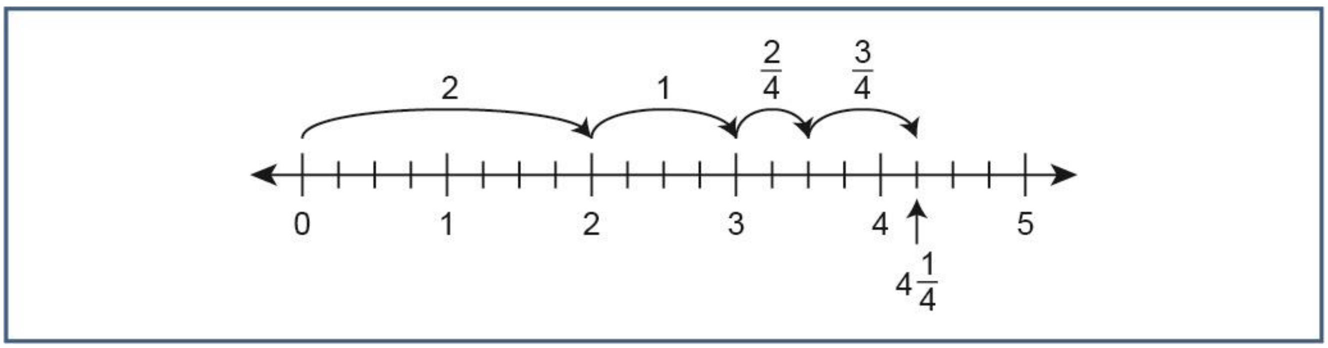
Solve problems involving fractions and whole numbers (straight computation or word problems).

▶ Eligible Content

A-F.2.1.4 Solve word problems involving addition and subtraction of fractions referring to the same whole or set and having like denominators (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100).

▶ Instructional Supports

- Ask students to solve a word problem using a visual model, manipulative, or equation. For example, provide this word problem: "Antoine did chores at his house. He cleaned his room for $1\frac{2}{4}$ hours, mowed the lawn for $\frac{3}{4}$ hour, and did laundry for 2 hours. How many hours did all of Antoine's chores take?" A possible model is shown.



  Students decompose the $1\frac{2}{4}$ as $1 + \frac{2}{4}$ and use the number line to add $2 + 1 + \frac{2}{4} + \frac{3}{4}$. The model shows a solution of $4\frac{1}{4}$ hours.

- Ask students to use equations or expressions to solve word problems. For example, give students this problem: "Melina and Amy have $\frac{9}{12}$ of a pan of lasagna. Melina eats $\frac{3}{12}$ of a pan of lasagna.

Amy also eats some of the lasagna. There is $\frac{4}{12}$ of a pan of lasagna left. What fraction of a pan of lasagna did Amy eat?" Students write the equation $\frac{9}{12} - \frac{3}{12} - \frac{4}{12} = \frac{2}{12}$ to find that Amy ate $\frac{2}{12}$ of a pan of lasagna.

► Identifying and Addressing Common Misconceptions

- Some students may misinterpret the whole that a fraction refers to. Give students this problem: "The students in Ms. Spence's class each go to the school play once during the week. On Monday, $\frac{1}{10}$ of her class attended the play, and on Tuesday, $\frac{2}{10}$ of her class attended." Ask students to explain what the sum $\frac{1}{10} + \frac{2}{10}$ means in this context. Some students may think it is the fraction of days that students attend the play. Ask students to note that both fractions in the problem are followed by the words "of her class." Help them recognize language cues that help indicate what the fractions refer to.

► Enrichment

- Give students specific values and ask them to use the values to write multiple word problems with different contexts and different operations.
- Ask students to write a word problem to represent a given visual model.
- Ask students to solve a multiplication word problem using repeated addition. For example, provide this word problem: "Janae is sewing hats and needs $\frac{1}{3}$ yard of fabric per hat. She wants to sew 5 hats. How many yards of fabric does Janae need?" Even if students have not learned to multiply fractions by whole numbers, students can still use repeated addition ($\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$) to find the solution.

► Key Terms

decompose, denominator, equation, expression, fraction, like denominators, mixed number, numerator, visual model, whole number, word problem

A-F.2.1.5

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Solve problems involving fractions and whole numbers (straight computation or word problems).

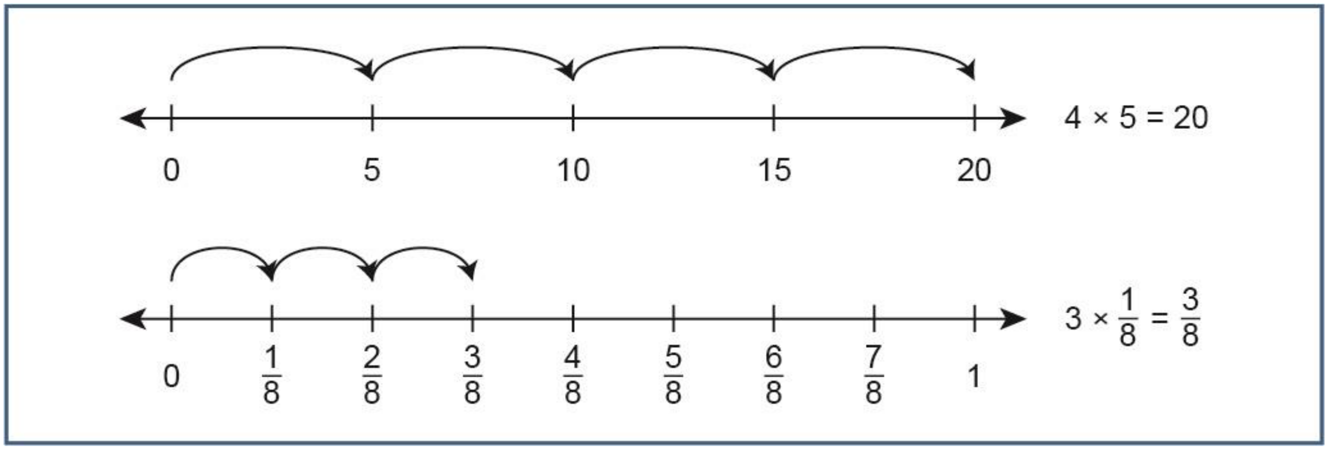
▶ Eligible Content

A-F.2.1.5 Multiply a whole number by a unit fraction (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100 and final answers do not need to be simplified or written as a mixed number).

▶ Instructional Supports

Extending whole number multiplication

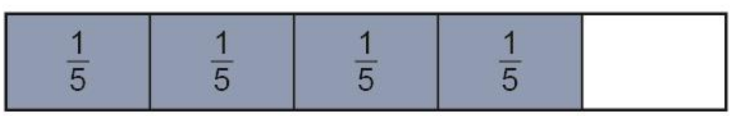
- Ask students to identify similarities between how multiplication is performed on whole numbers and how it is performed on fractions. For example, ask students to explain the meaning of 4×5 and extend that to explaining the meaning of $3 \times \frac{1}{8}$. Two possible responses are shown.
  - Students give a verbal description connecting whole number and fraction multiplication such as “4 times 5 is 4 groups of 5, and 3 times $\frac{1}{8}$ is 3 groups of $\frac{1}{8}$.”
  - Students create a visual representation such as a set of number lines to show similarities.

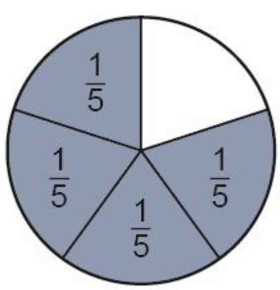


- Ask students to identify differences between multiplying whole numbers and multiplying whole numbers with unit fractions. For example, students may make statements such as “multiplying a whole number by a whole number results in a product that is also a whole number, but multiplying a whole number by a unit fraction can result in a whole number or a fraction.” Ask students to justify their statements with examples such as $3 \times 6 = 18$, $7 \times \frac{1}{10} = \frac{7}{10}$, and $10 \times \frac{1}{2} = \frac{10}{2}$ or 5.

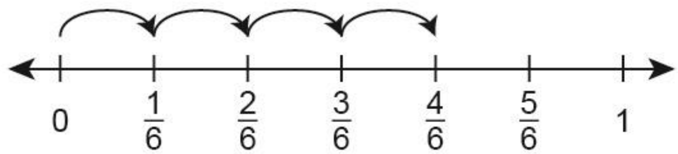
Non-unit fractions as products of unit fractions and whole numbers

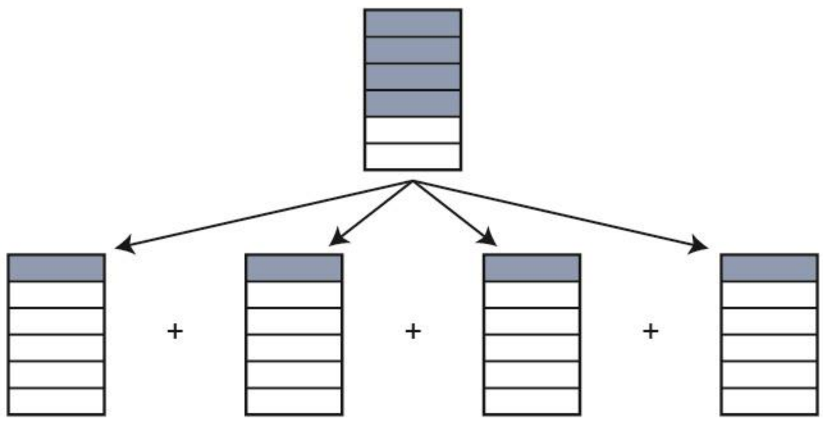
- Ask students to represent non-unit fractions in terms of unit fractions using a variety of manipulatives or models. For example, give students the fraction $\frac{4}{5}$. Two possible representations are shown.





- Give students a fraction and ask them to use manipulatives or visual models to represent it as the product of a whole number and a unit fraction. For example, give students the fraction $\frac{4}{6}$. Two possible models are shown.





Each model shows that $\frac{4}{6}$ is equal to $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$, which in turn is equal to $4 \times \frac{1}{6}$.

- Ask students to make generalizations regarding the relationship between unit fractions and non-unit fractions with the same denominator. For example, students may make notes such as "the numerator of a non-unit fraction is the number of unit fractions it could be separated into."
- Ask students to write equations to express non-unit fractions as a product of a whole number and a unit fraction. For example, give students the fraction $\frac{5}{8}$. Students can write the equation $5 \times \frac{1}{8} = \frac{5}{8}$.

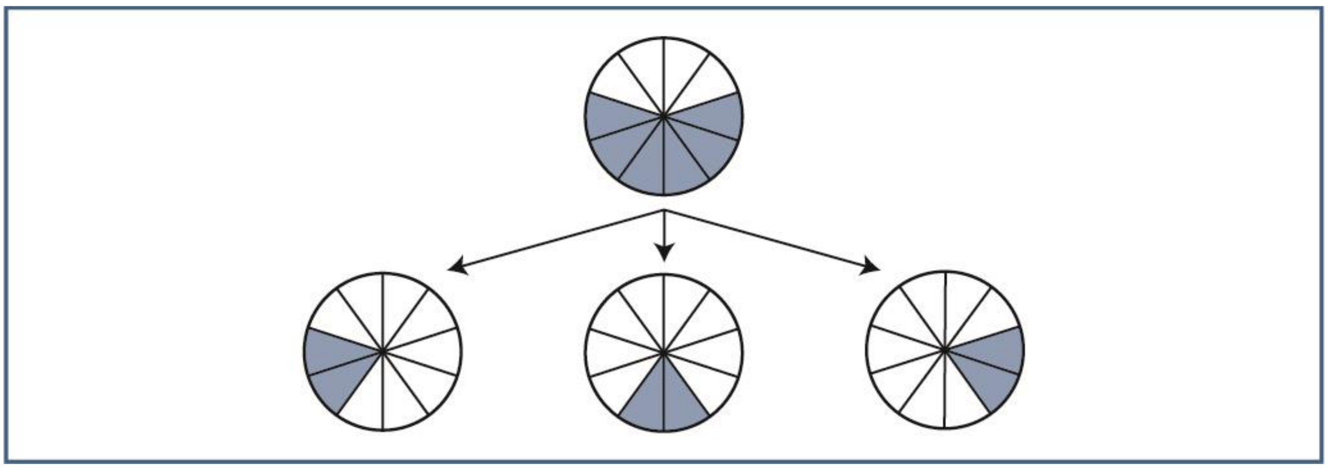
▶ Identifying and Addressing Common Misconceptions

- Some students may struggle to make the connection between multiplication and repeated addition when fractions are involved. Ask students to compare the expressions $4 \times \frac{1}{8}$ and

$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$. Some students may not recognize that the expressions are equivalent. Ask students to practice rewriting products of whole numbers as repeated sums and vice versa, and ask them to illustrate them using manipulatives and visual models. Help students transfer this understanding to fractions.

► Enrichment

- Ask students to write non-unit fractions as a product of a whole number and another non-unit fraction. For example, give students the fraction $\frac{6}{10}$. Students can create the following model to rewrite it as $3 \times \frac{2}{10}$.



► Key Terms

decompose, denominator, equation, mixed number, multiple, multiply, numerator, product, unit fraction, whole number

A-F.2.1.6

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

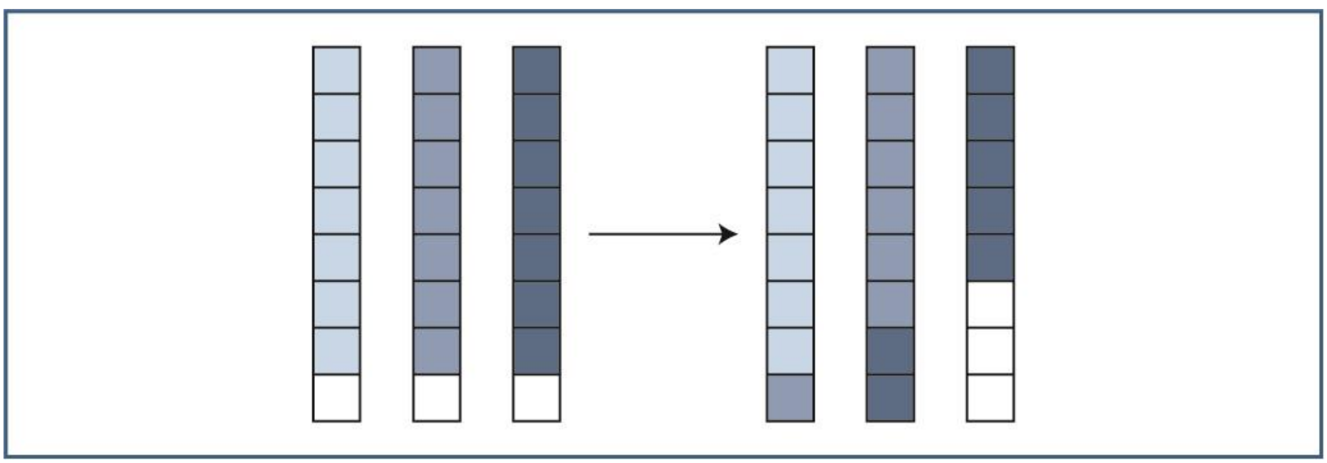
Solve problems involving fractions and whole numbers (straight computation or word problems).

▶ Eligible Content

A-F.2.1.6 Multiply a whole number by a non-unit fraction (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100 and final answers do not need to be reduced or written as a mixed number).

▶ Instructional Supports

- Ask students to use visual models to find the product of a whole number and a fraction. For example, give students the product $\frac{7}{8} \times 3$. A possible model is shown.



Using the diagram, students find the product to be equal to $\frac{21}{8}$ or $2\frac{5}{8}$.

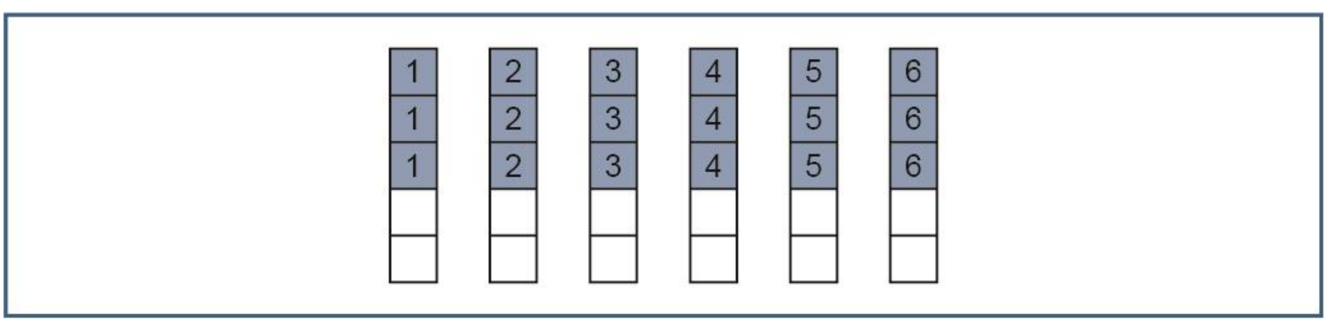
- Ask students to use properties of multiplication to multiply whole numbers by fractions. Students will use the associative, commutative, distributive, and identity properties in writing equivalent expressions. They do not, however, need to name or identify properties by name. For example, give students the expression $\frac{7}{12} \times 6$. Some possible responses are shown.

- Because $\frac{7}{12}$ is equal to $7 \times \frac{1}{12}$, the product can be written as $7 \times \frac{1}{12} \times 6$, which is equal to $42 \times \frac{1}{12}$. This, in turn, is equal to $\frac{42}{12}$.
- The product $\frac{7}{12} \times 6$ is equal to $\frac{7}{12} + \frac{7}{12} + \frac{7}{12} + \frac{7}{12} + \frac{7}{12} + \frac{7}{12}$. Students can use fraction addition to calculate the sum as $\frac{7+7+7+7+7+7}{12} = \frac{42}{12}$.

- Ask students to use different types of multiplication language to describe multiplication of a whole number by a fraction. For example, give students the expression $\frac{3}{10} \times 5$. Students may describe it as "three-tenths of 5" or "five groups of three-tenths."
- Ask students to recognize patterns in multiplying whole numbers and fractions and generalize them. For example, have students generate statements such as "When multiplying a whole number by a fraction, always multiply the numerator by the whole number and keep the denominator the same."

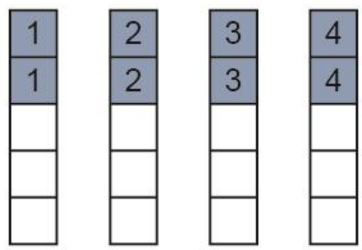
► Identifying and Addressing Common Misconceptions

- Students may misrepresent multiplication of a whole number with a fraction by mistaking the values of the numerator and denominator. Ask students to use a visual model to multiply $3 \times \frac{5}{6}$. Some students interchange the meanings of the numbers and create the following model representing 6 groups of $\frac{3}{5}$.



Encourage students to build off their previous understanding of multiplication and fractions instead of trying to memorize a rule for creating a model. Ask students to create an array to model 3×4 and help them recognize that they are showing 3 copies of a group of 4. Then ask students to create a model that shows three copies of the fraction $\frac{5}{6}$.

- Some students may multiply both the numerator and the denominator by the whole number. Ask students to multiply $\frac{2}{5} \times 4$. Some students may confuse multiplying by a whole number with multiplying to create equivalent fractions and multiply $\frac{2 \times 4}{5 \times 4} = \frac{8}{20}$. Ask students to explain how to multiply 3×7 using repeated addition. Then ask them to create a visual model showing 4 copies of $\frac{2}{5}$.



Ask students how many parts make up a whole. Students should note that there are 8 parts that are shaded and the whole is made up of 5 parts. Therefore, the product is $\frac{8}{5}$, not $\frac{8}{20}$.

► Enrichment

- Ask students to recognize when multiplying a whole number by a fraction results in a product that is a whole number. For example, students may notice that $a \times \frac{3}{8}$ is a whole number whenever a is a multiple of 8.
- Ask students to use properties of arithmetic or visual models to justify why $a \times \frac{b}{c} = b \times \frac{a}{c}$.
- Ask students to extend their understanding of factor pairs and find as many whole number and fraction pairs that multiply to a given fraction as they can. For example, give students the fraction $\frac{15}{8}$. Students may find the "factor pairs" $1 \times \frac{15}{8}$, $3 \times \frac{5}{8}$, $5 \times \frac{3}{8}$, and $15 \times \frac{1}{8}$. Students may notice that the number of "factor pairs" for the fraction is twice the number of factor pairs of the numerator unless the numerator is a perfect square.

► Key Terms

compose, decompose, denominator, equation, mixed number, multiple, multiply, numerator, product, unit fraction, whole number

A-F.2.1.7

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

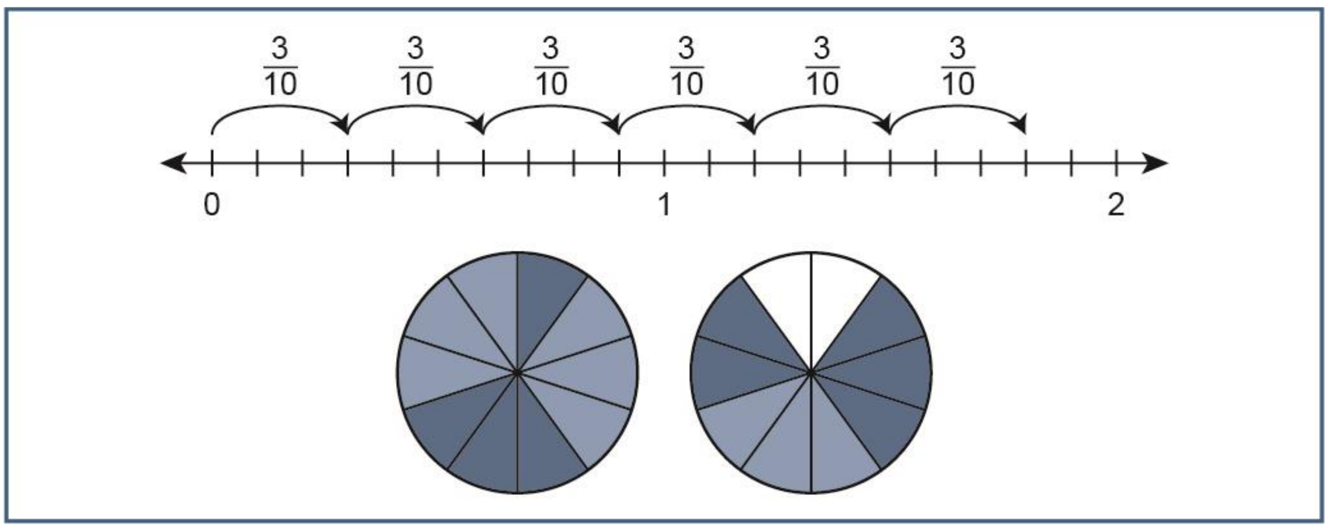
Solve problems involving fractions and whole numbers (straight computation or word problems).

▶ Eligible Content

A-F.2.1.7 Solve word problems involving multiplication of a whole number by a fraction (denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100).

▶ Instructional Supports

- Ask students to solve word problems containing multiplication of a whole number and a fraction by creating visual representations of the problems. For example, provide this problem: "Marc serves each of his 6 friends $\frac{3}{10}$ of a cake. How much cake does Marc serve his friends?" Possible student responses are shown.



Each model shows that $6 \times \frac{3}{10} = \frac{18}{10}$, or $1\frac{8}{10}$. Marc serves $1\frac{8}{10}$ cakes to his friends.

- Ask students to solve word problems containing multiplication of a whole number and a fraction by creating an equation. For example, provide this problem: "Marius reads a book with 5 chapters. Each chapter takes him $\frac{7}{8}$ hour to read. How long does it take him to read the entire book?" Students can give the response $5 \times \frac{7}{8} = \frac{35}{8}$.

► Identifying and Addressing Common Misconceptions

- Some students may misinterpret the operation to be used in the problem. Give students this problem: "Jaylen uses $\frac{1}{3}$ cup of brown sugar for each batch of cookies he bakes. He makes 2 batches. How much brown sugar does he need?" Some students may connect the idea of multiplication to repeated addition but misinterpret the situation to think they should add $2 + \frac{1}{3}$. Ask them to show 5×3 using repeated addition. Help them draw parallels to fraction multiplication. In particular, students should recognize that when writing multiplication as repeated addition, the same addend is being repeated, such as $\frac{1}{3} + \frac{1}{3}$. Also, ask them to create a visual model showing two batches of cookies. Then ask them to show a model of $\frac{1}{3}$ cup of brown sugar for each of the batches. Ask them to explain how this model shows the problem using repeated addition.

► Enrichment

- Provide students with a fraction and a whole number and ask them to write a word problem using those values.
- Ask students to find solutions to word problems presented with a missing factor. For example, provide this problem: "Malik has 5 boxes of pizza. Each box of pizza is $\frac{?}{4}$ full. What number can replace the '?' to show that Malik has $\frac{15}{4}$ pizzas?"
- Ask students to solve word problems that require multiplying two whole numbers and a fraction. Students can compare the strategy of multiplying the two whole numbers first with the strategy of multiplying a whole number and the fraction and then multiplying the second whole number. Students should recognize that both strategies give the same answer.

► Key Terms

denominator, equation, mixed number, multiple, multiply, numerator, product, unit fraction, whole number

A-F.3.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Understand decimal notation for fractions and compare decimal fractions.

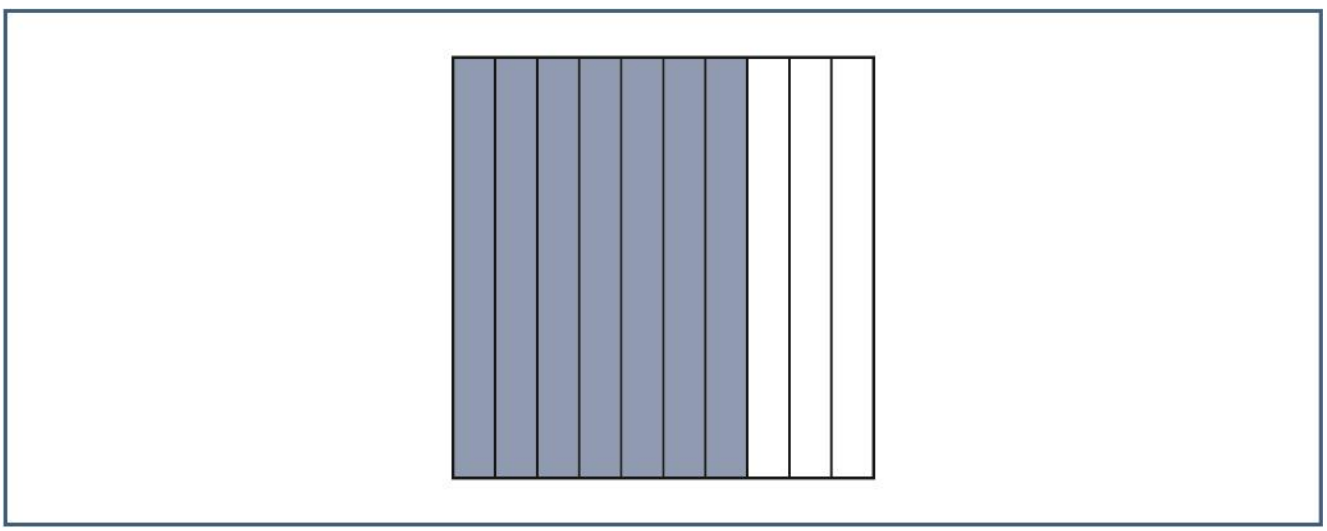
Use operations to solve problems involving decimals, including converting between fractions and decimals (may include word problems).

▶ Eligible Content

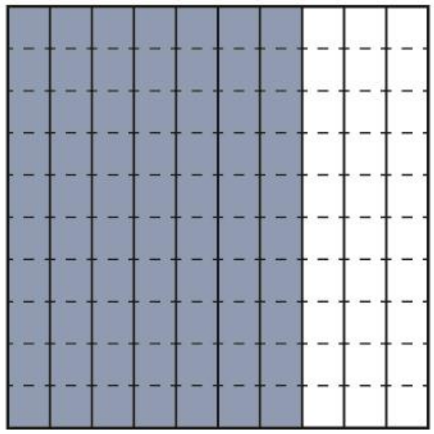
A-F.3.1.1 Add two fractions with respective denominators 10 and 100.

▶ Instructional Supports

- Ask students to use models to rewrite a fraction with 10 in the denominator as a fraction with 100 in the denominator. For example, give students the fraction $\frac{7}{10}$. Students can create the following model.

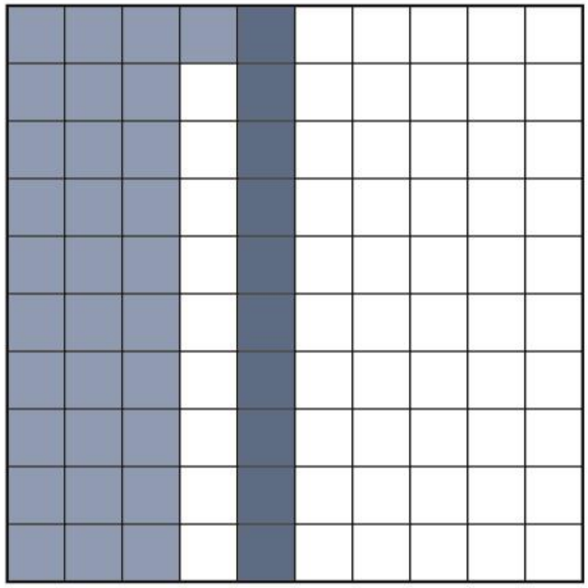


Students then divide each column into ten rows as shown.



The second model shows 70 shaded pieces out of 100 and represents the fraction $\frac{70}{100}$. Because both models show the same shaded area of the same size whole, this shows $\frac{70}{100} = \frac{7}{10}$.

- Ask students to use their understanding of equivalent fractions to rewrite a fraction with 10 in the denominator as a fraction with 100 in the denominator. For example, give students the fraction $\frac{3}{10}$. Students should recognize that $10 \times 10 = 100$. Students can multiply both the numerator and denominator to create the equivalent fraction $\frac{30}{100}$. If students have learned to multiply fractions, they can also recognize that $\frac{10}{10} = 1$ and write $\frac{3}{10} = \frac{3}{10} \times 1 = \frac{3}{10} \times \frac{10}{10} = \frac{30}{100}$.
- Ask students to use models to add fractions with 10 or 100 in the denominator. For example, ask students to add $\frac{31}{100} + \frac{1}{10}$. Students can create the following model.



The model shows 4 complete columns, which represent a total of $\frac{40}{100}$, and 1 additional square, which represents $\frac{1}{100}$, for a total of $\frac{41}{100}$.

- Ask students to solve addition problems involving fractions with denominators of 10 and 100 by rewriting the fractions with 10 in the denominator as fractions with 100 in the denominator, then adding the two fractions with like denominators. For example, give students the sum $\frac{51}{100} + \frac{3}{10}$. Students first find a fraction with a denominator of 100 that is equivalent to $\frac{3}{10}$ by multiplying both the numerator and denominator by 10. This gives the equivalent fraction $\frac{30}{100}$. Once both fractions have a common denominator of 100, the sum may be found by calculating $\frac{51}{100} + \frac{30}{100} = \frac{81}{100}$.

► Identifying and Addressing Common Misconceptions

- Some students may convert fractions with denominators of 10 to fractions with denominators of 100 using addition instead of multiplication. Ask students to rewrite $\frac{4}{10}$ as a fraction with a denominator of 100. Some students may add 90 to both the numerator and denominator, resulting in the fraction $\frac{94}{100}$. Ask students to create a model for $\frac{4}{10}$. Then ask them to describe how they can divide that same model into 100 equal parts. For all 100 parts to be equal, each of the original 10 parts needs to be divided into 10 equal parts. Help students recognize that this multiplies both the number of shaded parts by 10 and the number of total parts by 10.

- Some students may add a fraction with a denominator of 10 to a fraction with a denominator of 100 without first getting a common denominator. Ask students to find the sum of $\frac{2}{10}$ and $\frac{51}{100}$. Some students may respond that the sum is $\frac{53}{110}$, having added the numerators and then added the denominators. Give students a hundreds grid and ask them to use it to show the fraction $\frac{2}{10}$. Then ask them to show $\frac{51}{100}$ in the same grid. Help them extend their previous understanding of addition as joining parts and use it to conclude that their grid represents the sum $\frac{2}{10} + \frac{51}{100}$.
- Some students may not recognize that a single column (or row) of a model is equivalent to $\frac{1}{10}$ when using a hundreds grid. Give students a hundreds grid and ask them to show $\frac{3}{10}$. Some students may shade only 3 squares, thinking that each square represents $\frac{1}{10}$. Have students determine the total number of squares (100). Then ask them to sort the 100 squares into 10 equal parts. Students should determine that each of the 10 parts has 10 small squares in it, so $\frac{1}{10}$ of the diagram is equivalent to 10 small squares, not just 1.

► Enrichment

- Ask students to find fractions that are between two given fractions expressed with denominators of 10. For example, have students list fractions between $\frac{8}{10}$ and $\frac{9}{10}$ by first converting those fractions to $\frac{80}{100}$ and $\frac{90}{100}$ and then noting that fractions such as $\frac{81}{100}$, $\frac{85}{100}$, and $\frac{87}{100}$ all lie between $\frac{8}{10}$ and $\frac{9}{10}$ on a number line. This could be further extrapolated to find fractions between $\frac{81}{100}$ and $\frac{82}{100}$ using denominators of 1,000.
- Ask students to work backward and decompose a given fraction with a denominator of 100 into the sum of two different fractions, one with a denominator of 10 and the other with a

denominator of 100. For example, given the fraction $\frac{47}{100}$, students can decompose it in various ways, including $\frac{4}{10}+\frac{7}{100}$, $\frac{3}{10}+\frac{17}{100}$, $\frac{2}{10}+\frac{27}{100}$, and so on.

- Ask students to add fractions in which one denominator is a multiple of the other.

► Key Terms

addition, denominator, equivalent fraction, fraction models, fractions, hundredths, hundredths grid, numerator, tenths

A-F.3.1.2

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Understand decimal notation for fractions and compare decimal fractions.

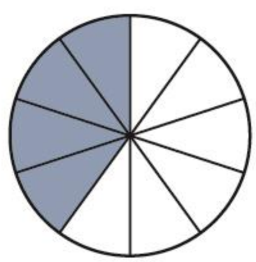
Use operations to solve problems involving decimals, including converting between fractions and decimals (may include word problems).

▶ Eligible Content

A-F.3.1.2 Use decimal notation for fractions with denominators 10 or 100.

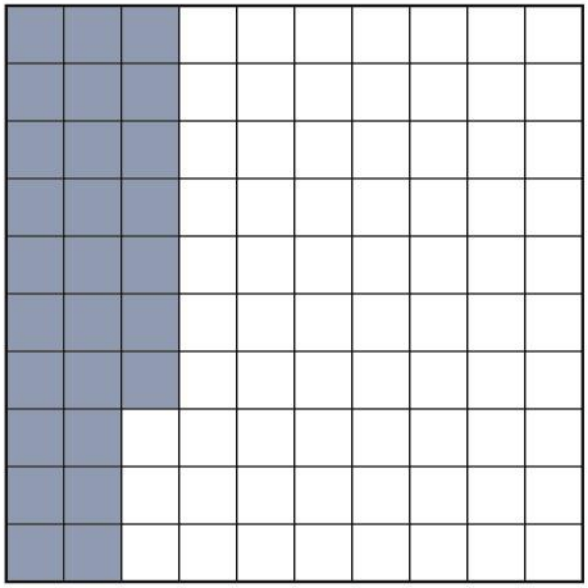
▶ Instructional Supports

- Ask students to use visual models for decimals to express decimals as fractions. Some examples are shown.
    - Give students the decimal 0.4. Students can create the following model.



As with fractions, a whole is divided into equal portions. Because the model shows 4 parts shaded out of 10 equal parts, the model can also be used to represent the fraction $\frac{4}{10}$.

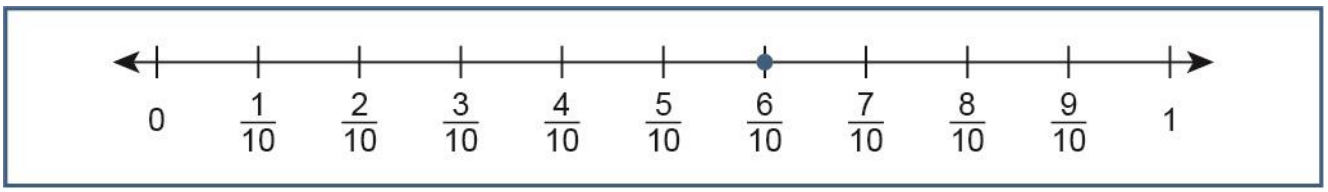
- Give students the decimal 0.27. Students can create the following model.



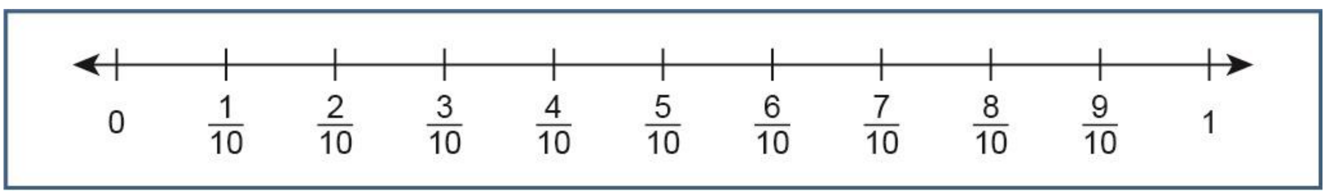
As with fractions, the whole is divided into equal portions. Because the model shows 27 parts shaded out of 100 equal parts, the model can also be used to represent the fraction $\frac{27}{100}$.

- Ask students to use their understanding of place value to write fractions with denominators of 10 or 100 as decimals. Some examples are shown.
  - Give students the fraction $\frac{8}{10}$. Because the fraction is "eight-tenths," the equivalent decimal, 0.8, has an 8 in the tenths place.
  - Give students the fraction $\frac{67}{100}$. Because the fraction is "sixty-seven hundredths," the equivalent decimal, 0.67, is 67 with the 7 in the hundredths place.
- Ask students to develop informal rules about how to read and write decimals and decimal fractions. Some examples include, "When a number has one digit after the decimal point, it is read as and written with tenths. When a number has two digits after the decimal point, it is read as and written with hundredths."

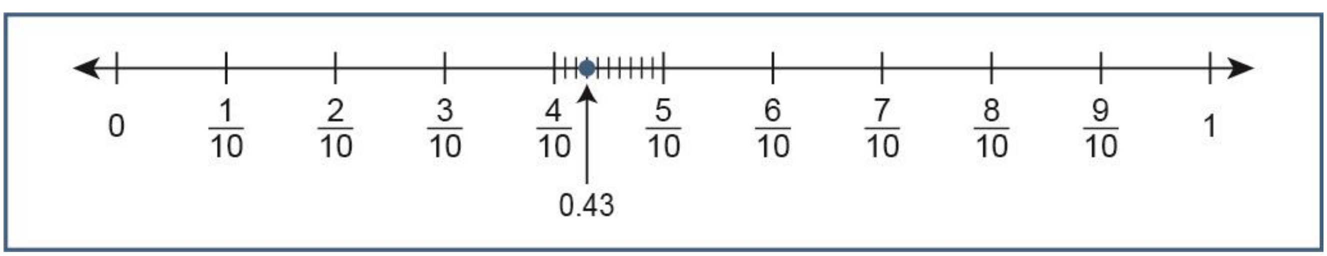
- Ask students to decompose decimals using products of decimal fractions in order to better understand decimal place values. For example, students write 2.9 as $2 \times 1 + 9 \times \frac{1}{10}$ or $29 \times \frac{1}{10}$ and read it as "two and nine-tenths" or "twenty-nine tenths."
- Ask students to use their understanding of fractions to place decimals and decimal fractions on number lines starting at 0. Some examples are shown.
    - Give students a number line from 0 to 1 and ask them to plot the decimal 0.6. Because 0.6 is equal to $\frac{6}{10}$, students divide the interval from 0 to 1 into 10 equal sections and mark a point that is 6 sections away from 0.



- Give students a number line from 0 to 1 and ask them to plot the decimal 0.43. Students can start by dividing the interval from 0 to 1 into 10 equal sections as shown.

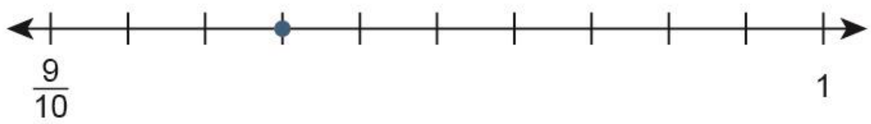


Because $\frac{4}{10} = \frac{40}{100} = 0.4$ and $\frac{5}{10} = \frac{50}{100} = 0.5$, the decimal 0.43 lies between $\frac{4}{10}$ and $\frac{5}{10}$. Students further divide the interval from $\frac{4}{10}$ to $\frac{5}{10}$ into 10 equal sections as shown.



Dividing a tenth into 10 equal sections results in parts that each represent one hundredth. The decimal 0.43 is located 3 sections past $\frac{4}{10}$.

- Ask students to locate decimals on a number line that does not start at 0. For example, ask students to plot the fraction $\frac{93}{100}$ on a number line from $\frac{9}{10}$ to 1.



Students may explain their reasoning as "The fraction $\frac{9}{10}$ is equal to $\frac{90}{100}$, or ninety hundredths. Because the number line goes from ninety hundredths to 1, each mark represents one hundredth, so I started at ninety hundredths and counted forward three tick marks."

▶ Identifying and Addressing Common Misconceptions

- Some students may use the tenths digit of a decimal as the denominator of a fraction. Ask students to write 0.7 as a fraction. Some students may write it as $\frac{1}{7}$. Have students read 0.7 as "seven tenths" and apply that language to writing a fraction where the first word describes the numerator and the second word describes the denominator.
- Some students may think that decimals with trailing zeros represent different values. Ask students to compare 0.2 and 0.20. Some students may say that 0.20 is greater than 0.2 because it looks like 20. Ask students to write each of the decimals as fractions: $\frac{2}{10}$ and $\frac{20}{100}$. Then help them recognize that the two fractions represent the same amount, either by using equivalent fractions or fraction models.
- Some students may ignore zeros immediately following the decimal point. For example, ask students to write 0.03 as a fraction. Some students may write $\frac{3}{10}$, having ignored the 0 between the decimal point and the digit 3. First, ask students if 205 and 25 and 250 all represent the same number. Use this as an illustration of the importance of zeros, and, particularly with 205, that sometimes zeroes are place holders. Have students demonstrate place value understanding by identifying both the tenths and hundredths place in the decimal 0.03. Help them recognize that the 3 is in the hundredths place and there are no tenths, so the decimal should be written as $\frac{3}{100}$.

► Enrichment

- Ask students to write decimals as fractions with a denominator of 1,000 and vice versa.
- Give students fractions with denominators that are factors of 10 or 100, ask them to write equivalent fractions with denominators of 10 or 100, and ask them to write the new fractions as decimals. For example, give students the fraction $\frac{3}{5}$. Students can write the equivalent fraction of $\frac{6}{10}$ and then write the equation $\frac{3}{5} = 0.6$.
- Ask students to write and describe decimals and decimal fractions in a variety of ways by using different place values. For example, give students the number 5.7. Students may write it as $5\frac{7}{10}$, $\frac{57}{10}$, or $\frac{570}{100}$ and describe it as "five and seven tenths," "fifty-seven tenths," or "five hundred seventy hundredths."
- Ask students to explain why writing the decimal equivalent of fractions such as $\frac{7}{9}$ is much more difficult than writing the decimal equivalent of a fraction with a denominator of 10, 100, or another number that is a factor of 10 or 100. A possible explanation is that our place value system is based on tens, and $\frac{7}{9}$ cannot be converted into a fraction with a denominator that is a power of 10 while keeping a whole number in the numerator.

► Key Terms

decimal notation, decimal point, denominator, fraction, hundredths, numerator, tenths

A-F.3.1.3

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations—Fractions

Understand decimal notation for fractions and compare decimal fractions.

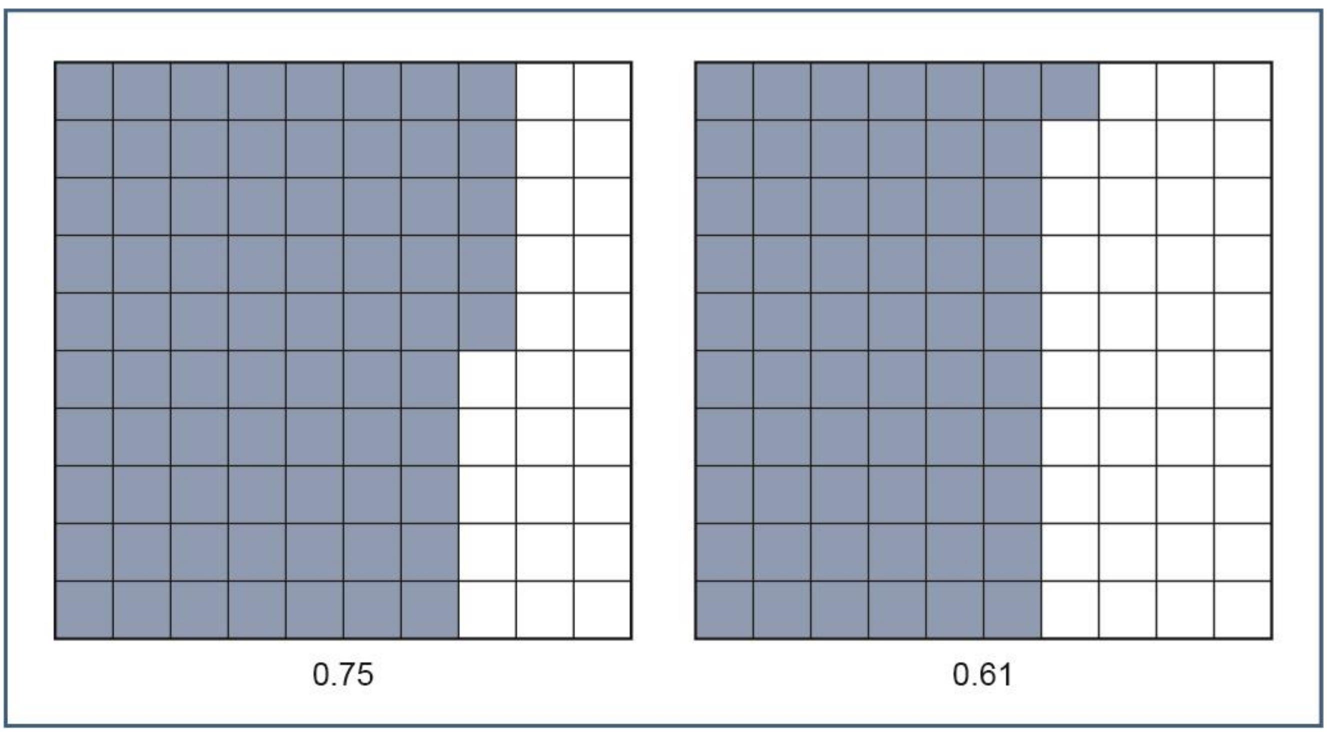
Use operations to solve problems involving decimals, including converting between fractions and decimals (may include word problems).

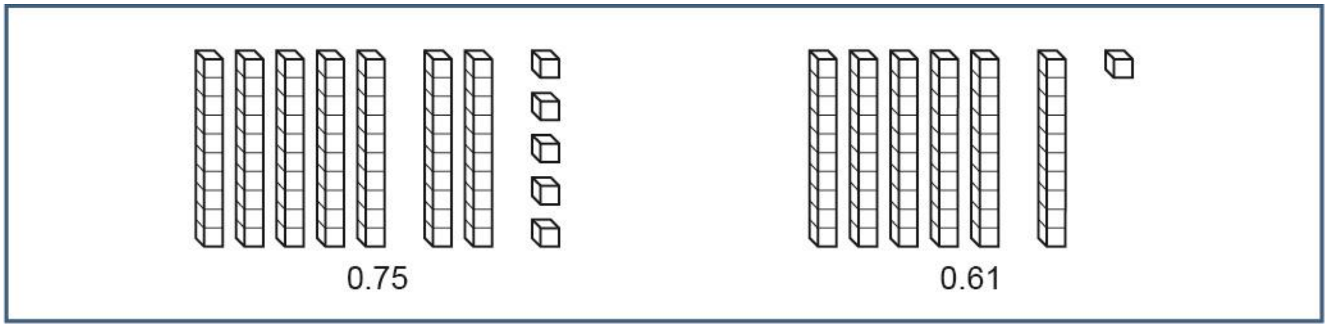
▶ Eligible Content

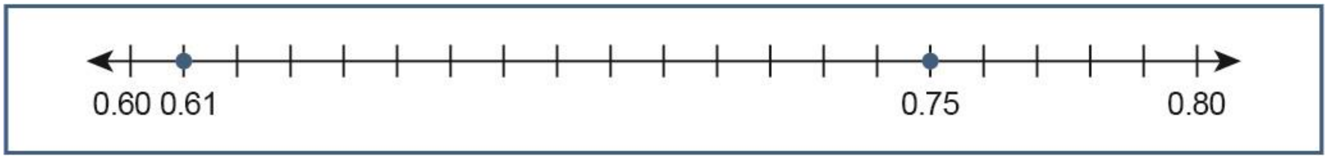
A-F.3.1.3 Compare two decimals to hundredths using the symbols >, =, or <, and justify the conclusions.

▶ Instructional Supports

- Ask students to use models or manipulatives to represent and compare decimals. For example, ask students to compare 0.75 and 0.61. Students may use the models or manipulatives shown in order to make the correct comparison.







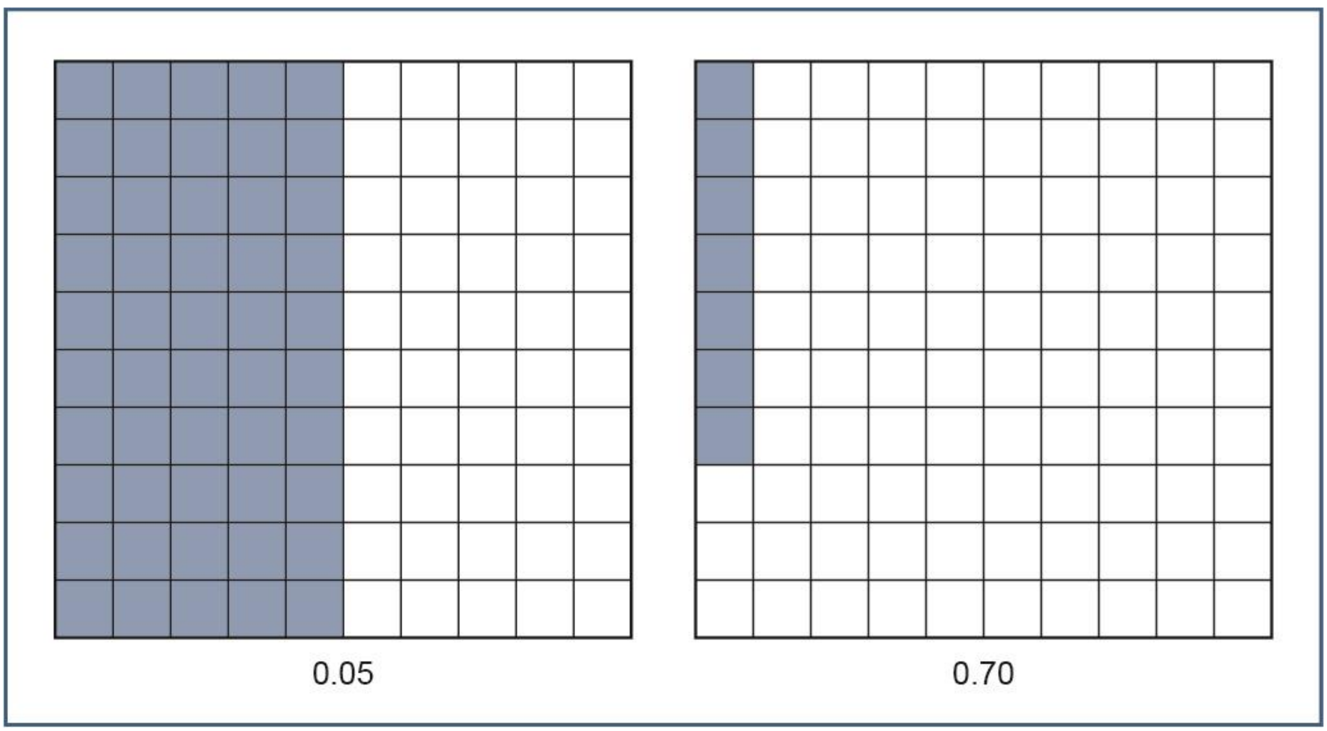
- Ask students to find patterns and generalizations that are useful when comparing decimals. Some possible examples include, "always compare the digits in order from left to right," or "for decimals less than 1, the digits in the hundredths place aren't needed for a comparison unless the digits in the tenths place are equal to one another."
- Ask students to consider the importance of the whole when comparing two decimals.
  - For example, when comparing 1.35 and 1.05 without context, the whole is assumed to be the same for both decimals, so 1.35 is greater than 1.05.
  - For example, give students this problem: "Ahmad and Bea each have a bag of granola. They each eat 0.5 of the bag. Explain how Ahmad could have eaten more than Bea." Students could create a drawing similar to the one shown as an explanation.

When given a context, it is necessary to consider the whole that each decimal is referring to before comparing the amounts represented by the decimals. Only if each decimal refers to the same whole can the decimals be easily and accurately compared.

- Ask students to complete decimal comparisons involving <, =, or >.
  - Give students the comparison 0.56 ☐ 0.62. Students choose the symbol < in order to make the true comparison 0.56 < 0.62.
  - Give students the comparison ☐ > 0.7. Students could complete the comparison with various decimals, including 0.8, 1.6, or 0.71.

▶ Identifying and Addressing Common Misconceptions

- Some students may create incorrect models, particularly with decimals that have zeros after the decimal point. Ask students to create models to represent 0.05 and 0.70. Some students may create models like the ones shown.



Ask students to identify the place values of the 5 and the 7 in the numbers. Help them connect the tenths place with a whole that is divided into 10 equal parts. The decimal seven-tenths is represented by shading 7 out of 10 equal parts. If necessary, draw darker vertical lines in the model to more clearly show the ten equal parts. Likewise, help them connect digits in the hundredths place with a whole that is divided into 100 equal parts. A number with a 5 in the hundredths place is represented by shading 5 out of 100 equal parts.

- Some students may compare decimals by comparing the digits from right to left, thinking that hundredths are greater than tenths. Ask students which decimal is greater: 0.79 or 0.81. Some students may respond that 0.79 is greater because 9 is greater than 1. Ask students to compare halves with tenths and help them recognize that a greater denominator results in more equal parts, which must be smaller parts. Therefore, tenths are greater than hundredths. In addition, ask students to consider how they would compare the whole numbers 79 and 81. Students should recognize that 81 is greater because there are 8 tens, while 79 only has 7 tens. Help students demonstrate this same reasoning when comparing decimals. They could demonstrate this reasoning by saying that 0.81 has 8 tenths and 0.79 only has 7 tenths.
- Students may assume that a decimal representing part of a larger whole must always represent more than a decimal representing part of a smaller whole. Ask students to compare 0.5 of a gallon bucket with 0.05 of a five-gallon bucket. Some students may respond that 0.05 of a five-gallon bucket is greater simply because the five-gallon bucket is a lot bigger than the gallon bucket. Explain that the size of the whole is only part of what must be considered when comparing quantities, and the decimal must also be considered. Demonstrate this by having two containers of different sizes. Fill the smaller one up almost the whole way and then add a quantity of water that is clearly less than the amount in the smaller container to the larger container.

► Enrichment

- Ask students to compare decimals with different wholes and determine the relationship between the size of the whole and the size of the decimal that represents the same quantity. For example, given two containers with capacities of 2 cups and 1 cup, fill the 1-cup container halfway with water. Students should note that the amount of water is 0.5 of the 1-cup container. Then, pour that water into the 2-cup container and have students estimate the part of the 2-cup container that has water. They should estimate it to be approximately 0.25. Therefore, students can conclude that 0.5 of a 1-cup container is the same as 0.25 of a 2-cup container. Ask students to make comparisons between both the decimals and the sizes of the containers and try to determine the relationships between all four quantities. That is, students should determine the relationships between the two capacities and the two decimals.

- Ask students to compare the same decimal in two different inequalities, and then have students write compound inequalities to combine their two inequalities. For example, students first determine some decimals that correctly complete the inequality ☐ > 0.36. Then, have students determine some decimals that correctly complete the inequality ☐ < 0.45. Have students determine some decimals that correctly complete both inequalities. Finally, have students write the compound inequality 0.36 < ☐ < 0.45.
- Ask students to discuss the differences between finding a fraction between two given fractions and finding a decimal between two given decimals. For example, ask students to find a fraction between $\frac{1}{5}$ and $\frac{1}{4}$. Then, ask students to find a decimal between 0.20 and 0.25. Explain that both problems are identical (because $\frac{1}{5}$ = 0.20 and $\frac{1}{4}$ = 0.25). Then, ask students to find a fraction between $\frac{75}{100}$ and $\frac{80}{100}$, and likewise to find a decimal between 0.75 and 0.80. Discuss why finding a decimal number between two given decimal numbers is often easier than finding a fraction between two given fractions.

► Key Terms

compare, equals, greater than, hundredths, hundredths grid, less than, tenths, whole

A-T.1.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

Apply place-value and numeration concepts to compare, find equivalencies, and round.

▶ Eligible Content

A-T.1.1.1 Demonstrate an understanding that in a multi-digit whole number (through 1,000,000), a digit in one place represents ten times what it represents in the place to its right.

▶ Instructional Supports

- Remind students that each place value represents a different-sized unit and that, when comparing the place values of two digits that are next to each other, the place value of the digit on the left is 10 times the place value of the digit on the right. Ask students to identify two digits in a given multi-digit number where one digit has a value that is 10 times as great as the value of the other digit. For example, given the number 4,552, students identify that the 5 in the hundreds place has a value that is 10 times as great as the value of the 5 in the tens place because 50 × 10 is 500.
- Ask students to determine the new value of all digits in a multi-digit whole number after the number is multiplied by 10. For example, given the product 384 × 10, students recognize that in the factor 384, the digit 3 represents 3 hundreds, the digit 8 represents 8 tens, and the digit 4 represents 4 ones. When multiplying 384 by 10, the value of all digits in the number shift so that the value of each digit becomes 10 times its original value:
    - 3 hundreds become 3 thousands
    - 8 tens become 8 hundreds
    - 4 ones become 4 tens

  Therefore, the product 384 × 10 = 3,840.

► Identifying and Addressing Common Misconceptions

- Students may think that a digit will always represent the same value regardless of its place in a number. For example, students may think that the 7 in 729 and the 7 in 273 represent the same value. Students should note that although the digits are both 7, each represents a different value, 700 and 70, respectively, because the digits are in different place values.
- Students may confuse or associate the negative (left) and positive (right) directions on a number line with the values of consecutive same digits in a multi-digit whole number. Students may think that a digit farther to the right has 10 times the value because numbers to the right on a number line are greater. Ask students which digit 5 in 7,554 has the greater value. Remind students that in a standard number, digits increase in value to the left. The value of the digit 5 in the hundreds place is 10 times the value of the digit 5 in the tens place, as the digit 5 in the hundreds place has a value of 500 and the digit 5 in the tens place has a value of 50.

► Enrichment

- Ask students to compare the values of repeated digits in a multi-digit whole number that are two or more places away from each other. For example, ask students to find how many times as great the value of the 2 in the hundreds place is relative to the 2 in the ones place of the number 1,232. Students should note that the value of a digit two places to the left is 100 or 10×10 times as great as the same digit two places to the right, since each place value in a multi-digit number is ten times as great as the place to its right.
- Challenge students to find a missing digit in a multi-digit number when given its value relative to another digit in the same number. For example, ask students to find the value of the missing digit in the tens place in 5_2, given that the value of the missing digit is 30 times as great as the value of the digit in the ones place.

- Ask students to find a number that matches a given set of characteristics. For example, ask them to find a four-digit number that meets the following criteria:
  - contains only the digits 3 and 8,
  - has one 3 that has 1,000 times the value of the other 3, and
  - has one 8 that has 10 times the value of the other 8.

► Key Terms

division, multi-digit, multiplication, place value

A-T.1.1.2 & A-T.1.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

Apply place-value and numeration concepts to compare, find equivalencies, and round.

► Eligible Content

A-T.1.1.2 Read and write whole numbers in expanded, standard and/or word form through 1,000,000.

A-T.1.1.3 Compare two multi-digit numbers through 1,000,000 based on meanings of the digits in each place, using >, =, and < symbols.

► Instructional Supports

Number names and commas

- Ask students to recognize the structure of the base-ten number system by noticing that the digits in multi-digit numbers are separated by commas in groups called "periods." For example, in the number 168,524, the first group of three digits to the left of the decimal (524) is called the "ones period." The second group of three digits (168) is called the "thousands period."
- Ask students to recognize that within each period, the place values repeat: hundreds, tens, and ones. For example, in the number 592,683,741, the 5, 6, and 7 each represent **hundreds** of the amount corresponding to their respective period.
  - The 5 represents 5 **hundred** million.
  - The 6 represents 6 **hundred** thousand.
  - The 7 represents 7 **hundred**.

  Similarly, the 9 represents 90 million (where 90 is equivalent to 9 **tens**), the 8 represents 80 thousand (where 80 is equivalent to 8 **tens**), and the 4 represents 40 or 4 **tens**. Finally, the 2, 3, and 1 represent 2 million, 3 thousand, and 1.

- Ask students to identify the place value of each digit of a multi-digit number in standard form. For example, given the number 479,356, use a place value chart to identify the place value of each digit.

Thousands Period			**,**	**Ones Period**		
Hundred Thousands	**Ten Thousands**	**One Thousands**	**,**	**Hundreds**	**Tens**	**Ones**
4	7	9	,	3	5	6

- Ask students to read multiples of 1,000. For example, in the number 634,000, students should recognize that 634 is in the period relating to thousands, so the number is read as "six hundred thirty-four thousand." Students should know that the digits in the ones period do not need to be read if they have a value of 0.

- Ask students to give the number name for a six-digit number. For example, 741,596 is read as "seven hundred forty-one thousand, five-hundred ninety-six." Students should note that the thousands period is read as a three-digit number with the period name immediately following.

Expanded form

- Ask students to write a multi-digit number in expanded form. For example, give students the number 736,518. There are two common ways to write expanded forms of numbers. In the first way, the value of each digit is written as addends.

$$700{,}000 + 30{,}000 + 6{,}000 + 500 + 10 + 8$$

  In the other way, each addend is written as a single digit multiplied by a power of 10.

$$(7 \times 100{,}000) + (3 \times 10{,}000) + (6 \times 1{,}000) + (5 \times 100) + (1 \times 10) + (8 \times 1)$$

- Ask students to write each period as the product of a three-digit number and a power of 10 and then connect this structure to number names. For example, given the number 649,835, students should consider the thousands period (containing the digits 6, 4, and 9), and the ones period (containing the digits 8, 3, and 5).
  - The thousands period of 649 can be represented as $649 \times 1{,}000$.
  - The ones period of 835 can be represented as 835×1.

The number 649,835 can then be written as (649 × 1,000) + (835 × 1). Observe that this format parallels the structure of the number name:

(649	×	1,000)	+	(835	×	1)
"six hundred forty-nine		thousand,		eight hundred thirty-five		(ones)"

Comparing numbers

- Ask students to compare two numbers that have different numbers of digits. Students should record the results using the correct inequality sign. For example, in the given numbers 278,436 and 91,054, the leading digit in 278,436 is in the hundred thousands place and 91,054 does not contain a digit in the hundred thousands place, so 278,436 must be greater than 91,054. The comparison can be written as 278,436 > 91,054.
- Ask students to compare two numbers that have the same number of digits with unequal leading digits. Students should record the results using the appropriate inequality sign. For example, ask students to compare the five-digit numbers 29,865 and 30,741. Both numbers have leading digits in the ten-thousands place. Since 2 < 3, the inequality 29,865 < 30,741 correctly compares the two numbers. Students should note that 30,741 > 29,865 is also a correct comparison.
- Ask students to compare two numbers that have the same number of digits with equal leading digits. Students should record the results using the appropriate inequality sign. For example, ask students to compare the five-digit numbers 47,205 and 47,198, where the leading digits (both in the ten-thousands place) are both 4. Since both digits in the ten-thousands place are 4, students must move to the thousands place to continue comparing. The comparison shows that both digits in the thousands place are 7. When the comparison is then made of the digits in the hundreds place, the students should find that 2 > 1. Therefore, the number containing 2 in the hundreds place (which is 47,205), is greater than the number containing 1 in the hundreds place (which is 47,198), so 47,205 > 47,198.

▶ Identifying and Addressing Common Misconceptions

- In a comparison of two multi-digit numbers, students may make their comparison based on which leading digit is greatest without considering the place value of the leading digit. Ask students to compare the numbers 12,573 and 9,648. If students pick 9,648 as the greater number, they should be encouraged to align both numbers with a vertical line on the right. Then, they should write the numbers from right to left starting with the ones place digit, then the tens place digit, and so on. Grid paper can help students align their numbers. One number should be written directly above the other number. This format will allow the students to see how the place values align. This format will more importantly allow students to see that the 1 in 12,573 represents 10,000, whereas the 9 in 9,648 represents 9,000; therefore, 12,573 must be greater than 9,648 even though 1 is less than 9.

1	2	5	7	3
	9	6	4	8

- Students may confuse the greater than (>) and less than (<) symbols. Give students single-digit numbers to compare using the greater than and less than symbols. Remind students that the open side of the symbol opens to the greater number and the closed side of the symbol points to the lesser number. Some teachers sketch an alligator with the inequality symbol integrated in the sketch as the mouth of the alligator. The alligator is sketched so that its open mouth "eats" the greater number.
- Students may think that an equal sign will not be used in a comparison problem. Ask students to solve a comparison problem where the numbers are equal but are not represented in the same way. Some examples are 03 = 3 and 3 = 3.0.

► Enrichment

- Ask students to use the digits 2, 5, 3, 7, and 4 to create the greatest number possible and the least number possible. Students should be able to explain how they know their answers are correct.
- Create partial comparisons with multi-digit numbers and have students fill in the blanks with digits or comparison symbols to complete each comparison statement. More missing components in the comparison statement give students more possibilities for correct answers. For the two examples shown, ask students to list or describe all possible correct options for the empty boxes.

$$4\square 2 \ \square \ \square 32$$

$$657 > 6\square 8$$

Further, ask students to identify the greatest digit that makes 657 > 6_8 a true comparison. Also, ask students to identify the least digit that makes 657 > 6_8 a true comparison. Have students explain their reasoning.

- Ask students to compare two six-digit numbers in jumbled expanded form. The numbers should have one or two of the leading digits the same. For example, ask students to compare the number in the top row with the number in the bottom row.

$$(5 \times 100) + (8 \times 10{,}000) + (4 \times 1) + (9 \times 100{,}000) + (2 \times 1{,}000) + (7 \times 10)$$
$$(2 \times 1{,}000) + (4 \times 10{,}000) + (5 \times 10) + (7 \times 1) + (8 \times 100) + (9 \times 100{,}000)$$

► Key Terms

base-ten numerals, comma, expanded form, number name, period, place value, properties of operations

A-T.1.1.2 & A-T.1.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

Apply place-value and numeration concepts to compare, find equivalencies, and round.

► Eligible Content

A-T.1.1.2 Read and write whole numbers in expanded, standard and/or word form through 1,000,000.

A-T.1.1.3 Compare two multi-digit numbers through 1,000,000 based on meanings of the digits in each place, using >, =, and < symbols.

► Instructional Supports

Number names and commas

- Ask students to recognize the structure of the base-ten number system by noticing that the digits in multi-digit numbers are separated by commas in groups called "periods." For example, in the number 168,524, the first group of three digits to the left of the decimal (524) is called the "ones period." The second group of three digits (168) is called the "thousands period."
- Ask students to recognize that within each period, the place values repeat: hundreds, tens, and ones. For example, in the number 592,683,741, the 5, 6, and 7 each represent **hundreds** of the amount corresponding to their respective period.
    - The 5 represents 5 **hundred** million.
    - The 6 represents 6 **hundred** thousand.
    - The 7 represents 7 **hundred**.

  Similarly, the 9 represents 90 million (where 90 is equivalent to 9 **tens**), the 8 represents 80 thousand (where 80 is equivalent to 8 **tens**), and the 4 represents 40 or 4 **tens**. Finally, the 2, 3, and 1 represent 2 million, 3 thousand, and 1.

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Thousands Period			,	Ones Period		
Hundred Thousands	**Ten Thousands**	**One Thousands**	,	**Hundreds**	**Tens**	**Ones**
4	7	9	,	3	5	6

- Ask students to read multiples of 1,000. For example, in the number 634,000, students should recognize that 634 is in the period relating to thousands, so the number is read as "six hundred thirty-four thousand." Students should know that the digits in the ones period do not need to be read if they have a value of 0.

- Ask students to give the number name for a six-digit number. For example, 741,596 is read as "seven hundred forty-one thousand, five-hundred ninety-six." Students should note that the thousands period is read as a three-digit number with the period name immediately following.

Expanded form

- Ask students to write a multi-digit number in expanded form. For example, give students the number 736,518. There are two common ways to write expanded forms of numbers. In the first way, the value of each digit is written as addends.

$$700{,}000 + 30{,}000 + 6{,}000 + 500 + 10 + 8$$

  In the other way, each addend is written as a single digit multiplied by a power of 10.

$$(7 \times 100{,}000) + (3 \times 10{,}000) + (6 \times 1{,}000) + (5 \times 100) + (1 \times 10) + (8 \times 1)$$

- Ask students to write each period as the product of a three-digit number and a power of 10 and then connect this structure to number names. For example, given the number 649,835, students should consider the thousands period (containing the digits 6, 4, and 9), and the ones period (containing the digits 8, 3, and 5).
  - The thousands period of 649 can be represented as $649 \times 1{,}000$.
  - The ones period of 835 can be represented as 835×1.

The number 649,835 can then be written as (649 × 1,000) + (835 × 1). Observe that this format parallels the structure of the number name:

(649	×	1,000)	+	(835	×	1)
"six hundred forty-nine		thousand,		eight hundred thirty-five		(ones)"

Comparing numbers

- Ask students to compare two numbers that have different numbers of digits. Students should record the results using the correct inequality sign. For example, in the given numbers 278,436 and 91,054, the leading digit in 278,436 is in the hundred thousands place and 91,054 does not contain a digit in the hundred thousands place, so 278,436 must be greater than 91,054. The comparison can be written as 278,436 > 91,054.

- Ask students to compare two numbers that have the same number of digits with unequal leading digits. Students should record the results using the appropriate inequality sign. For example, ask students to compare the five-digit numbers 29,865 and 30,741. Both numbers have leading digits in the ten-thousands place. Since 2 < 3, the inequality 29,865 < 30,741 correctly compares the two numbers. Students should note that 30,741 > 29,865 is also a correct comparison.

- Ask students to compare two numbers that have the same number of digits with equal leading digits. Students should record the results using the appropriate inequality sign. For example, ask students to compare the five-digit numbers 47,205 and 47,198, where the leading digits (both in the ten-thousands place) are both 4. Since both digits in the ten-thousands place are 4, students must move to the thousands place to continue comparing. The comparison shows that both digits in the thousands place are 7. When the comparison is then made of the digits in the hundreds place, the students should find that 2 > 1. Therefore, the number containing 2 in the hundreds place (which is 47,205), is greater than the number containing 1 in the hundreds place (which is 47,198), so 47,205 > 47,198.

► Identifying and Addressing Common Misconceptions

- In a comparison of two multi-digit numbers, students may make their comparison based on which leading digit is greatest without considering the place value of the leading digit. Ask students to compare the numbers 12,573 and 9,648. If students pick 9,648 as the greater number, they should be encouraged to align both numbers with a vertical line on the right. Then, they should write the numbers from right to left starting with the ones place digit, then the tens place digit, and so on. Grid paper can help students align their numbers. One number should be written directly above the other number. This format will allow the students to see how the place values align. This format will more importantly allow students to see that the 1 in 12,573 represents 10,000, whereas the 9 in 9,648 represents 9,000; therefore, 12,573 must be greater than 9,648 even though 1 is less than 9.

1	2	5	7	3
	9	6	4	8

- Students may confuse the greater than (>) and less than (<) symbols. Give students single-digit numbers to compare using the greater than and less than symbols. Remind students that the open side of the symbol opens to the greater number and the closed side of the symbol points to the lesser number. Some teachers sketch an alligator with the inequality symbol integrated in the sketch as the mouth of the alligator. The alligator is sketched so that its open mouth "eats" the greater number.
- Students may think that an equal sign will not be used in a comparison problem. Ask students to solve a comparison problem where the numbers are equal but are not represented in the same way. Some examples are 03 = 3 and 3 = 3.0.

► Enrichment

- Ask students to use the digits 2, 5, 3, 7, and 4 to create the greatest number possible and the least number possible. Students should be able to explain how they know their answers are correct.
- Create partial comparisons with multi-digit numbers and have students fill in the blanks with digits or comparison symbols to complete each comparison statement. More missing components in the comparison statement give students more possibilities for correct answers. For the two examples shown, ask students to list or describe all possible correct options for the empty boxes.

$$4\square 2 \quad \square \quad \square 32$$

$$657 > 6\square 8$$

Further, ask students to identify the greatest digit that makes 657 > 6_8 a true comparison. Also, ask students to identify the least digit that makes 657 > 6_8 a true comparison. Have students explain their reasoning.

- Ask students to compare two six-digit numbers in jumbled expanded form. The numbers should have one or two of the leading digits the same. For example, ask students to compare the number in the top row with the number in the bottom row.

$$(5 \times 100) + (8 \times 10{,}000) + (4 \times 1) + (9 \times 100{,}000) + (2 \times 1{,}000) + (7 \times 10)$$
$$(2 \times 1{,}000) + (4 \times 10{,}000) + (5 \times 10) + (7 \times 1) + (8 \times 100) + (9 \times 100{,}000)$$

► Key Terms

base-ten numerals, comma, expanded form, number name, period, place value, properties of operations

A-T.1.1.4

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

Apply place-value and numeration concepts to compare, find equivalencies, and round.

► Eligible Content

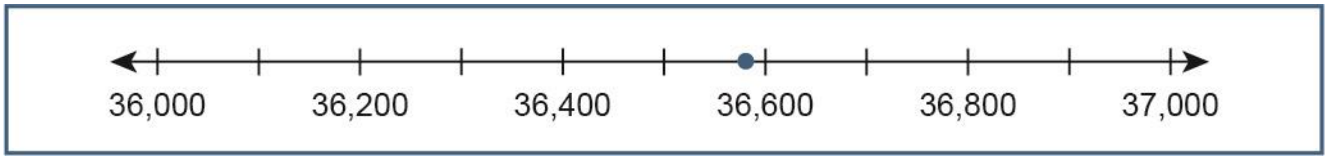
A-T.1.1.4 Round multi-digit whole numbers (through 1,000,000) to any place.

► Instructional Supports

- Ask students to recognize the determining digit for rounding a number. For example, when students are rounding 47,623 to the nearest thousand, they should realize that the place value that determines how the number is rounded is not the thousands place but the next place to the right. In this case, the 6 in the hundreds place is the determining digit for rounding to the nearest thousand. When rounding a six-digit number such as 245,976 to the nearest hundred thousand, the determining digit for rounding is in the ten thousands place. In this case, the 4 is the determining digit for rounding to the nearest hundred thousand.

- Ask students to round a given number to a given place value. For example, when rounding 364,729 to the nearest hundred thousand, students should recognize that 364,729 should be rounded up since the digit in the place to the right of the hundred thousands place, the 6 in the ten thousands place, is greater than 5. Therefore, 364,729 rounded to the nearest hundred thousand is 400,000.

- Ask students to round a given number located on a number line to a given place based on the location of the number on the number line. For example, when rounding 36,581 to the nearest thousand using a number line, students should first create a number line, marked off in hundreds, containing the multiples of 1,000 just before and after the number 36,581. In this case, the number line should go from 36,000 to 37,000. The relevant part of the number line should have a point plotted approximately at 36,581 as shown.

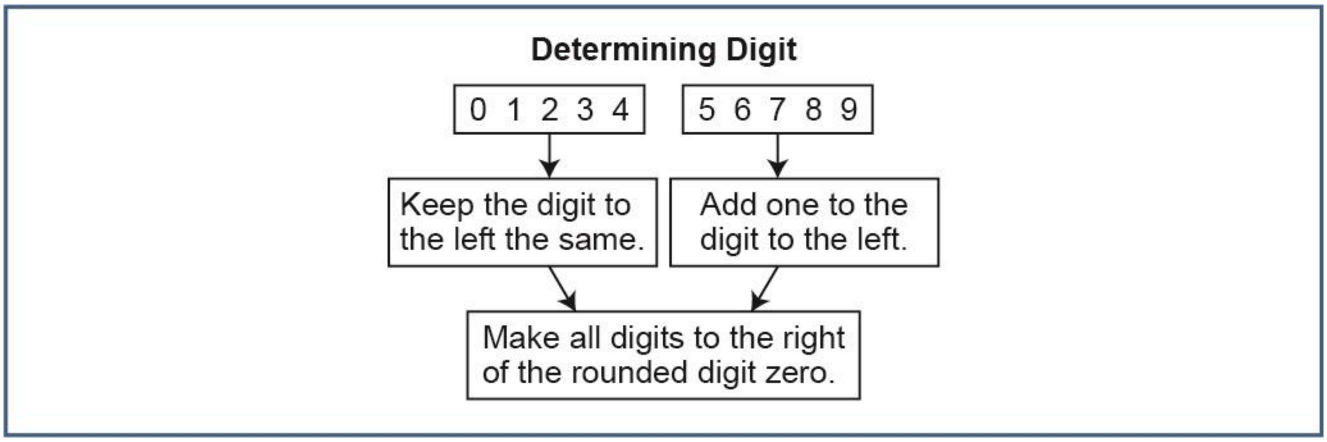


Since the distance from 36,581 to 37,000 is less than 5 hundred and the distance from 36,581 to 36,000 is more than 5 hundred, the number 36,581 rounded to the nearest thousand is 37,000.

- Ask students to round numbers with a 5 in the hundreds place to the nearest thousand. For example, when rounding 3,582 to the nearest thousand, students should recognize the hundreds place as the determining digit. Also, students should recognize that 3,582 is between 3,000 and 4,000. Since there is a 5 in the hundreds place, 3,582 is rounded up to the nearest thousand. Students should round 3,582 to 4,000. As an additional example, ask students to round 73,564 to the nearest thousand. Students should recognize the hundreds place as the determining digit (rounding is to the nearest thousand, even though the number 73,564 reaches the ten thousands place). Since there is a 5 in the hundreds place, students should round 73,564 to 74,000, the nearest thousand.

► Identifying and Addressing Common Misconceptions

- Students may think that when a number is rounded to a given place value, it is that place value that determines whether to round up or down. Ask students to round each of the numbers 46,387 and 73,241 to the nearest ten thousand. Some students may use the digits in the ten thousands place to determine whether to round up or down and will round to 40,000 and 80,000, respectively. Remind students that the determining digit is in the place to the right of the place being rounded to and have students follow the flow of the following graphic.



- Students may be confused about the phrase “round down” and decrease the value of the place instead of rounding to the nearest multiple. Ask students to round the number 28,413 to the nearest hundred. Students not understanding the phrase “round down” may round 28,413 to 28,300. Students may benefit most from using a number line to help visualize the closest multiple of one hundred.

- Students may round correctly but to the wrong place value because they may not know their place values well. Ask students to round 482,631 to the nearest ten thousand. Students who round the number to 483,000 or 500,000 may not know their place values well enough to be consistently correct with rounding. Students with this misconception may benefit from review of a table with place values to help solve the problem.

► Enrichment

- Create a rounding problem for students where there is only one possible solution, such as a single digit missing in a given number. For example, give students the number 83,_54 with the digit in the hundreds place missing. If the number is rounded to the nearest hundred and the rounded number is 83,700, ask students to determine the digit missing from the hundreds place in the given number.

- Ask students to find all numbers that will round to a given number. For example, ask students to find the interval of whole numbers that round to 76,000 when rounding to the nearest thousand. Challenge students by encouraging them to think backwards and find multiple possible numbers to fit the parameters given.

- Ask students to round the number 99,965 to the nearest hundred and 9,981 to the nearest hundred. Ask students to look for and describe patterns in numbers for which rounding results in all digits being changed and an additional digit being written. These special cases occur for numbers less than, but near powers of 10.

► Key Terms

about, approximately, hundred thousands, hundreds, millions, ones, place value, round, ten thousands, tens, thousands

A-T.2.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Use operations to solve problems.

▶ Eligible Content

A-T.2.1.1 Add and subtract multi-digit whole numbers (limit sums and subtrahends up to and including 1,000,000).

▶ Instructional Supports

- Ask students to add two four-digit numbers using the standard algorithm where regrouping is required. For example, ask students to calculate 1,574 + 6,239. First, students add the digits in the ones place. Since 4 + 9 = 13, 10 of the 13 ones are bundled to make an additional ten and 3 ones remain. Then, the digits in the tens place are added, including the ten that was regrouped from the addition in the ones place. Therefore, the number of tens in the sum is 1 + 7 + 3, resulting in a total of 11 tens. The 11 tens are regrouped to make an additional one hundred and one ten remains. Next, students add the digits in the hundreds place, which gives 8 because 1 + 5 + 2 = 8. Finally, students add the digits in the thousands place, which gives 7 because 1 + 6 = 7. Therefore, 1,574 + 6,239 = 7,813.
- Ask students to subtract two four-digit numbers using the standard algorithm where regrouping is required. For example, ask students to calculate 8,914 – 7,523. Students should first find the difference of the digits in the ones place, 4 – 3 = 1. Then, students should notice that the algorithm would require subtracting 2 from 1 to find the difference of the digits in the tens place. As it is not possible to yield a positive number by taking 2 from 1, students should take 1 of the 9 hundreds from 8,914 and make it 10 tens because 1 hundred is equal to 10 tens. The number 8,914 could then be thought of as having 8 thousands, 8 hundreds, 11 tens, and 4 ones. The difference of the tens digits would then be 9 because 11 – 2 = 9. Next, students find the

difference of the hundreds digits by solving 8 – 5 = 3, as 9 hundreds were changed to 8 hundreds because 1 of the hundreds was bundled into 10 tens. Finally, the difference of the thousands digits is 1 because 8 – 7 = 1. Therefore, 8,914 – 7,523 = 1,391.

► Identifying and Addressing Common Misconceptions

- Students may not recognize that when adding using the standard algorithm, regrouping is sometimes necessary. Ask students to find 236 + 547. If students have 7 in the tens place in the sum and do not have a 1 above the top addend in the tens place, that is an indication that students did not regroup the ones correctly into the tens place. Show students that the sum from the ones place, 6 + 7 = 13, is represented by the one at the top of the tens column and the three in the sum at the bottom of the ones column. Remind students that this representation matches what they have seen while regrouping using base-ten blocks.
- Students may find the difference between two digits in a certain place value even when the subtrahend is greater than the minuend. Ask students to solve 275 – 148. When subtracting the values in the ones place, students may think they can use 8 – 5 = 3 to calculate the ones place of the difference instead of regrouping from the tens place to make the difference in the ones place 15 – 8. Use a smaller difference, like 25 – 18, and show with manipulatives or visual aids why subtracting 8 – 5 in the ones place does not give the correct answer.
- Students may rewrite numbers incorrectly when regrouping in a subtraction problem. Ask students to calculate 2,000 – 345, showing all work. Students may correctly regroup a value from one digit to the next digit to the right but may continue to do so in an automatic manner, simply changing all zeros to nines, including the 0 in the ones place. When dealing with whole numbers, there is no value past the ones place, so the 0 in the ones place must be changed to a 10 and must stay a 10. Check student understanding by asking students to explain each change from a 0 to a 10 and a 10 to a 9 when regrouping.

► Enrichment

- Ask students to explain the notation written above the addends in the standard algorithm for addition. For example, give students the following work for the calculation of 786 + 459.

$$\begin{array}{r} {\scriptstyle 11} \\ 786 \\ +\,459 \\ \hline 1245 \end{array}$$

  Students should describe the meanings of the ones written above the tens column and the hundreds column as a regrouping of 10 ones and 10 tens respectively.

- Ask students to explain any notation above the minuend in the standard algorithm for subtraction. For example, give students the following work for the problem 452 – 267.

$$\begin{array}{r} {\scriptstyle 14} \\ {\scriptstyle 3\,\cancel{4}\,12} \\ \cancel{4}\cancel{5}\cancel{2} \\ -\,267 \\ \hline 185 \end{array}$$

  Ask students to explain what each of the written numbers 12, 4, 14, and 3 represent.

- Ask students to find all numbers that can be subtracted from a given minuend that result in a difference with a specific digit in a certain place value. For example, challenge students to find all numbers that can be subtracted from 1,000 that result in a difference with a 4 in the tens place. Ask students to explain how they found their answers.

- Ask students to create an addition problem using exactly 4 addends, where they must regroup bundles that are carried to each new place value in the order 1 ten, 2 hundreds, and 3 thousands without the use of zero in any of the addends.

- Ask students to create an addition problem where two three-digit numbers create a four-digit sum where digits in the addends and sum are not repeated (i.e., all ten digits are used exactly once in the addends and sum). An example is 246 + 789 = 1,035. The complement would be for students to create a subtraction problem where a three-digit number is subtracted from a four-digit number for a three-digit difference where the digits in the minuend, subtrahend, and difference are not repeated (i.e., all ten digits are used exactly once). An example from the same fact family using subtraction is 1,035 – 246 = 789.

► Key Terms

addition, place value, regroup, standard algorithm, subtraction

A-T.2.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Use operations to solve problems.

► Eligible Content

A-T.2.1.2 Multiply a whole number of up to four digits by a one-digit whole number and multiply 2 two-digit numbers.

► Instructional Supports

Using multiples of 10

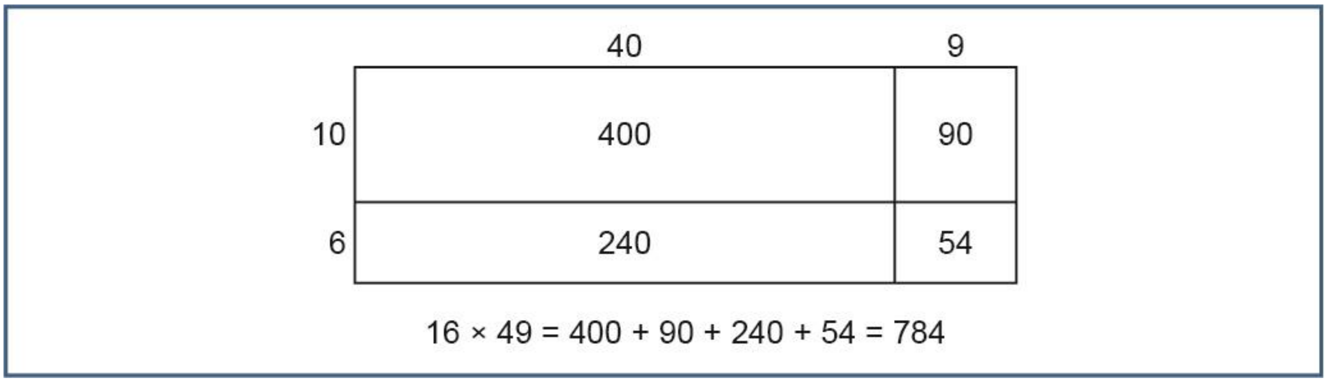
- Ask students to use a given product to determine the value of that product after one factor has been multiplied by ten. For example, ask students to use the product 62 × 7 to determine the value of 62 × 70. Since 62 × 7 = 434, the product 62 × 70 can be expressed as (62 × 7) × 10 and the value of 62 × 70 = 434 × 10. Students should recognize this as 434 tens, which has a value of 4,340. Therefore, 62 × 70 = 4,340.

- Ask students to rewrite the product of a four-digit number and a single-digit number using properties of operations and place value. For example, 2,465 × 8 is equivalent to (2,000 × 8) + (400 × 8) + (60 × 8) + (5 × 8). The first three products in this expression can be decomposed further using place value to create the equivalent expression (2 × 1,000 × 8) + (4 × 100 × 8) + (6 × 10 × 8) + (5 × 8). Each of these products can be evaluated using single-digit multiplication combined with an understanding of place value of powers of 10 to get 16,000 + 3,200 + 480 + 40. Therefore, 2,465 × 8 = 19,720.

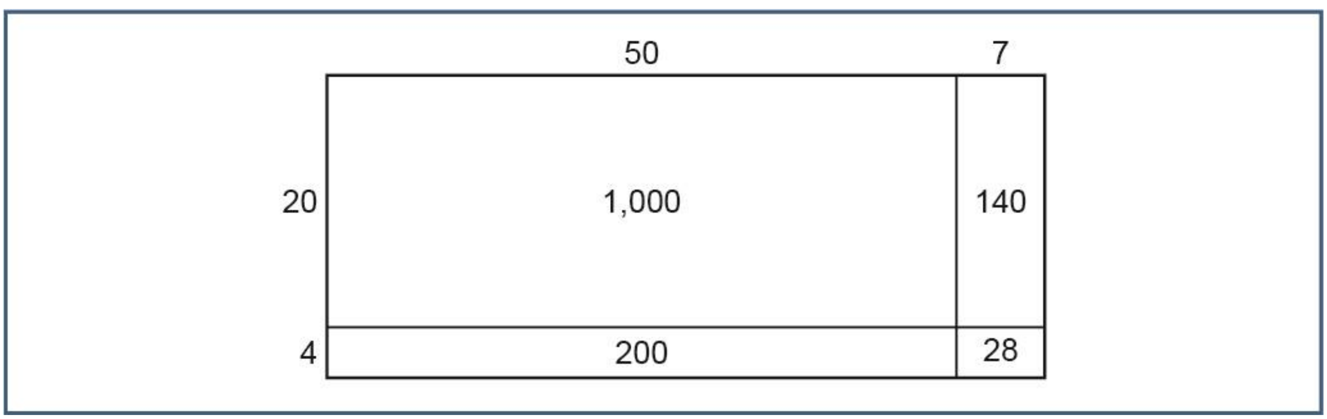
- Ask students to think of valid strategies for multiplication problems in which one of the factors is close to a multiple of ten. For example, given 67 × 25, students could share the strategy of rounding 67 up to 70, performing the multiplication, and then subtracting 75 from the product, since 3 × 25 = 75. Similarly, given 81 × 68, students may use the strategy of rounding 81 down to 80, performing the multiplication, and then adding 68 to the product, since 1 × 68 = 68.

Using area models

- Ask students to create an area model for a given multiplication problem by decomposing each factor based on place value. For example, give students the problem 16 × 49. The factors can be decomposed into 10 + 6 and 40 + 9. The area model shown depicts the product (10 + 6) × (40 + 9) using the decomposed factors. The partial product expression is (10 × 40) + (10 × 9) + (6 × 40) + (6 × 9).



- Ask students to write the original expression from a given area model in addition to the partial product expression and then use the partial product expression to find the product. For example, consider the area model shown.



The original expression is 24 × 57, since 20 + 4 = 24 and 50 + 7 = 57. The partial product expression is (20 × 50) + (20 × 7) + (4 × 50) + (4 × 7). When each part of the partial product expression is evaluated, the sum is 1,000 + 140 + 200 + 28, which is equal to 1,368. Therefore, the product is 24 × 57 = 1,368.

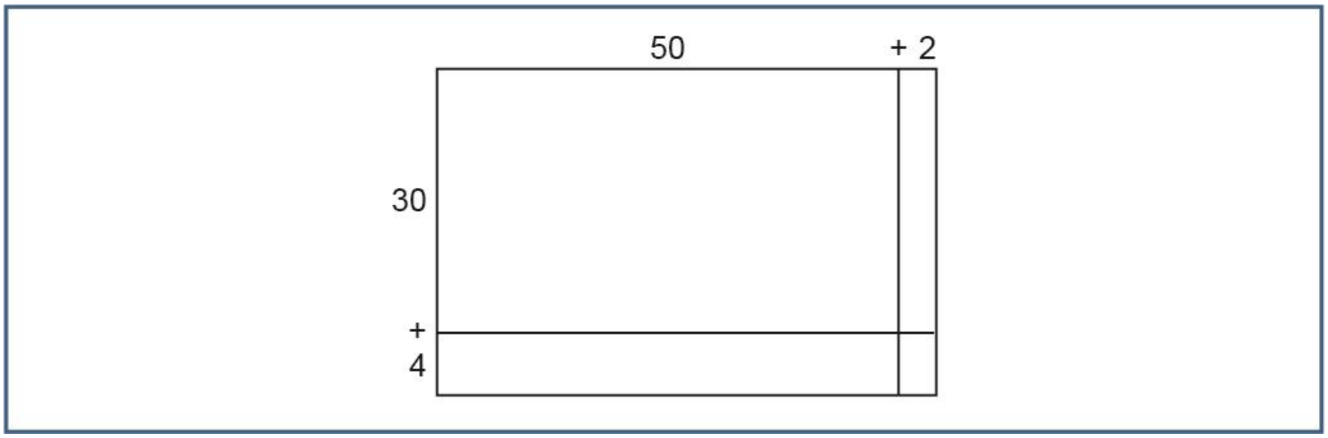
Using decomposition of numbers

- Ask students to describe different ways of decomposing one or both factors of a multiplication problem. For example, when solving 38×62, students might describe the strategy of decomposing one or both factors.
  - Decomposing 38 as $30 + 8$ allows the product to be rewritten as $(30 \times 62) + (8 \times 62)$.
  - Decomposing 62 as $60 + 2$ allows the product to be rewritten as $(38 \times 60) + (38 \times 2)$.
  - Decomposing both 38 and 62 allows the product to be rewritten as $(30 \times 60) + (30 \times 2) + (8 \times 60) + (8 \times 2)$.

  In each case, the decomposition results in numbers that are easier to multiply because they are either single-digit numbers or multiples of ten.
- Ask students to rewrite the product of 2 two-digit numbers using properties of operations. For example, 64×47 is equivalent to $(60 + 4) \times 47$, which can be rewritten as $(60 \times 47) + (4 \times 47)$. Each of these products can be decomposed further to create, for example, the expression $(60 \times 40) + (60 \times 7) + (4 \times 40) + (4 \times 7)$. Simplifying each partial product gives $2{,}400 + 420 + 160 + 28$. Therefore, $64 \times 47 = 3{,}008$.
- Ask students to describe valid strategies involving doubling or halving one of the factors. For example, given 65×18, students might find the product 65×9 and then double the product, since $2 \times 9 = 18$.

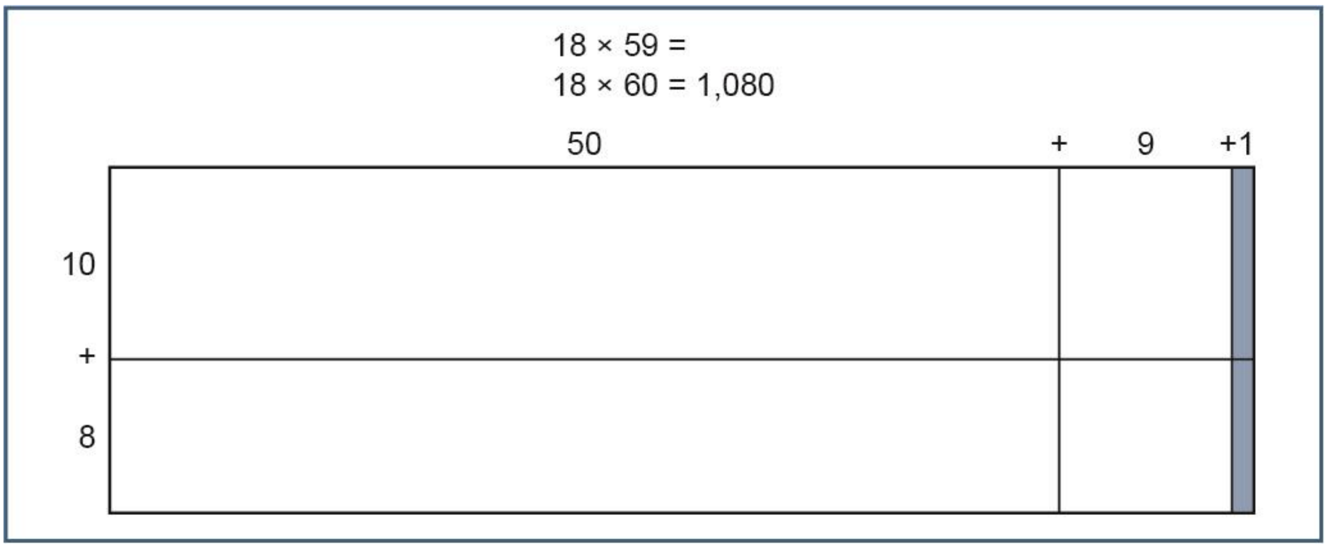
► Identifying and Addressing Common Misconceptions

- Students may think that when multiplying 2 two-digit numbers, they only need to multiply the tens values from each factor together, multiply the ones values from each factor together, and then add the two partial products. Ask students to multiply 34×52. Students may multiply 30×50 to get 1,500, multiply 4×2 to get 8, and then add $1{,}500 + 8$ to get 1,508. Encourage students to set up the multiplication problem with an area model of $(30 + 4) \times (50 + 2)$ so that students will visually see that there are four regions that need partial products to be added together to get the final product.

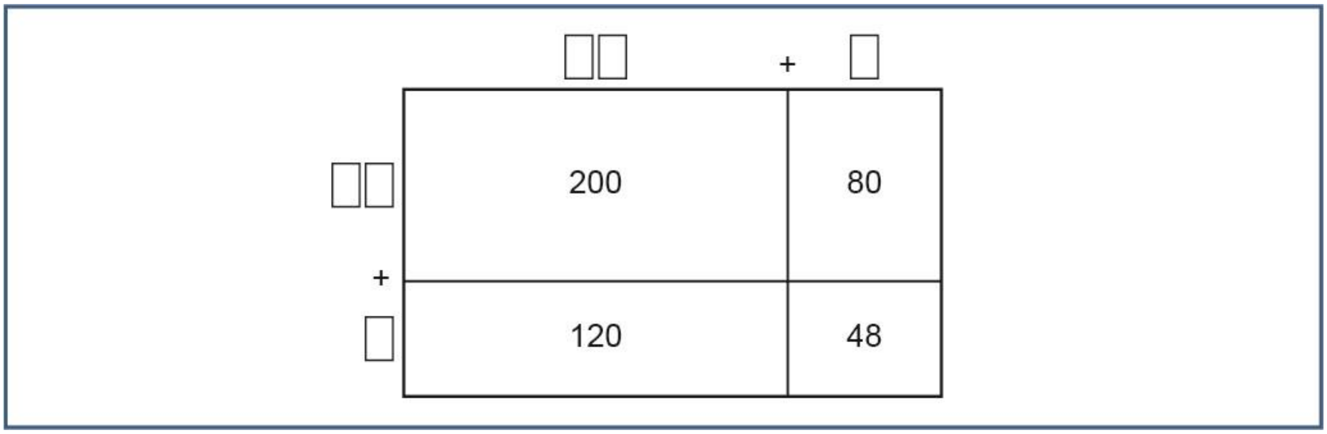


► Enrichment

- Ask students to find a product using an area model with subtraction. For example, ask students to determine 18 × 59. The area model is set up to the nearest multiple of ten and the area that needs to be subtracted from the product is shaded.



- Ask students to find the factors in an area model of a multiplication problem. For example, give students the following incomplete area model.



- Ask students to show their work for a two-digit by two-digit multiplication problem with an area model and the standard algorithm. Students should relate the two problems by adding the columns or rows in the area model and comparing the relevant column or row to the partial products in the standard algorithm. The sums at the bottom (after the addition of the columns) or the far right (after the addition of the rows) of the area model will match the partial products in the rows in the standard algorithm when completed correctly.
- Ask students to create an area model for a four-digit by four-digit multiplication problem. Students should notice that since they are multiplying a four-digit number by a four-digit number, they will have an area model with 16 boxes to fill with partial products. Students should also know that they will need to add all the partial products together to find the final product.

► Key Terms

area model, array, equation, multiple of 10, multiply, partial product, place value, strategy

A-T.2.1.3

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Use operations to solve problems.

▶ Eligible Content

A-T.2.1.3 Divide up to four-digit dividends by one-digit divisors with answers written as whole-number quotients and remainders.

▶ Instructional Supports

Connecting multiplication and division

- Ask students to use the equation *quotient* × *divisor* + *remainder* = *dividend* to check their calculation of 26 ÷ 3 = 8 R2. Students should identify the quotient as 8, the divisor as 3, the remainder as 2, and the dividend as 26.

- Ask students to use their knowledge of multiples to solve division problems after rewriting those problems as related multiplication problems. For example, the quotient 308 ÷ 5 can be rewritten as 5 × n = 308. To solve this multiplication equation, students should determine that the greatest multiple of 5 that goes into 30 tens and 8 ones is 6 tens (since 5 × 6 = 30) and 1 five (since 5 × 1 = 5 and 5 × 2 = 10, which is greater than 8). Since 1 five is the greatest multiple of 5 that goes into 8 ones, there are still 3 ones that remain. Therefore, the answer is 61 with a remainder of 3.

Decomposing Dividends

- Ask students to use their place value understanding when dividing multi-digit whole numbers ending in at least one zero by a single-digit number. For example, in the quotient 4,200 ÷ 7, the number 4,200 can be thought of as 42 hundreds. Since 42 ÷ 7 = 6, 42 hundreds divided by 7 is equal to 6 hundreds. Therefore, 4,200 ÷ 7 = 600.

- Ask students to decompose dividends by breaking down larger division problems into smaller division problems. For example, to determine the quotient of 1,287 ÷ 4, the number 1,287 can be decomposed to 1,200 + 80 + 7. Then, each addend can be divided by 4. The last addend has a remainder of 3.

$$1{,}200 \div 4 = 300$$

$$80 \div 4 = 20$$

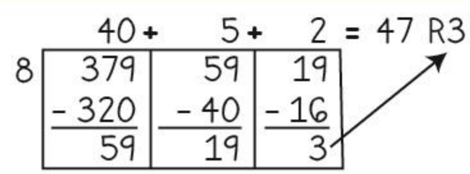
$$7 \div 4 = 1 \text{ R3}$$

Therefore, the quotient of 1,287 ÷ 4 = 300 + 20 + 1 R3, or 321 R3.

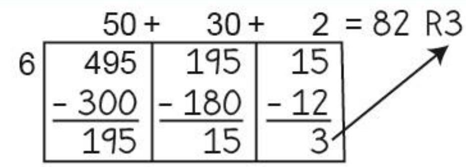
- Give students two division problems in which the two dividends share the same leading digits and ask students to use what they know about the leading digits in the smaller number to help them divide the larger number. For example, ask students to find the quotients of 63 ÷ 3 and 639 ÷ 3. Once students have found that 63 ÷ 3 = 21 (possibly by decomposing 63 into 60 + 3), students should decompose 639 into 630 + 9. Students should use the fact that 63 ÷ 3 = 21, as well as the fact that 9 ÷ 3 = 3, to show that 639 ÷ 3 = 213.

Area models for division

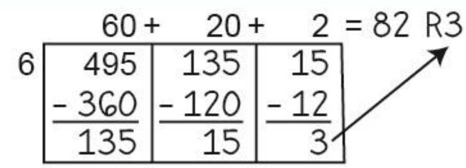
- Ask students to create an area model to solve a division problem. For example, give students the problem 379 ÷ 8. A possible student response is shown.



- Ask students to complete several different area models for the same division problem and show that each area model gives the same quotient. For example, ask students to complete two area models for 495 ÷ 6 by finding the equations that correspond to each portion of the models shown. The dividend, divisor, and partial quotients of the diagrams are set up to guide students. The first area model shows one method with partial quotients to determine the overall quotient of 495 ÷ 6.

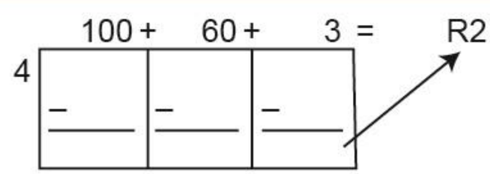


The second area model shows a different method with partial quotients for determining the same overall quotient.



Students should notice that each model has different quantities being subtracted from the dividend and different numbers decomposed in the boxes to make the different partial quotients above the boxes. However, the sum of the partial quotients with the remainder is the same overall quotient.

- Give students an incomplete area model and ask them to find the dividend of the division problem being modeled. For example, give students the following diagram.



Start with the box on the right. The remainder is taken from the result of the subtraction in the last box. Therefore, the difference in that box is 2. The subtrahend is the product of 4 and 3, which is 12. Therefore, the minuend in the box on the right is 14. Next, look at the middle box. The difference in this box, the middle box, will become the minuend, 14, of the right box. Therefore, the difference in the middle box is 14. Students should solve for the minuend of the middle box and repeat the process to the left until the minuend of the left box is found. That minuend, in the left box, is the dividend of the division problem. A completed student response is shown.

	100 +	60 +	3 = 163 R2
4	654	254	14
	− 400	− 240	− 12
	254	14	2

► Identifying and Addressing Common Misconceptions

- Students may think that remainders are decimal answers and can be written as such. Ask students to interpret the remainder in the quotient 45 ÷ 6 = 7 R3. Students may think that 7 R3 could be written as 7.3. Emphasize that the remainder is the fraction of the divisor that remains after the greatest whole number multiple of the divisor has been subtracted from the dividend. For example, there are 7 sixes in 45 for a total of 42. The remaining 3 is a fractional part of the divisor, 6, so the remainder becomes the numerator and the divisor is the denominator of the fraction when the quotient is expressed as a mixed number. In this case, 3 over 6 is $\frac{1}{2}$, therefore the quotient is the mixed number $7\frac{1}{2}$.
- When dividing, students may think they can just get as close as possible to the dividend and then take the difference of the dividend and that number as the remainder. Ask students to divide 26 by 7. Some students may think the quotient is 4 and the remainder is 2 since the greater multiple of 7 (28) is closer to 26 than the previous multiple of 7 (21). Students should note that when dividing, the quotient is determined by going as close as possible to the dividend with a multiple of the divisor without going over the value of the dividend. The remainder is then determined by finding the difference between the dividend and the closest multiple of the divisor that is less than the dividend.
- Students may not recognize repeated subtraction as a strategy for division. Ask students to explain how to use subtraction to calculate 67 ÷ 9 by asking students to subtract 9 from 67 as many times as possible until the number remaining is less than 9. In this case, students should end up with a remainder of 4. It may help students to use *quotient* × *divisor* + *remainder* = *dividend* to check both their quotient and remainder.

► Enrichment

- Ask students to create a division problem that has specified digits in the quotient and remainder. For example, ask students to find a four-digit dividend that can be divided by a one-digit divisor and result in a quotient with a zero in the tens place and a remainder.
- Ask students to find a number that is divisible by each of the digits 1–9 without a remainder. As an additional challenge, ask students to find the least number that is divisible by the all the digits 1–9 without a remainder. Any multiple of 2,520 would be a solution to the first challenge, since it is the least number that is divisible by all the digits 1–9.
- Ask students to find a number that has specified remainders when divided by specific numbers. For example, ask students to find a number that has a remainder of 1 when divided by 7 and a remainder of 2 when divided by 9.
- Ask students to divide numbers by decomposing into numbers other than multiples of the divisor. For example, ask students to compute 137 ÷ 9 by decomposing 137 into expanded form as 100 + 30 + 7. The quotient is found by dividing each part by 9 and then adding the partial quotients together: (100 ÷ 9) + (30 ÷ 9) + (7 ÷ 9). However, each partial quotient has its own remainder that needs to be combined with the other remainders.

► Key Terms

area model, array, compose, decompose, divide, dividend, divisor, equation, greatest multiple, multiple, multiply, properties of operations, quotient, remainder, unknown factor

B-O.1.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

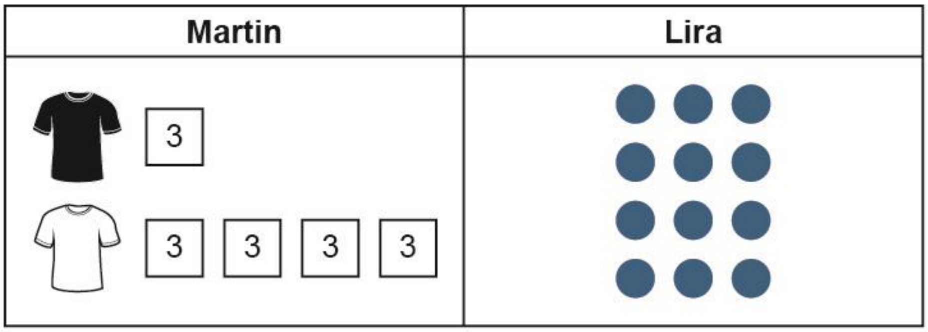
Use numbers and symbols to model the concepts of expressions and equations.

► Eligible Content

B-O.1.1.1 Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.

► Instructional Supports

- Ask students to explain the difference between an additive comparison and a multiplicative comparison and give examples. For example, saying that 12 is 8 more than 4 is an additive comparison because it indicates what amount must be added to 4 to get 12. Similarly, saying that 12 is 3 times as many as 4 is a multiplicative comparison because it indicates what amount must be multiplied by 4 (or how many times 4 must be repeated) to get 12.
- Ask students to compare a word problem involving a multiplicative comparison with a word problem involving equal groups. For example, give students these word problems: "Martin has 4 times as many white T-shirts as black T-shirts. He has 3 black T-shirts. How many white T-shirts does Martin have?" and "Lira has 3 bags that each contain 4 pieces of sidewalk chalk. How many pieces of sidewalk chalk does she have?" Two visual models for the problems are shown.



Both problems can be solved using the same equation: $3 \times 4 = 12$. However, the first problem compares two separate objects, black T-shirts and white T-shirts, while the second problem explains how two objects, bags and sidewalk chalk, are interrelated. In the first problem, the 4 does not have a physical representation in the context, but the 4 has a more immediate meaning in the second context. The first problem also has an additional set of 3 (shirts) that are not included in the answer.

- Ask students to determine whether a given word problem involves equal groups or a multiplicative comparison. For example, give students the following word problems.
  - Javier has 2 times as many crackers as slices of cheese. He has 6 slices of cheese. How many crackers does Javier have?
  - Aaliyah earns \$6 per week for allowance for 2 weeks. How much money does she earn?

  The first problem involves a multiplicative comparison. The second problem involves equal groups.

- Ask students to solve problems involving multiplicative comparisons. For example, give students this problem: “At an airport, there are 3 times as many airplanes as helicopters. The airport has 5 helicopters. How many airplanes are at the airport?” A possible student model is shown.

Helicopters	5		
Airplanes	5	5	5

The model shows a total of 15 airplanes at the airport.

- Ask students to write an equation to represent a multiplicative comparison. For example, give students this statement: “45 is 5 times as many as 9.” Students can write the equations $45 = 5 \times 9$ or $45 = 9 \times 5$.
- Ask students to write a multiplicative comparison to represent an equation. For example, give students the equation $88 = 8 \times 11$. A possible response is that “88 is 8 times as many as 11.”

► Identifying and Addressing Common Misconceptions

- Some students may incorrectly translate a multiplicative comparison into an equation. Ask students to write an equation for this statement: “32 is 8 times as many as 4.” Some students may write the equation $32 \times 8 = 4$ because they are accustomed to always writing operations on the left side of the equation. Ask them to illustrate the product, 32×8, and help them notice that this product is much greater than 4. Help them practice writing equations in which the operation is on the right side of the equation.
- Some students may misinterpret the answer to a problem involving multiplicative comparison. Give students this problem: “Alayna needs 4 times as many forks as spoons for a dinner party. She needs 8 spoons. How many forks does Alayna need?” Some students may give an answer of 40 because they think the 8 spoons must be included in the answer just because the spoons are mentioned in the problem. Ask students to draw a picture to represent the situation and note that even though the 8 spoons are part of the problem, the question only asks about forks, so the 8 spoons are not included in the answer.

► Enrichment

- Give students a word problem involving equal groups and ask them to reinterpret it as a multiplicative comparison. For example, give students this problem: “Madeline eats 5 strawberries each day for 7 days. How many strawberries does she eat?” Students can reinterpret the problem like this: “Madeline eats 5 strawberries in a day. She eats 7 times as many strawberries in a week as she eats in a day. How many strawberries does she eat in a week?”
- Ask students to find a pair of numbers whose multiplicative comparison is the same as their

additive comparison. For example, 4 is 2 more than 2 while also being 2 times as many as 2.

- Give students a multiplication statement and ask them to write a comparative multiplication word problem that can be solved with that equation.

► Key Terms

array, factor, multiple, multiplication equation, multiplicative comparison, product, repeated addition, times as many, verbal statement

B-O.1.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

Use numbers and symbols to model the concepts of expressions and equations.

► Eligible Content

B-O.1.1.2 Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.

► Instructional Supports

- Ask students to identify key words in word problems that indicate whether a comparison is additive or multiplicative. For example, give students the following word problems and have them compare key words from each.
    - In a restaurant, there are 3 times as many people wearing short-sleeved shirts as people wearing long-sleeved shirts. There are 18 people wearing short-sleeved shirts. How many people are wearing long-sleeved shirts?

      "3 times as many" means a multiplicative comparison

    - Gabrielle has 3 siblings. Cameron has 2 more siblings than Gabrielle has. How many siblings does Cameron have?

      "2 more siblings" means an additive comparison

- Ask students to determine whether a given word problem shows an additive comparison or a multiplicative comparison and explain how they know. For example, give students the following word problems.

- Leilani observes 15 birds and 3 deer on a nature walk. How many times as many birds as deer does she observe?
- Omni is 5 years older than his brother. Omni is 11 years old. How old is Omni's brother?
- Amir has 6 bookmarks. Celeste has 9 bookmarks. How many more bookmarks does Celeste have than Amir?
- Pete has 4 times as many books as he has magazines. He has 3 magazines. How many books does Pete have?

Students should determine that the first and last word problems use multiplicative comparisons, while the other two problems use additive comparisons.

- Ask students to solve word problems involving multiplicative comparisons by creating models. For example, give students this problem: "There are 30 white houses on a street. There are 6 times as many white houses on the street as houses that are not white. How many houses on the street are not white?" A possible student representation and solution is shown.

White houses: $\boxed{p}\ \boxed{p}\ \boxed{p}\ \boxed{p}\ \boxed{p}\ \boxed{p}$ = 30 houses

Not white houses: $\boxed{p}$ = 5 houses

Therefore there are 5 houses that are not white.

- Ask students to create equations to solve word problems involving multiplicative comparisons. For example, give students this problem: "A plant store sells oak trees and ash trees. The store sold 3 times as many oak trees as ash trees. The store sold 12 ash trees. How many oak trees did the store sell?" Students can write the equation $3 \times 12 =$ ___ to represent the problem. Ask students to explain how they created their equations. Students should then arrive at the answer of 36 oak trees.

▶ Identifying and Addressing Common Misconceptions

- Some students may always write the unknown by itself on one side of the equation. Ask students to write an equation with an unknown to model this problem: "Brianna practiced the flute for 3 times as many minutes as Justin practiced. Brianna practiced the flute for 12 minutes. How many minutes did Justin practice?" Some students may write "$3 \times 12 = ?$" Ask students to determine based on the context of the problem whether the answer should be greater than 12 or less than 12. Then ask students to determine whether the solution to their equation is greater than 12 or less than 12.
- Some students may confuse additive and multiplicative comparisons. Ask students what number is 5 times as great as 6. Some students may overlook the word "times" and think of the problem as "5 greater than 6," resulting in an answer of 11. Ask students to explain their reasoning. Emphasize the importance of the word "times" and the fact that "5 times as great" indicates multiplication, while "5 greater than 6" indicates addition.
- Some students may not be sure whether multiplication or division should be used to solve a problem involving a multiplicative comparison. Ask students to solve this problem: "Thaddeus's brother is 2 times as old as Thaddeus. Thaddeus is 12 years old. How old is Thaddeus's brother?" Some students may think that Thaddeus's brother is 6 years old. Ask students whether "2 times as old" indicates that Thaddeus's brother is older or younger than Thaddeus. Then ask them whether multiplying by 2 or dividing by 2 gives a larger result.

▶ Enrichment

- Ask students to combine two or more multiplicative comparisons. For example, give students this problem: "Person A has 3 times as many beads as person B, and person B has 5 times as many beads as person C." Ask students to write a multiplicative comparison between the number of beads that person A has and the number of beads that person C has.
- Ask students to write a multiplicative comparison word problem for a given visual model or equation.

- Ask students to solve problems involving both an additive comparison and a multiplicative comparison. For example, give students this problem: “Josie’s dog weighs 18 pounds more than Veronica’s dog. Josie’s dog weighs 3 times as much as Veronica’s dog. What is the combined weight, in pounds, of both dogs?”

► Key Terms

additive comparison, equation, factor, multiplication, multiplicative comparison, product, symbol, times as many

B-O.1.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

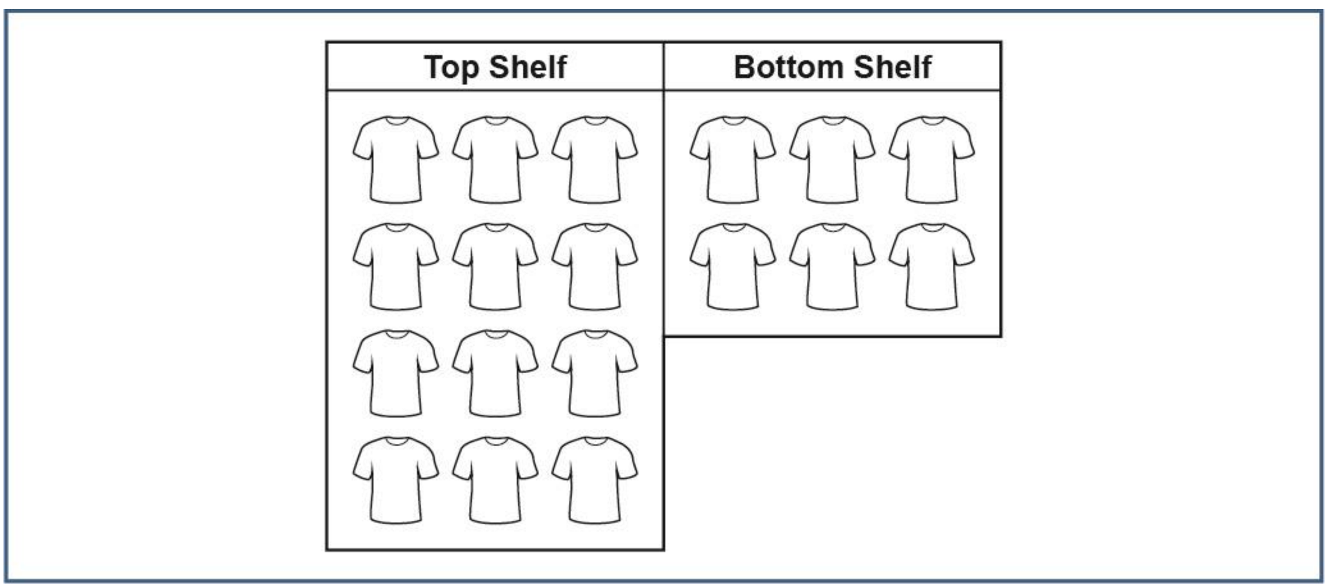
Use numbers and symbols to model the concepts of expressions and equations.

► Eligible Content

B-O.1.1.3 Solve multi-step word problems posed with whole numbers using the four operations. Answers will be either whole numbers or have remainders that must be interpreted yielding a final answer that is a whole number. Represent these problems using equations with a symbol or letter standing for the unknown quantity.

► Instructional Supports

- Ask students to use drawings and models to solve word problems. For example, ask students to create a visual model and use it to solve this problem: "A store has 12 shirts on the top shelf, which is 2 times as many shirts as it has on the bottom shelf. How many shirts are on both shelves altogether?" A possible student response is shown.



I drew 12 shirts since the problem told me that the top shelf has 12 shirts, then I drew 6 shirts for the bottom shelf since 6 is half of 12. When I count all of the shirts, I see that they have 18 shirts altogether.

- Ask students to identify key phrases within multistep word problems and explain the mathematical operations suggested by those phrases. For example, give students the following word problem. Ask them to highlight key phrases and underline the question to be answered.

  Julian has a lemonade stand where he sells cookies for $2 each and cups of lemonade for $1 each. One day, he sold 6 cookies and 11 cups of lemonade. <u>How much money did Julian collect from sales that day?</u>

  Ask students to discuss the information that was highlighted in order to check their understanding of key phrases. The first two highlighted phrases indicate multiplication. The third highlighted phrase indicates addition.

- Ask students to determine the order in which operations in multistep problems should be performed, with or without providing a solution. For example, give students the following problems. Potential student answers are shown after the problems.
  - The students in fourth grade create a game by tossing bean bags into cups that are located at distances of far, medium, and close. They score 6 points for each successful toss into the far cup, 3 points for each successful toss into the medium cup, and 1 point for each successful toss into the close cup. A student successfully tosses 4 beanbags into the far cup, 4 bean bags into the medium cup, and 2 bean bags into the close cup. How many points does the student earn?

    multiplication (three times), then addition

- There were 33 students enrolled in a class. Five of the students moved away, and the remaining students were split into 2 groups. How many students were in each group?

  subtraction, then division:

  $33 - 5 = 28$; $28 \div 2 = 14$ students

- Ask students to use context in order to decide what to do with remainders in word problems that require division and explain how they decided. Choices may include whether to round up, round down, or partition the remainder. Several examples are shown.
  - There are 51 students going on a field trip. For each group of at most 4 students, 1 parent volunteer is needed. What is the minimum number of parent volunteers needed?

    Round up to the next number, since a group smaller than 4 students still requires a parent volunteer. Round up 12 R3 to 13 parent volunteers for this group of students.

  - Ava has cupcakes to give to 12 of her teammates. She has 10 vanilla cupcakes and 8 chocolate cupcakes. She wants each teammate to have the same size portion of cupcakes. How many cupcakes do each of Ava's teammates receive?

    Partition the remainder, since a cupcake can be cut into pieces. In this case, $10 + 8 = 18$ and $18 \div 12 = 1\frac{1}{2}$ cupcakes per teammate.

  - Peter has $37 to buy school lunches. A school lunch costs $3. How many school lunches can Peter buy?

    Round down or ignore the remainder. Since Peter cannot buy a part of a lunch, he will just have leftover money.

- Give students a word problem and an equation with a variable that may be used to solve the word problem and ask them to explain the meaning of the variable in the equation. For example, give students this problem: “A coffee shop sells two different sizes of coffee. A large cup holds 12 ounces, and a small cup holds 8 ounces. The coffee shop sells a total of 164 ounces of coffee, including 7 small cups of coffee. How many large cups of coffee does the coffee shop sell?” Give students the equation $(12 \times x) + (8 \times 7) = 164$ and ask them to explain the meaning of the variable. In the equation, x represents the number of large cups of coffee the coffee shop sells.

- Ask students to write an equation for a given word problem and solve the word problem. For example, give students this problem: “Nya sold three different sizes of shirts: small, medium, and large. She sold 85 shirts in all, and she sold 3 times as many medium shirts as small shirts. She sold 42 medium shirts. How many large shirts did Nya sell?” A possible student response is shown.

  An equation could be $(42 \div 3) + 42 + x = 85$ or $14 + 42 + x = 85$. Nya sold 29 large shirts.

► Identifying and Addressing Common Misconceptions

- Some students may not correctly consider the order in which they calculate values when solving a multistep problem. Ask students to solve this problem: “Augustin has 5 more than 2 times as many marbles as Brett. Brett has 3 marbles. How many marbles does Augustin have?” Some students may interpret the problem as $(5 + 3) \times 2$, five more than what Brett has times 2, rather than $5 + (2 \times 3)$, and give an answer of 16. Ask students to add parentheses and brackets to the sentence as “grouping symbols” in order to help determine the sequencing of the operations. Students can rewrite the problem as shown.

  Augustin has [5 more than (2 times as many marbles as Brett.)]

  Help students recognize that the structure of the sentence indicates that “2 times as many” refers to Brett’s marbles and that “5 more than” refers to the number of marbles after multiplying by 2.

- Some students may struggle to understand how the context of a problem indicates how to handle remainders in division problems. Ask students to explain how to answer the question “How many vans are needed to transport 66 people when each van seats 9 people?" Some students may say that the remainder should be ignored or that the answer is between 7 and 8. Ask students to draw a picture to represent the situation. Then ask them to explain what the quotient represents (in this case, a number of vans) and what the remainder of 3 represents (in this case, 3 people). Ask them to consider whether the quotient represents something that can be divided, what would happen to the remainder if the quotient is left as is, and what would happen to the remainder if the quotient is rounded up. Help students to assess each of the three options in the context of the problem.

► Enrichment

- Give students an outline of a multistep word problem and ask them to fill in values that result in a particular remainder. For example, give students this problem: "Aviana has 21 bananas and ___ oranges and wants to hand an equal number of pieces of fruit to ___ people." Ask students to fill in values that result in a remainder of 8.
- Give students a multistep word problem with rounded values instead of specific values and ask students to estimate a range of values for the answer. For example, give students this problem: "Lamar's mother has 90 holiday cards when rounded to the nearest 10. Lamar needs approximately 15 cards, rounded to the nearest 5, to give to his classmates. His mother gives him the cards he needs and she uses exactly 28 for her coworkers. About how many cards does his mother have left?" The number of holiday cards is between 85 and 94, inclusive, and the number that Lamar needs is between 13 and 17, inclusive. The most his mother can have left is $94 - 13 - 28 = 53$. The least she can have left is $85 - 17 - 28 = 40$.
- Give students an equation and ask them to write a multistep word problem that matches the equation.

► Key Terms

addition, division, equation, mental strategy, multiplication, multistep problem, operation, partition, remainder, rounding, subtraction, unknown quantity

B-O.2.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Gain familiarity with factors and multiples.

Develop and apply number theory concepts to represent numbers in various ways.

► Eligible Content

B-O.2.1.1 Find all factor pairs for a whole number in the interval 1 through 100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the interval 1 through 100 is a multiple of a given one-digit number. Determine whether a given whole number in the interval 1 through 100 is prime or composite.

► Instructional Supports

Factors and multiples

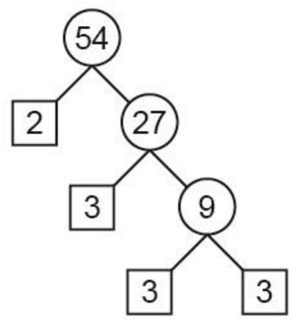
- Ask students to determine whether a number in the range of 1–100 is a multiple of a given 1-digit number. For example, ask students if 26 is a multiple of 4. By writing the multiples of 4 (4, 8, 12, 16, 20, 24, 28, . . .) students can reason that 26 is not a multiple of 4. Additionally, help students recognize that division is also a method to determine whether a number is a multiple of another. To determine if 78 is a multiple of 6, students divide 78 by 6. Solving 78 ÷ 6 yields 13 with no remainder, which means 78 is a multiple of 6.

- Ask students to systematically find all factor pairs of a number. For example, ask students to find all factor pairs of 24. Ask students to determine whether 1 is a factor of 24, then determine whether 2 is a factor of 24, then whether 3 is a factor or 24, etc. As students find a factor, they find the number it gets paired with and include both in the list shown.

1, 2, 3, 4, 6, 8, 12, 24

Help students recognize that they do not need to check all numbers in order to know that all factors have been found. After finding 4 × 6, students check that 5 is not a factor, then recognize that 6 has already been paired with 4. There is no need to continue checking for other factors.

- Ask students to use a factor tree to find all the factor pairs of a number. For example, ask students to find the factors of 54. Have students create a factor tree for 54 and find the prime factors: 2, 3, 3, and 3.

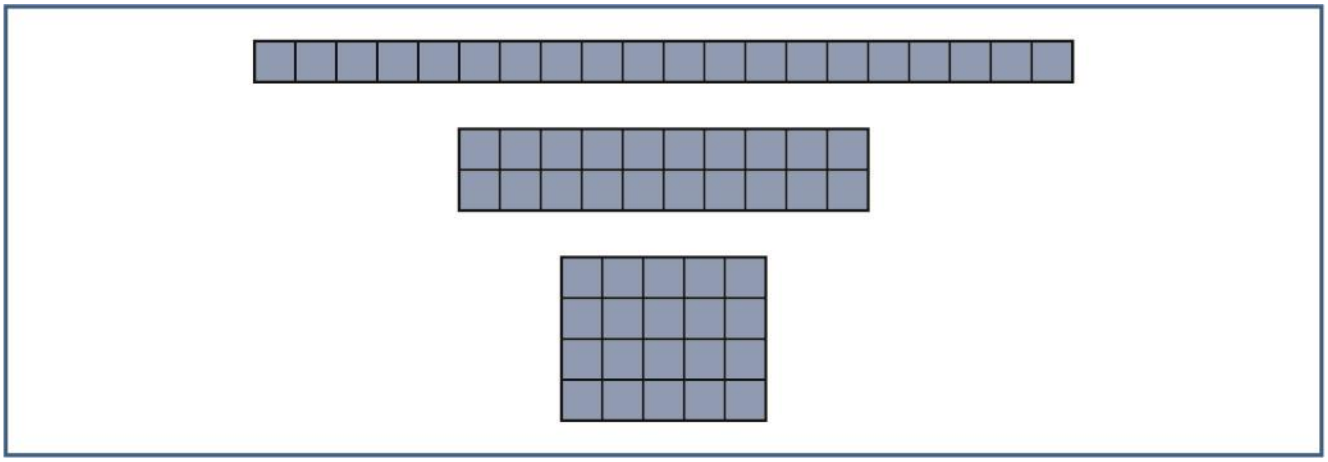


From the prime factors 2, 3, 3, and 3, all the factors may be found using varying combinations of products of the prime factors as shown.

1 (a combination of no 2’s and no 3’s)	=	1
2	=	2
2 × 3	=	6
2 × 3 × 3	=	18
2 × 3 × 3 × 3	=	54
3	=	3
3 × 3	=	9
3 × 3 × 3	=	27

The factors for 54 are 1, 2, 3, 6, 9, 18, 27, and 54.

- Ask students to use area as a visual tool to find factor pairs. For example, ask students to find the factors of 20 by creating rectangles with whole number side lengths and an area of 20 square units.



There are three ways to create a rectangle with an area of 20 square units, and the factors of 20 are the lengths and widths of those rectangles: 1, 2, 4, 5, 10, and 20.

Prime and composite

- Ask students to describe why a number is prime or composite. For example, given the numbers 23 and 25, students determine that 23 is prime since it has exactly two factors: 1 and 23. Students determine that 25 is composite because it has more than two factors: 1, 5, and 25. Additionally, help students recognize that 1 is neither prime nor composite as it only has a single factor.
- Ask students to use areas of rectangles to determine whether a number is prime or composite. For example, ask students to use rectangles to determine whether 18 and 19 are prime or composite.

Since there is more than one rectangle that can be made with 18 square units, 18 is a composite number, and since there is only one rectangle that can be made with 19 square units, 19 is prime.

Note that an exception to this is the number 1. Even though there is only one rectangle with whole number side lengths and an area of 1, 1 is not prime (nor is it composite).

- Ask students to use the Sieve of Eratosthenes to determine prime and composite numbers between 1 and 100. First, the number 1 on a hundred grid is crossed out, as it is neither prime nor composite. The first unmarked number, 2, is then circled. Next, students shade all the multiples of 2. The next unmarked number remaining, 3, is circled. Then, students shade all the multiples of 3. The next number 4 is already shaded, and the next unmarked number is 5. Therefore, it is circled and the multiples of 5 are shaded. This pattern of circling the next unmarked number and shading its multiples continues through the grid until it looks like the grid shown.

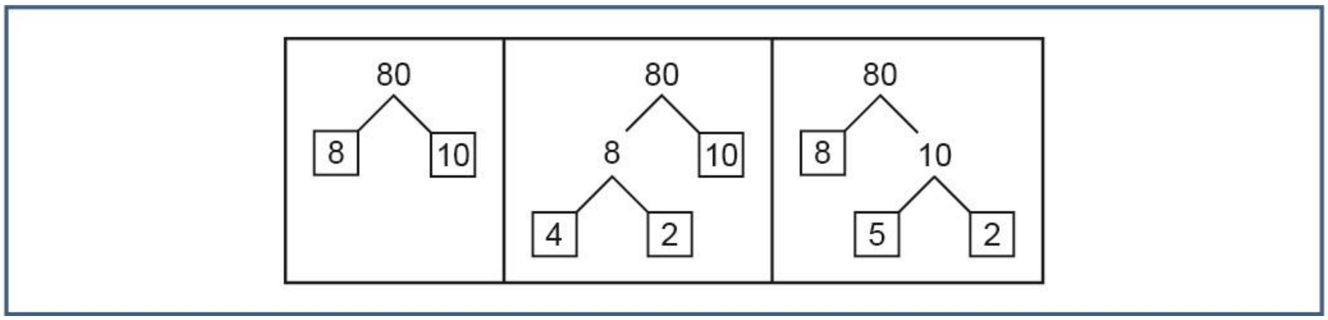
~~1~~	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The circled numbers are prime, and the shaded numbers are composite.

► Identifying and Addressing Common Misconceptions

- Some students may not recognize that not all numbers need to be checked to find all factors. Ask students to find the factors of 34. Some students may check all numbers from 1 to 34. Help students recognize that because factors occur in pairs, not all numbers need to be checked. After students check the numbers 1 through 5, when checking whether 6 is a factor they may notice that 6 × 6 is greater than 34. Therefore, 6 needs to be multiplied by a number that is less than 6 to be a factor of 34. However, all numbers less than 6 have already been checked. Any other numbers that remain to be checked would also need to be paired with a number less than 6. Therefore, no other numbers need to be checked.

- Some students may confuse the concept of factors with multiples. Ask students to list factors of 10 and multiples of 10. Some students may reverse the answers. Emphasize to students the connection between multiple and multiply to help them differentiate between the terms.
- Some students may stop their factor tree before reaching the prime factors. Ask students to create a factor tree for the number 80. Some students may draw a tree similar to one of the trees shown.



For each "leaf" of their tree, ask students to identify the factors of the boxed numbers. Emphasize that creating a factor tree means continuing to divide as far as is possible until all "leaves" are prime.

► Enrichment

- Give students the factor pairs of a number and ask them to use those factor pairs to find factor pairs of a related number. For example, the factor pairs of 80 are 1 × 80, 2 × 40, 4 × 20, 5 × 16, and 8 × 10. Ask students to use the factors of 80 to find the factors of 40. Since 40 is half of 80, the factors of 40 are the factors of 80 with one of the factors divided by 2: 1 × 40, 2 × 20, 4 × 10, and 5 × 8.
- Ask students to determine factor pairs for a set of numbers and identify the relationship between those factor pairs. For example, students may create a table showing factor pairs for 16, 32, and 48. Observe that three of the pairs in each list have the same first number. However, the second number of each factor pair for 32 is two times the second number of each factor pair for 16 because 32 is two times 16. Similarly, the second number of each factor pair for 48 is three times

the second number of each factor pair for 16 because 48 is three times 16.

Factor Pairs for 16	Factor Pairs for 32	Factor Pairs for 48
1×16	1×32	1×48
2×8	2×16	2×24
		3×16
4×4	4×8	4×12
		6×8

Students may use different groupings of numbers to see if the relationships between the factor pairs continue to happen in the same way and discuss why that is or is not the case.

- Introduce students to the concept of numbers that are relatively prime and ask them to determine whether a given pair of numbers is relatively prime. For example, give students the numbers 24 and 35. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The factors of 35 are 1, 5, 7, and 35. Although neither number is prime, the numbers 24 and 35 as a pair are relatively prime because they have no factors in common except for 1.
- Ask students to discuss why it is possible to list all factors of a number but impossible to list all multiples of a number.
- Ask students to identify factors and multiples of 0.

► Key Terms:

composite, factor, factor pair, multiple, prime, whole number

B-O.3.1.1 & B-O.3.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Generate and analyze patterns.

Recognize, describe, extend, create, and replicate a variety of patterns.

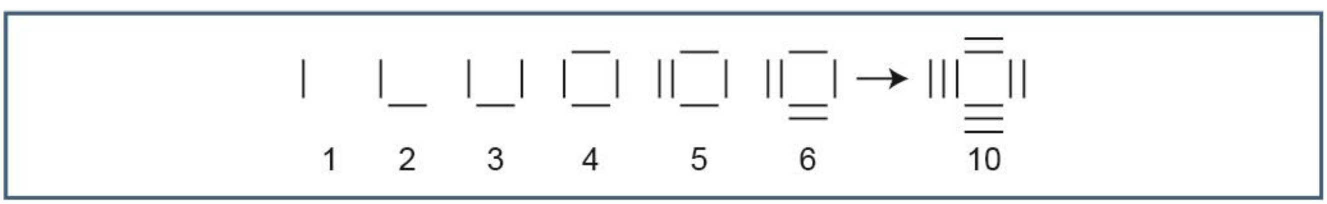
► Eligible Content

B-O.3.1.1 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

B-O.3.1.3 Determine the rule for a function given a table (limit to +, –, or × and to whole numbers).

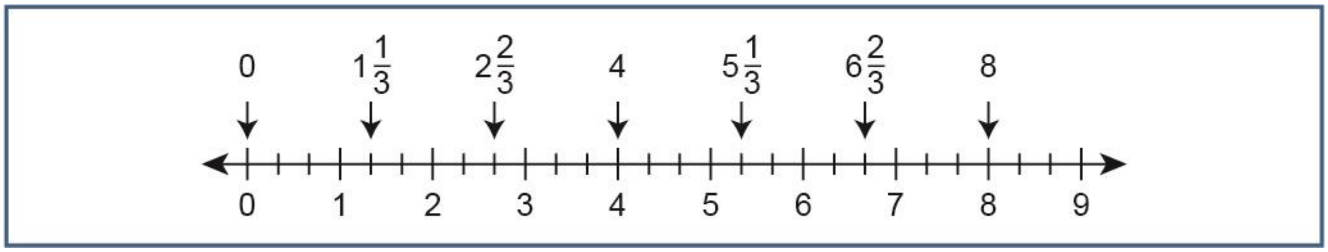
► Instructional Supports

- Ask students to create a sequence of numbers when given a starting number and a rule. For example, give students the description "starting at 5, multiply by 3." Students then create the sequence: 5, 15, 45, 135, . . .
- Ask students to find missing values in a sequence that increases at a constant rate. For example, give students the sequence 0, 15, 30, 45, ___, ___, 90. Students recognize that the numbers in the sequence increase by 15 and that the missing numbers are 60 and 75.
- Ask students to identify a rule from a given shape pattern. For example, provide a drawing of a pattern as shown and ask students to generate a rule for the pattern.



A student response may be that the pattern adds one stick each time and rotates around a square.

- Ask students to use various tools to extend a pattern. For example, give students the pattern "starting at 0, add $1\frac{1}{3}$" along with a number line. Ask students to plot the numbers in the pattern. Students should recognize that the pattern can be continued even if they have not yet learned to add fractions. Help students recognize that the numbers in the sequence are evenly spaced. Then give them a strip of paper and ask them to mark off on the strip the distance between consecutive numbers in the sequence. Students can use the marks to identify the locations on the number line of the next terms in the sequence.



- Give students a rule, ask them to generate the sequence, and then ask them to find additional patterns in the sequence. For example, give students a sequence that starts at 12 and follows the rule "add 11." Students write the sequence 12, 23, 34, 45, ... Some other possible patterns students may notice are shown.
    - The ones digit and tens digit both increase by 1.
    - The ones digit of each number is the tens digit of the following number.
    - The different digits in the sequence appear in increasing order.

► Identifying and Addressing Common Misconceptions

- Some students may assume that a sequence must always start with 1 or 0. Ask students to write a sequence from a given rule and starting value. Some students may ignore the starting value and start with 1 or 0, while others may append a 1 or 0 before the given starting value. Show students a sequence that does not start with 1 or 0 and ask them to describe the sequence. Then help them apply the same reasoning when given a description.
- Some students may not recognize numeric patterns that utilize multiplication versus addition. Ask students to describe the pattern in the sequence 2, 4, 8, 16, 32, ... Some students may only

look at the first two numbers and respond that the pattern is to add 2. Ask them to continue their stated pattern and help them recognize that their pattern does not continue to match the sequence. Help them understand that the goal is to describe a rule that describes the entire sequence, not just part of it.

- Some students may not recognize how many numbers must be given in order for a pattern to be established. Give students the sequence 1, 2, 4, . . . and ask them to find as many possible rules for the sequence as they can. Some students may think that there is only one rule. Help students find other possible rules for the sequence. Then show that in order to distinguish between different rules, it may be necessary to extend the amount of the sequence that is shown.

► Enrichment

- Have students create sequences that follow specific patterns. For example, ask students to create a sequence where every other number is odd and the other numbers are even. As an additional example, ask students to create a sequence where the digit in the tens place of each number is always a multiple of 3.
- Ask students to investigate sequences that are made of sums of other sequences. For example, give students the sequence of odd numbers. Then have students create a new sequence using partial sums of the odd numbers. The new sequence is 1, 1 + 3, 1 + 3 + 5, 1 + 3 + 5 + 7, 1 + 3 + 5 + 7 + 9, ... Help students recognize the pattern that the terms in this new sequence can all be shown as a square array of dots.
- Ask students to investigate sequences in which each term of the sequence is determined by the previous two terms, known as a Fibonacci sequence. For example, give students sequences like 1, 3, 4, 7, 11, 18, 29, . . ., in which the sum of two consecutive terms determines the following term.

► Key Terms

continue, explicit, generate, identify, pattern, rule, sequence, term

B-O.3.1.1 & B-O.3.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Generate and analyze patterns.

Recognize, describe, extend, create, and replicate a variety of patterns.

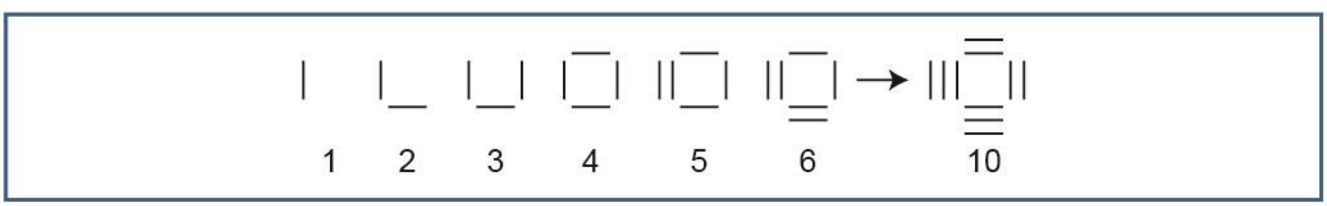
► Eligible Content

B-O.3.1.1 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

B-O.3.1.3 Determine the rule for a function given a table (limit to +, –, or × and to whole numbers).

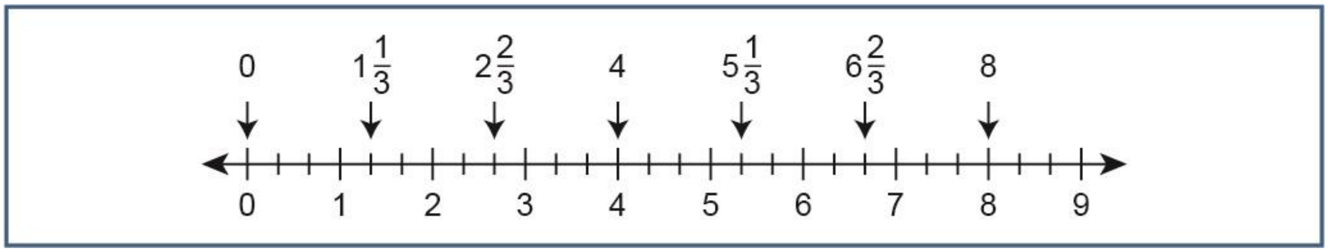
► Instructional Supports

- Ask students to create a sequence of numbers when given a starting number and a rule. For example, give students the description “starting at 5, multiply by 3.” Students then create the sequence: 5, 15, 45, 135, ...
- Ask students to find missing values in a sequence that increases at a constant rate. For example, give students the sequence 0, 15, 30, 45, ___, ___, 90. Students recognize that the numbers in the sequence increase by 15 and that the missing numbers are 60 and 75.
- Ask students to identify a rule from a given shape pattern. For example, provide a drawing of a pattern as shown and ask students to generate a rule for the pattern.



A student response may be that the pattern adds one stick each time and rotates around a square.

- Ask students to use various tools to extend a pattern. For example, give students the pattern "starting at 0, add $1\frac{1}{3}$" along with a number line. Ask students to plot the numbers in the pattern. Students should recognize that the pattern can be continued even if they have not yet learned to add fractions. Help students recognize that the numbers in the sequence are evenly spaced. Then give them a strip of paper and ask them to mark off on the strip the distance between consecutive numbers in the sequence. Students can use the marks to identify the locations on the number line of the next terms in the sequence.



- Give students a rule, ask them to generate the sequence, and then ask them to find additional patterns in the sequence. For example, give students a sequence that starts at 12 and follows the rule "add 11." Students write the sequence 12, 23, 34, 45, ... Some other possible patterns students may notice are shown.
    - The ones digit and tens digit both increase by 1.
    - The ones digit of each number is the tens digit of the following number.
    - The different digits in the sequence appear in increasing order.

► Identifying and Addressing Common Misconceptions

- Some students may assume that a sequence must always start with 1 or 0. Ask students to write a sequence from a given rule and starting value. Some students may ignore the starting value and start with 1 or 0, while others may append a 1 or 0 before the given starting value. Show students a sequence that does not start with 1 or 0 and ask them to describe the sequence. Then help them apply the same reasoning when given a description.
- Some students may not recognize numeric patterns that utilize multiplication versus addition. Ask students to describe the pattern in the sequence 2, 4, 8, 16, 32, ... Some students may only

look at the first two numbers and respond that the pattern is to add 2. Ask them to continue their stated pattern and help them recognize that their pattern does not continue to match the sequence. Help them understand that the goal is to describe a rule that describes the entire sequence, not just part of it.

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► Enrichment

- Have students create sequences that follow specific patterns. For example, ask students to create a sequence where every other number is odd and the other numbers are even. As an additional example, ask students to create a sequence where the digit in the tens place of each number is always a multiple of 3.
- Ask students to investigate sequences that are made of sums of other sequences. For example, give students the sequence of odd numbers. Then have students create a new sequence using partial sums of the odd numbers. The new sequence is 1, 1 + 3, 1 + 3 + 5, 1 + 3 + 5 + 7, 1 + 3 + 5 + 7 + 9, ... Help students recognize the pattern that the terms in this new sequence can all be shown as a square array of dots.
- Ask students to investigate sequences in which each term of the sequence is determined by the previous two terms, known as a Fibonacci sequence. For example, give students sequences like 1, 3, 4, 7, 11, 18, 29, . . . in which the sum of two consecutive terms determines the following term.

► Key Terms

continue, explicit, generate, identify, pattern, rule, sequence, term

C-G.1.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

List properties, classify, draw, and identify geometric figures in two dimensions.

▶ Eligible Content

C-G.1.1.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

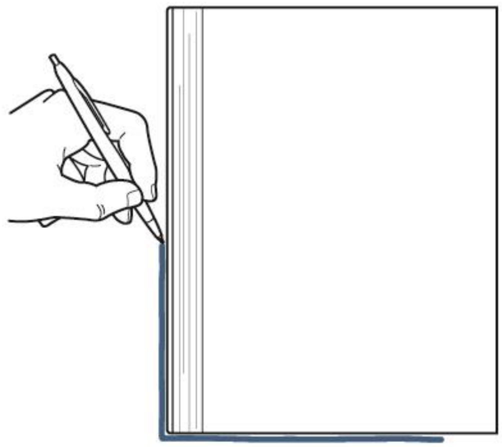
▶ Instructional Supports

Lines, line segments, and rays

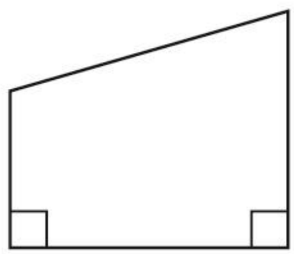
- Ask students to draw examples of points, line segments, lines, and rays and explain the differences among the last three.

The line segment has a finite length. Only a portion of the line and only a position of the ray are shown. The line, if fully drawn, would extend forever in both directions as indicated by the arrows. The complete ray would extend forever in the direction of the arrow.

- Ask students to use a pencil, paper, and ruler or other straightedge object (a textbook, the cardboard cover of a notebook, etc.) to draw a set of parallel line segments or a set of perpendicular line segments. For example, students can trace along two adjacent sides of a textbook to create perpendicular line segments because the angle between the two sides of the textbook has a measure of 90 degrees.



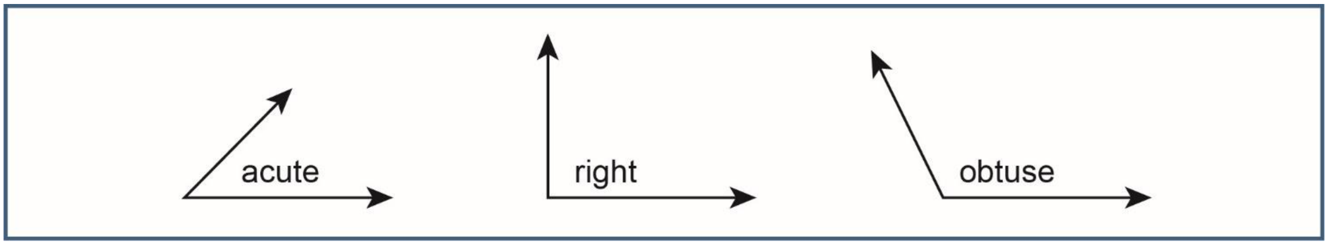
- Ask students to classify pairs of line segments in a figure as perpendicular, parallel, or neither. For example, give students the following trapezoid.



The trapezoid has two parallel line segments, each of which is perpendicular to one side of the trapezoid. The fourth side of the trapezoid is neither parallel nor perpendicular to the other sides of the trapezoid.

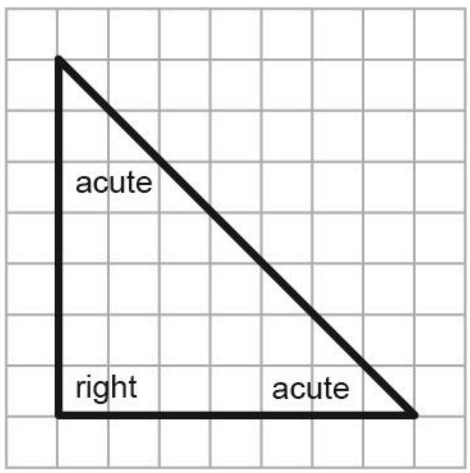
Angles

- Ask students to define and create examples of acute, right, and obtuse angles. Acute angles have measures that are less than 90 degrees, right angles have measures that are equal to 90 degrees, and obtuse angles have measures that are between 90 degrees and 180 degrees. Some possible examples are shown.



Have students exchange their drawn angles with another student. Each student should classify the angles they are given as acute, right, or obtuse.

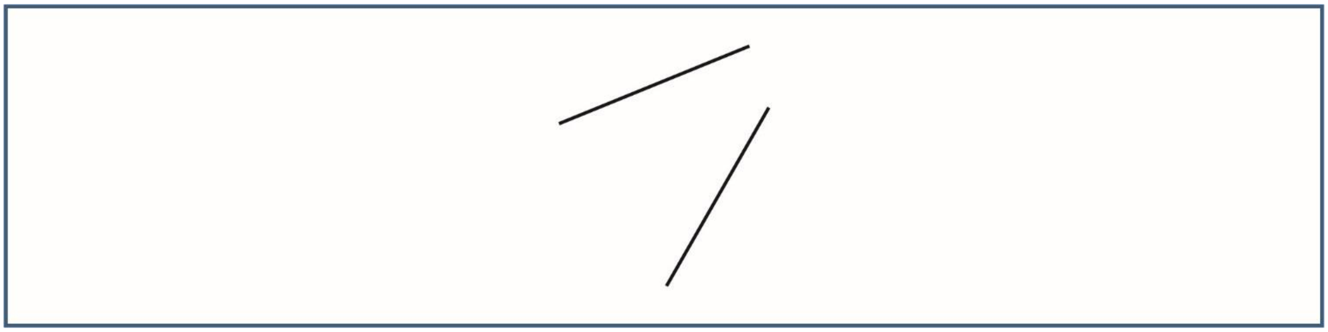
- Ask students to classify each angle in a figure as acute, right, or obtuse. For example, give students the following figure.



The triangle has three angles: one right angle and two acute angles.

► Identifying and Addressing Common Misconceptions

- Some students may think that any non-intersecting line segments are parallel. Give students the following figure and ask them whether the segments are parallel.



Some students may think that the line segments are parallel because they do not intersect. Ask students to use a ruler to extend the line segments. Remind students that line segments are only parallel if the lines containing them are parallel.

- Some students may think that points only exist where they are explicitly shown. Show students a square and ask them how many points are in the figure. Some students may think that there are only 4 points in the figure or possibly 0 points if the 4 vertices of the square are not marked with points. Point to the midpoint of a side and ask students if there is a point at that location. Help them understand that every part of a line or line segment is a point and that there are infinitely many points on any line or line segment.
- Some students may think that a triangle cannot have both obtuse and acute angles. Ask students how many acute angles are in a triangle that has an obtuse angle. Some students may only be able to visualize acute and right triangles and say none. Ask students to draw an obtuse angle and connect the ends of the rays they have drawn. Ask them to identify each angle in the triangle as acute, right, or obtuse.

► Enrichment

- Ask students to try to create a triangle that contains more than one obtuse angle. Alternatively, ask students to try to create a triangle that contains two right angles or a right angle and an obtuse angle.
- Ask students to create a quadrilateral that has specified angle types. For example, ask students to draw a quadrilateral with 2 right angles, an acute angle, and an obtuse angle.
- Ask students to give real-world examples of parallel lines, perpendicular lines, acute angles, right angles, and obtuse angles.

► Key Terms

acute, angle, characteristics, line, line segment, obtuse, parallel, perpendicular, point, ray, right, straight angle, two-dimensional, vertex

C-G.1.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

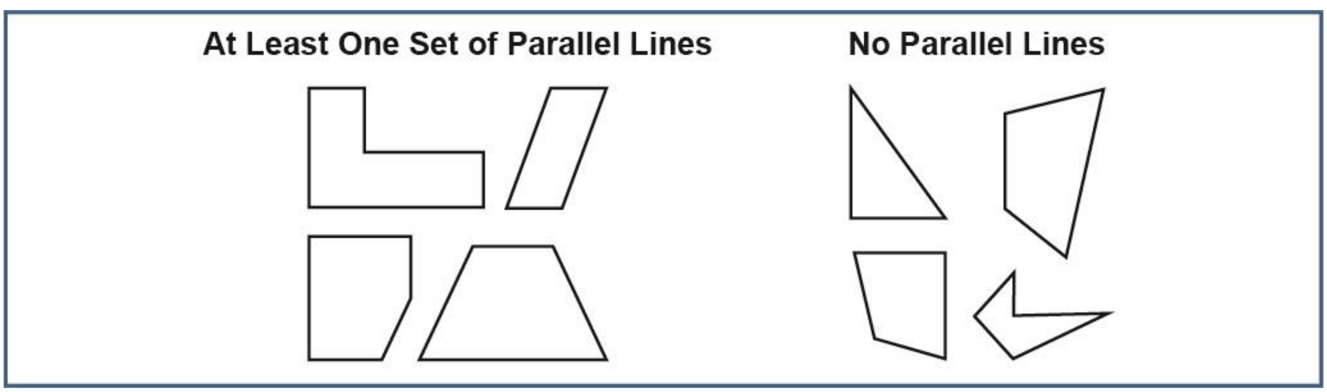
List properties, classify, draw, and identify geometric figures in two dimensions.

► Eligible Content

C-G.1.1.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

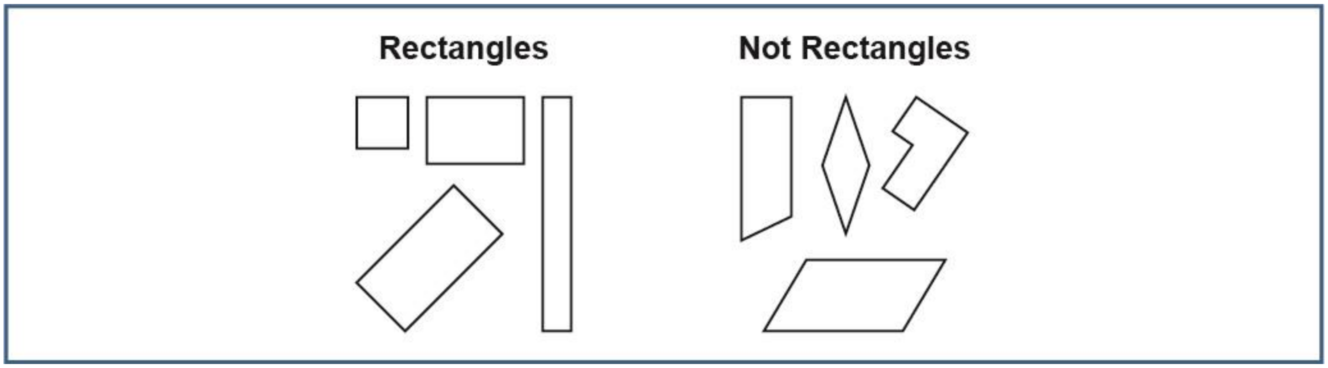
► Instructional Supports

- Ask students to sort shapes based on the number of sets of parallel lines in the shape. For example, give students a set of shapes and ask students to determine for each shape whether there is at least one set of parallel lines in the shape. The following figure shows eight shapes that have been sorted based on whether they have at least one set of parallel lines.

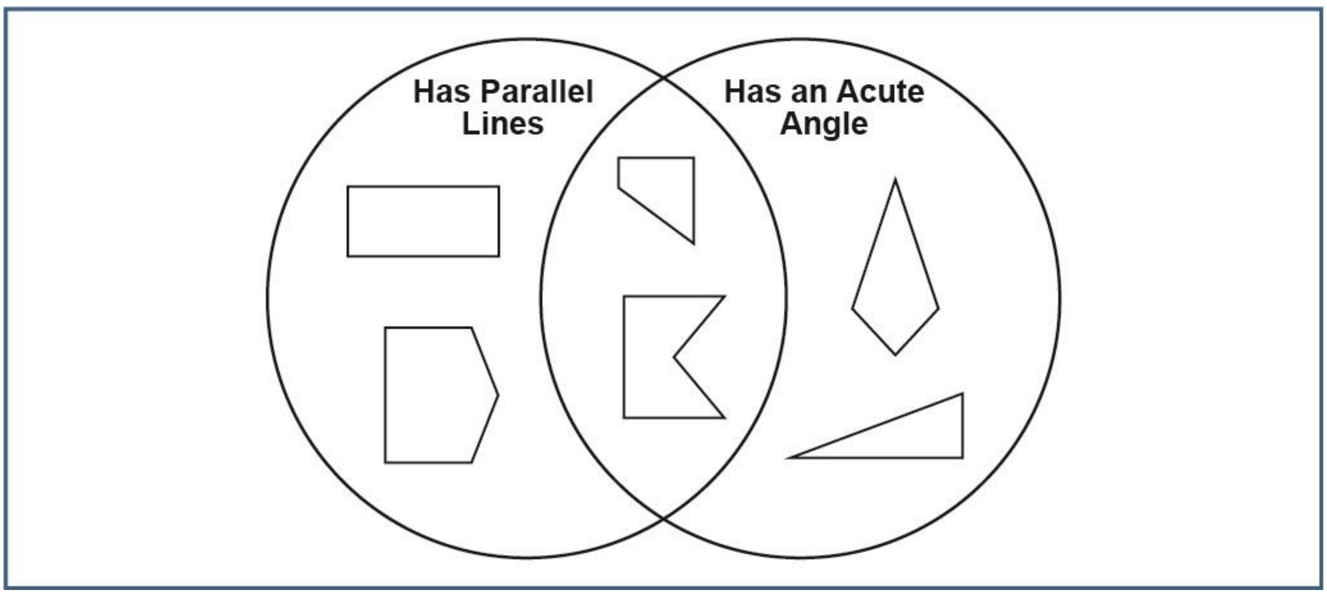


Additionally, ask students to identify quadrilaterals with two pairs of parallel lines as parallelograms.

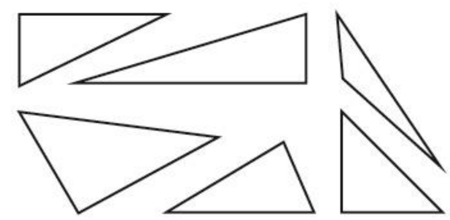
- Ask students to sort shapes based on the number of pairs of perpendicular lines in the shape. For example, ask students to determine which quadrilaterals in a set have four pairs of perpendicular lines (rectangles). The following figure shows eight shapes that have been sorted by a student.



- Ask students to create shapes that meet two different characteristics. For example, ask students to create shapes for a Venn diagram that have parallel lines, an acute angle, or both. A possible response is shown.

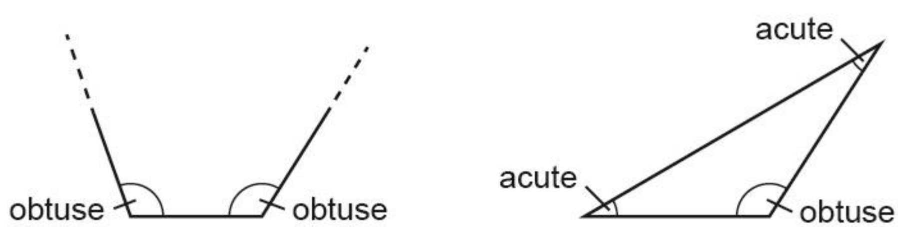


- Ask students to identify right triangles. For example, give students the following set of triangles.



The first two triangles in the top row and the first and third triangle in the bottom row are right triangles.

- Ask students to construct triangles with certain combinations of angle measures and recognize that some combinations are not possible. For example, ask students to draw a triangle with 2 obtuse angles and a triangle with 1 obtuse angle and 2 acute angles. It is not possible to draw a triangle with two obtuse angles. A possible triangle with 1 obtuse angle and 2 acute angles is shown.



▶ Identifying and Addressing Common Misconceptions

- Some students may think all parallelograms are rectangles. Show students a set of parallelograms, some of which have right angles and some of which do not, and ask students which shapes are parallelograms. Some may identify only the rectangles as parallelograms. Ask students what must be true of a shape for it to be a parallelogram. Use a visual model like four sticks connected with tape to represent a rectangle. Illustrate that the sticks can form both a rectangle and a parallelogram. By holding opposite corners of the rectangle and gently pulling the corners apart, demonstrate that the shape stops being a rectangle but still maintains two pairs of parallel sides. The transformed shape is therefore still a parallelogram but is no longer a rectangle.

- Some students may think that an acute triangle is any triangle that has at least one acute angle. Draw an obtuse triangle and ask students whether the triangle is acute, since it has two acute angles, or obtuse, since it has one obtuse angle. Some students may respond that the triangle is acute. Ask students to draw an obtuse angle and observe that obtuse triangles have only one obtuse angle and will always have two acute angles. Similarly, right triangles have only one right angle and two acute angles. Triangles are only acute if all three angles are acute.

► Enrichment

- Ask students to make observations about the size of the two acute angles in a right triangle. Students should observe that as one of the acute angles increases in size, the other acute angle decreases in size.
- Ask students to look for patterns and relationships between side lengths and angle measures in triangles. In particular, students should notice that in a triangle, the smallest angle is always opposite the shortest side and the largest angle is always opposite the longest side.
- Ask students to create Venn diagrams in which one of the sections is empty. For example, in a Venn diagram for triangles in which the left circle represents right triangles and the right circle represents triangles with at least one acute angle, there are no shapes that would fit in the left section. All right triangles would fit in the intersection of the two circles of the Venn diagram.

► Key Terms

acute, angle, categorize, characteristics, classify, equiangular, equilateral, isosceles, obtuse, parallel, parallelogram, perpendicular, polygon, regular polygon, rhombus, right angle, scalene, two-dimensional

C-G.1.1.3

► Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

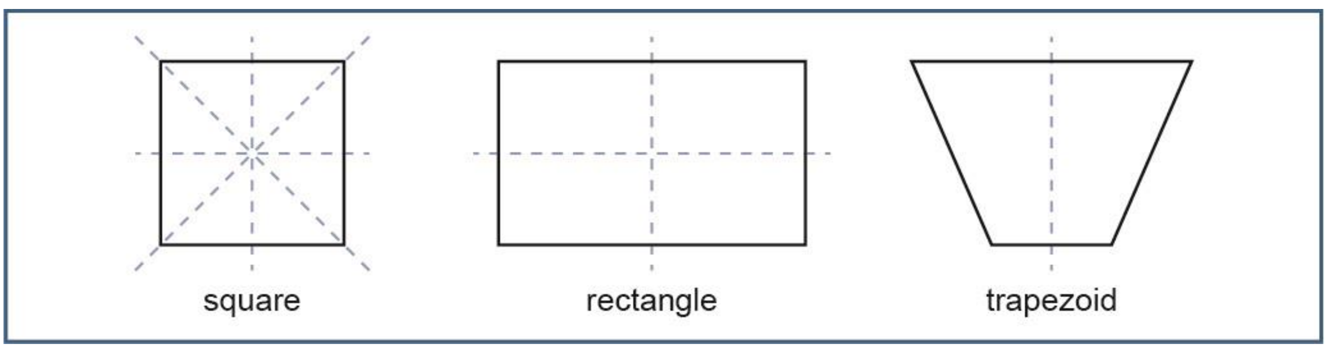
List properties, classify, draw, and identify geometric figures in two dimensions.

► Eligible Content

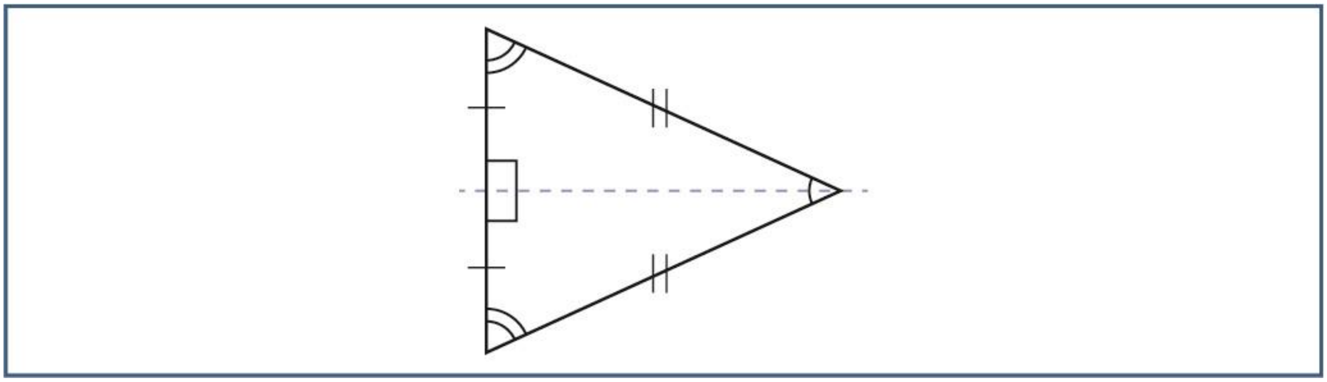
C-G.1.1.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into mirroring parts. Identify line-symmetric figures and draw lines of symmetry (up to two lines of symmetry).

► Instructional Supports

- Ask students to find lines of symmetry by folding paper figures in half. For example, give students a square, a rectangle, and an isosceles trapezoid. Help students observe that the square has four lines of symmetry, the rectangle has two, and the isosceles trapezoid has one.



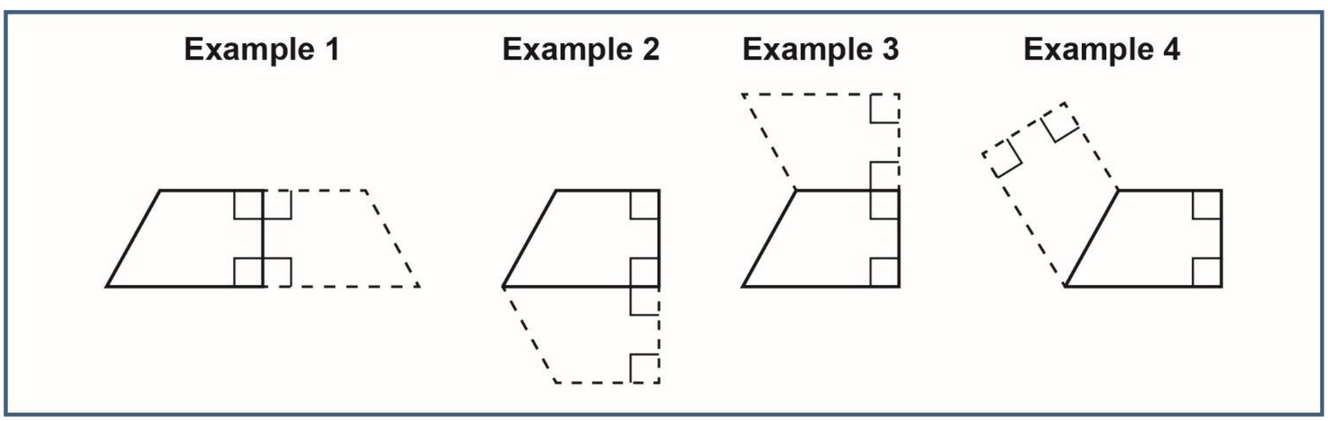
- Ask students to identify pairs of angles and pairs of sides or parts of sides that correspond when a line of symmetry is drawn on a figure. For example, when a line of symmetry is drawn on an equilateral triangle, each half has one right angle and two acute angles. Also, each part of the base of the original triangle is the same length, and the two other sides are the same length as each other.



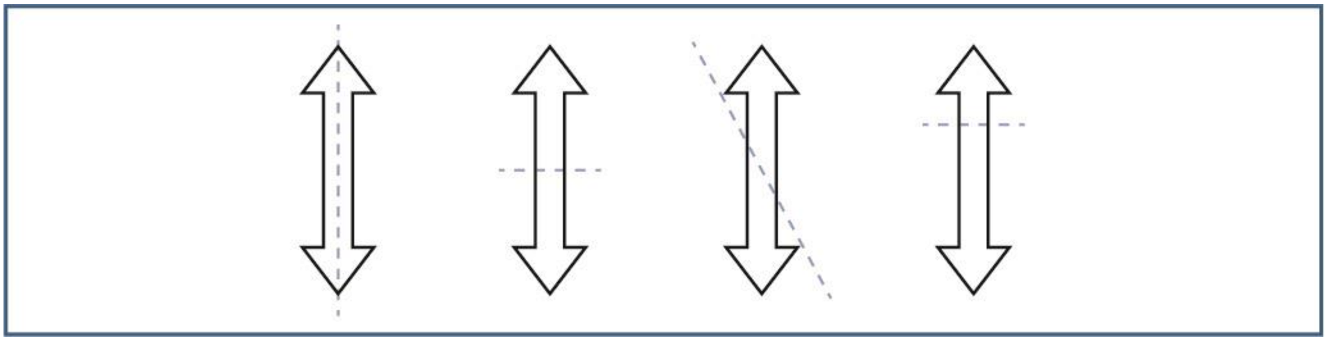
- Ask students to identify lines of symmetry in line figures. For example, give students the following letters of the alphabet.

The letter X has two lines of symmetry, the letters A, C, E, M, and T have one line of symmetry each, and the letters F and P do not have any lines of symmetry.

- Ask students to complete a figure when given half the figure and a line of symmetry for the entire, incomplete figure. For example, give students the following trapezoid and ask them to draw the resulting figure when each side of the trapezoid is used as a line of symmetry.

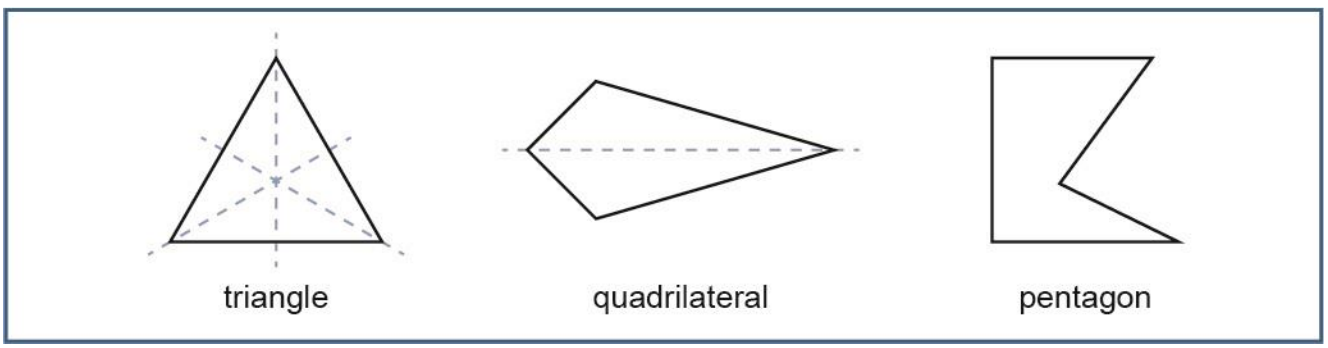


- Ask students to identify which lines shown in a figure are lines of symmetry. For example, give students the following figures.



The dashed lines in the first and second figures are lines of symmetry, and the dashed lines in the third and fourth figures are not lines of symmetry.

- Ask students to draw lines of symmetry for a figure, if possible, and determine how many lines of symmetry exist for the figure. In the following examples, the triangle has three lines of symmetry, the quadrilateral has one line of symmetry, and the pentagon has no lines of symmetry.



► Identifying and Addressing Common Misconceptions

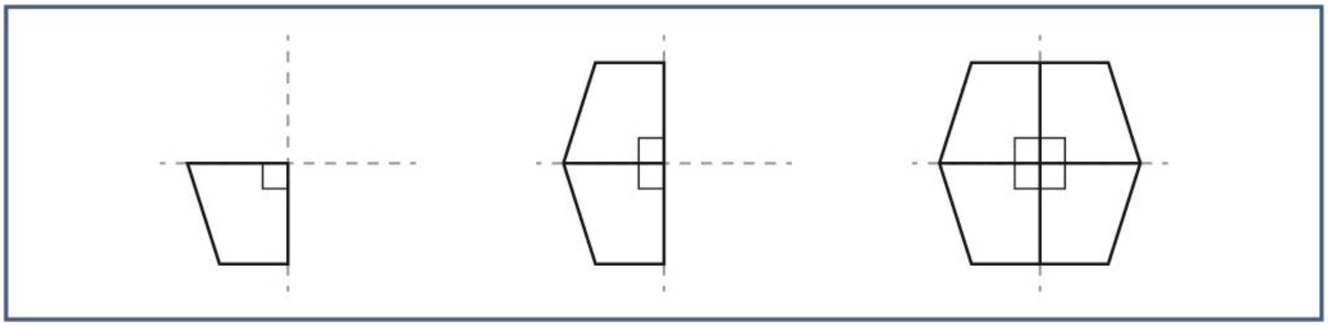
- Some students may think that lines of symmetry must always be perpendicular to the side of a given figure. Ask students to draw all the lines of symmetry of a regular hexagon. Some may omit the lines of symmetry that pass through the vertices. Ask students to fold the hexagon along one of the long diagonals and determine whether the crease is perpendicular to any of the sides of the hexagon.

- Some students may think the diagonals of all rectangles are lines of symmetry. Ask students to draw all the lines of symmetry in a rectangle that is not a square. Some students may include the diagonals. Have students cut out their rectangles and fold them along the lines of symmetry—in particular, any diagonal lines that the students indicated were lines of symmetry. The students should see that the two halves of the figure do not match up with each other, so the diagonals are not lines of symmetry.
- Some students may think that, when given an incomplete figure and a line of symmetry, the number of sides of the completed figure is always twice the number of sides of the incomplete figure. Give students half of an equilateral triangle and a line of symmetry of the triangle, then ask the students to count the number of sides of the incomplete figure (not counting the line of symmetry) and predict how many sides the completed figure will have. Some students may notice that the incomplete figure has two sides and assume that the completed figure will have four sides. Ask students to complete the triangle and notice that there are only three sides in the completed figure. Use additional examples to show that when the line of symmetry is perpendicular to a side of the incomplete figure, the number of sides does not double when completing the figure.

► Enrichment

- Ask students to make observations about the size or type of angle that is created by a line of symmetry. For example, when a straight angle (or line segment) is bisected by a line of symmetry, the result is two right angles. When an obtuse angle is bisected by a line of symmetry, the result must be two acute angles.

- Ask students to finish an incomplete figure that is shown with two different lines of symmetry. For example, give students the figure on the left and ask them to use the lines of symmetry to create the complete figure, shown on the right.



- Ask students to draw all the lines of symmetry in a figure with multiple lines of symmetry—for example, a regular polygon—and ask them to make observations about the point of intersection of the lines of symmetry. Ask them to explain why all the lines of symmetry of a figure must intersect at the same point.

► Key Terms

equal halves, line of symmetry, mirror image, symmetry, two-dimensional figure

D-M.1.1.1

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Solve problems involving length, weight (mass), liquid volume, time, area, and perimeter.

▶ Eligible Content

D-M.1.1.1 Know relative sizes of measurement units within one system of units including standard units (in., ft, yd, mi; oz., lb; c, pt, qt, gal), metric units (cm, m, km; g, kg; mL, L), and time (sec, min, hr, day, wk, mo, yr). Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. A table of equivalencies will be provided.

▶ Instructional Supports

- Ask students to record personal benchmarks for common units of measurement. Students can record the benchmark items in a table as shown.

Measurement	Benchmark
1 meter	*height of whiteboard*
1 gram	*pencil eraser*
1 pound	*my math book*
1 liter	*small water bottle*
1 foot	*length of my notebook*

- Ask students to compare the relative sizes of units within the same measurement system. For example, ask students to find how many grams are in a kilogram by placing a 1-kilogram mass on one side of a balance scale and adding 100-gram masses to the other side of the scale until the scale is balanced. To find the number of feet in a yard, have students measure an object that is 1 yard in length with a 12-inch ruler and count how many "rulers" long the object is. To determine the number of milliliters in a liter, use an empty water bottle that has a volume of 500 milliliters and have students fill it (i.e., with water or sand) and then pour the entire contents into an empty 1-liter container twice.

- Ask students to convert measurements from a larger unit to a smaller unit within both the US Customary system and the metric system.
    - A length of 8 yards can be converted to feet by multiplying the number of yards (8) by the number of feet per yard (3) to get 24 because 8 yards is 8 groups of 3 feet each.
    - To convert 24 feet to inches, multiply the number of feet (24) by the number of inches per foot (12) to get 288 because 24 feet is 24 groups of 12 inches each.
    - To convert 5 kilograms to grams, multiply the number of kilograms (5) by the number of grams per kilogram (1,000) to get 5,000 because 5 kilograms is 5 groups of 1,000 grams each.
- Ask students to understand and use the prefixes associated with the metric system, along with an understanding of place value, in order to perform conversions within the metric system. For example, since “milli” represents one thousandth, a millimeter is one thousandth of a meter and there are 1,000 millimeters in a meter. A measurement of 8 meters can be written as 8,000 millimeters. Likewise, since “kilo” represents one thousand, 3 kilograms can be written as 3,000 grams.
- Ask students to use a table to show measurement conversions. For example, ask students to create a table that shows the pattern when converting hours to minutes. Students should notice that each time the number of hours increases by 1, the number of minutes increases by 60. Therefore, there are 60 minutes in each hour as shown in the table.

Hours	Minutes
1	60
2	120
3	180
4	240
5	300

▶ Identifying and Addressing Common Misconceptions

- Some students may not be certain about what types of quantities are measured by which types of units. Ask students to identify units that could be used to measure the volume of a container. Students may respond with non-volume measures such as grams or pounds. Assist the students in creating lists or tables that separate the different measurements based on the type of quantity they measure.
- Some students may complete tables showing conversions by applying the pattern in one column (i.e., adding 1) to the second column. For example, ask students to complete the table shown.

Feet	Inches
1	12
2	
3	
4	

Some students may complete the table with the numbers 13, 14, and 15, since 1 + 1 = 2 and 12 + 1 = 13, and so on. The student assumes that since the given values in the "Feet" column were increasing by 1, then the values in the "Inches" column must also be increasing by 1. Use visual models such as a ruler to demonstrate that the number of inches increases by 12 each time the number of feet increases by 1.

- Some students may confuse the prefixes “kilo-” and “milli-” since they refer to the similar-sounding “thousand” and “thousandth.” Ask students what each prefix represents. Some students may say that both prefixes represent “thousand.” Emphasize the pronunciation of both “thousand” and “thousandth” and connect the ending sound of “thousandth” to other fractional quantities such as “fourth” and “fifth” (as opposed to “four” and “five”).

► Enrichment

- Ask students to convert non-whole numbers of units. For example, ask students to determine the number of inches in $3\frac{1}{2}$ feet by first determining the number of inches in 3 feet, then determining the number of inches in half a foot, and then combining those two results.
- Ask students to combine two given conversions into one “larger” conversion. For example, given that there are 60 minutes in 1 hour and 60 seconds in 1 minute, ask students to create a conversion that can be used to convert hours directly into seconds. In this case, students can multiply 60 × 60 to determine that there are 3,600 seconds in 1 hour.
- Ask students to use their understanding of arithmetic to explain why multiplication is used to convert from larger units to smaller units rather than addition, subtraction, or division.
- Ask students to create their own alternative units and create conversion tables for their alternative unit and a standard unit. For example, a student may create an alternative unit of “pencils” and the conversion that 1 pencil = 4 inches. Then, the student may create the conversion table shown.

Pencils	Inches
1	4
2	8
3	12
4	16

Students can also create incomplete tables and exchange them with other students to learn about other students’ alternative units.

► Key Terms

centimeter, comparisons, conversion table, equivalent measurement, estimate, gram, hour, kilometer, liter, mass, meter, metric system, milliliter, minute, ounce, personal benchmark, pound, prefix, relative size, second, US Customary system

D-M.1.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Solve problems involving length, weight (mass), liquid volume, time, area, and perimeter.

► Eligible Content

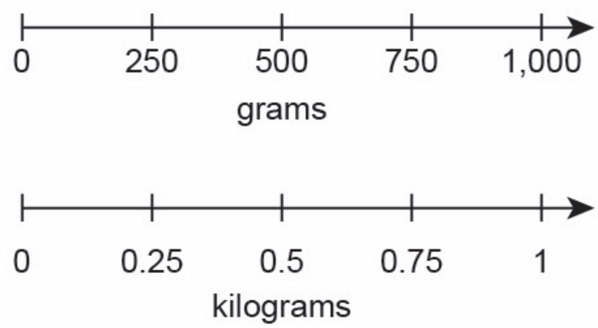
D-M.1.1.2 Use the four operations to solve word problems involving distances, intervals of time (such as elapsed time), liquid volumes, masses of objects; money, including problems involving simple fractions or decimals; and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

► Instructional Supports

- Ask students to solve a measurement problem by determining which operation is required, formulating a strategy to solve the problem, and then executing that strategy. For example, give students this problem: "Mr. Henson teaches piano to 4 students. He teaches for a total of 60 minutes. He teaches each student for the same amount of time. How many minutes did he teach piano to each student?" Students determine that since the problem involves dividing the total amount of time into 4 equal groups, the word problem requires division. Dividing the total amount of time Mr. Henson teaches by 4 will give the amount of time he taught each student. Since $60 \div 4 = 15$, Mr. Henson teaches piano to each student for 15 minutes.
- Ask students to algebraically solve a word problem that requires a larger unit to be expressed in terms of a smaller unit. For example, give students this problem: "Jeremy brings $2\frac{1}{4}$ pounds of potato salad to the picnic. Kayley brings 12 ounces of potato salad to the picnic. How many more ounces of potato salad does Jeremy bring than Kayley?" Students should recognize that the units must be the same. In this case, ounces are a convenient unit for answering the question. The student knows 16 ounces is equivalent to 1 pound, and so the 2 whole pounds Jeremy brings is

equivalent to 32 ounces, and the $\frac{1}{4}$ pound is equivalent to 4 ounces. Therefore, Jeremy brings 36 ounces because $32 + 4 = 36$. Since Kayley brings 12 ounces, Jeremy brings 24 more ounces than Kayley does.

- Ask students to create a visual model to solve a measurement problem. For example, give students this problem: "Andrea found two stones. One stone has a mass of 0.5 kilograms. The other stone has a mass that is 4 times the mass of the first stone. What is the total mass, in grams, of Andrea's two stones?" Students can use a double number line to determine that 500 grams is the same as 0.5 kilograms.



Andrea's smaller stone has a mass of 500 grams. Her larger stone has a mass of 2,000 grams because $4 \times 500 = 2{,}000$. Therefore, the total mass of Andrea's two stones is 2,500 grams because $500 + 2{,}000 = 2{,}500$.

▶ Identifying and Addressing Common Misconceptions

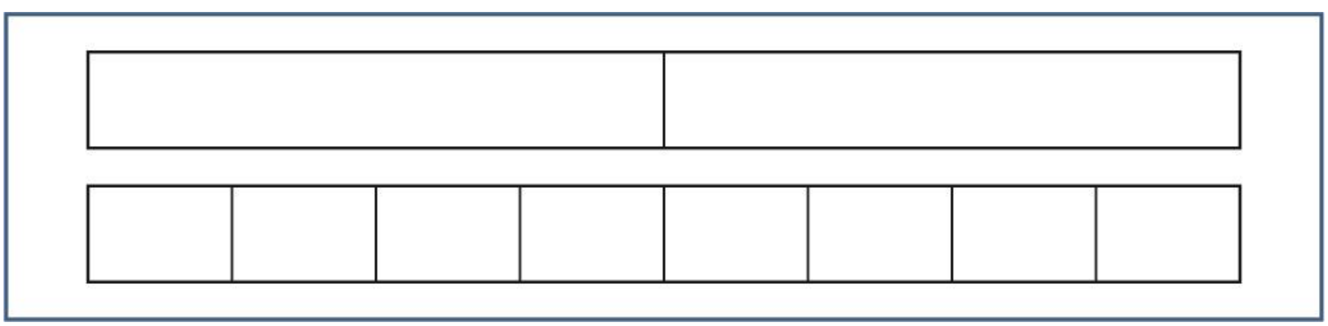
- Some students may select the incorrect operation(s) to solve a word problem. Ask students which operation or operations are necessary to solve this problem: "Lara and Mel each exercise. Lara exercises for 30 minutes. Mel exercises for 3 times as long as Lara. How many hours, in total, do Lara and Mel exercise?" Some students may not correctly identify that addition and multiplication (and, depending on the student, division) are necessary to solve the problem. Encourage students to highlight key words or phrases to help identify the necessary operations. In the given prompt, students could highlight the phrases "times as long" and "in total" to help them identify the operations of multiplication and addition, respectively.

- Some students may incorrectly convert units when fractions are involved. Ask students how many seconds are equivalent to $\frac{1}{4}$ minute. Some students may give answers including 30 (based on half a minute) or 56 (subtracting 4 from 60). Help students practice creating visual models for fractions and visual models for converting units. Then help them incorporate both concepts together into a visual model that can be used for converting fractional units.

- Some students may not know whether to multiply or divide when converting units. Ask students how many quarts are equivalent to 8 gallons. Some students may recall that 4 quarts are equivalent to 1 gallon, divide 8 by 4, and give an incorrect answer of 2. Ask students, “How many quarts are equivalent to 1 gallon?” Students should respond that 4 quarts are equivalent to 1 gallon. Then ask them whether, in that comparison, the number of quarts is greater or whether the number of gallons is greater. Help them recognize that because the number of quarts (4) is greater than the number of gallons (1) in the conversion rule, the number of quarts should always be greater than the number of gallons. Then ask the students whether multiplying by 4 or dividing by 4 would yield a greater number.

► Enrichment

- Give students blank double tape diagrams and ask them to try to complete the double tape diagrams they are given with as many different units and fractions of units as possible. For example, give students the following diagrams.



Some possible labels are shown.

- The top tape diagram is labeled with 1-foot increments and the bottom tape diagram is labeled with 3-inch increments.

- The top tape diagram is labeled with 1-pint increments and the bottom tape diagram is labeled with $\frac{1}{2}$-cup increments.

- The top tape diagram is labeled with 1-pound increments and the bottom tape diagram is labeled with 4-ounce increments.

- Provide students with word problems or statements with some missing values and ask them to fill in the possible missing values. For example, "Viola exercised for ___ of an hour. Xavier exercised for ___ times as long as Viola. Xavier exercised for $1\frac{1}{4}$ hours." For this example, the student must pick a time for Viola that is less than Xavier's time and then also correctly compare the two times. As a further challenge, ask students to find multiple correct solutions.
- Ask students to create an improvised measurement system and write word problems using their improvised measurement system. For example, an improvised measurement system with 3 levels might include "the length of one notebook," "the length of one pencil," and "the length of one eraser." The improvised system should include reasonable conversions such as "2 pencils = 1 notebook" and "4 erasers = 1 pencil."

► Key Terms

distance, double number line, intervals of time, liquid volume, mass, measurement scale, money, number line diagram, operations, table, tape diagram

D-M.1.1.3

▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

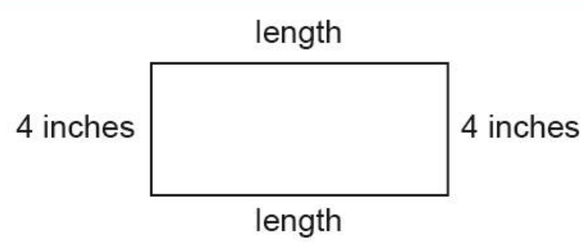
Solve problems involving length, weight (mass), liquid volume, time, area, and perimeter.

▶ Eligible Content

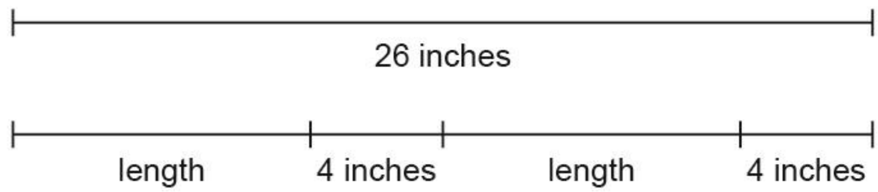
D-M.1.1.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems (may include finding a missing side length). Whole numbers only. The formulas will be provided.

▶ Instructional Supports

- Ask students to create and label a diagram to represent a given problem. For example, give students this problem: “A rectangle has a perimeter of 26 inches and a width of 4 inches. What is the length, in inches, of the rectangle?”



In addition to showing the rectangle, students may find it helpful to show the rectangle “unfolded” to form a straight line.



- Ask students to determine an unknown quantity in the formula for the perimeter of a rectangle when given other quantities in the formula. For example, give students this problem: “A rectangle has a perimeter of 18 inches. The length of the rectangle is 7 inches. What is the width of the rectangle?” Students should recall that the formula $P = l + l + w + w$ can be used to determine the perimeter (P) of a rectangle with a length of l units and a width of w units. Ask students to substitute the given quantities in the formula, giving the equation $18 = 7 + 7 + w + w$. The right side of the equation is the sum of 14 and $w + w$, and that sum is equal to 18. Therefore, $w + w$ must be equal to 4 because $18 = 14 + 4$. Because the 4 represents two identical widths of the rectangle, each width must be 2 inches.

- Ask students to determine an unknown quantity in the formula for the area of a rectangle when given the values of the other two quantities. For example, ask students to find the length of a rectangle whose area is 60 square inches and whose width is 6 inches. Students should recall that the formula $A = l \times w$ can be used to find the area (A) of a rectangle with a length of l inches and a width of w inches. Ask students to substitute the given quantities into the formula to get $60 = l \times 6$. The length of the rectangle must be a number such that when it is multiplied by 6, the product is 60. This value can be found by dividing 60 by 6 or using the student’s knowledge of multiplication facts. Because $60 = 10 \times 6$, the length of the rectangle must be 10 inches.

► Identifying and Addressing Common Misconceptions

- Some students may think there is only one possible area for a rectangle with a given perimeter. Ask students to determine the areas of two different rectangles with a perimeter of 20 units. Some students may think there is only one area possible, such as 25 square units, for a 5×5 square. Draw or create multiple rectangles with a perimeter of 20 and point out the variety of possibilities in their areas. The possible areas are as small as 9 square units and as large as 25 square units when the side lengths are restricted to whole numbers of units.

- Some students may forget to divide the unknown dimension in a “missing-length” or “missing-width” perimeter problem by 2. For example, ask students to find the width of a rectangle with a perimeter of 22 units and a length of 8 units. Students may perform the calculation $22 - 8 - 8 = 6$ and conclude the width is 6 units. Ask students to draw their rectangle and find the perimeter. Remind students that the missing value of 6 represents the combined lengths of both widths of the rectangle, and therefore the 6 must be divided by 2 to get a width of 3 units.

► Enrichment

- Ask students to find rectangles for which the perimeter is equal to the area when ignoring units. The only rectangles with whole-number side lengths with this property are 4 × 4 rectangles and 3 × 6 rectangles.
- Ask students to explore the semi-perimeter (half of the perimeter of a shape) and solve "missing-value" perimeter problems using the concept of semi-perimeter rather than perimeter. For example, the perimeter of a rectangle is 40 units and the length is 15 units. Students can find that the semi-perimeter is 20 units, and so the width of the rectangle is 20 – 15 = 5 units. Have students discuss which approach is easier for them and why.
- Give students a perimeter and ask them to find the number of unique rectangles with whole-number side lengths that have that perimeter.

► Key Terms

area, equation, formula, length, perimeter, rectangle, square units, width

D-M.2.1.1 & D-M.2.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Organize, display, and answer questions based on data.

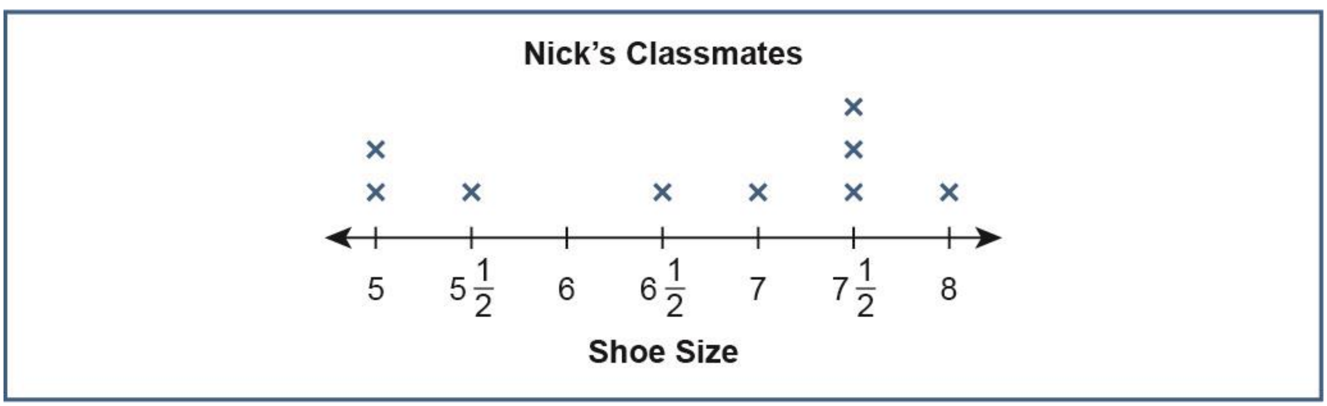
► Eligible Content

D-M.2.1.1 Make a line plot to display a data set of measurements in fractions of a unit (e.g., intervals of 1/2, 1/4, or 1/8).

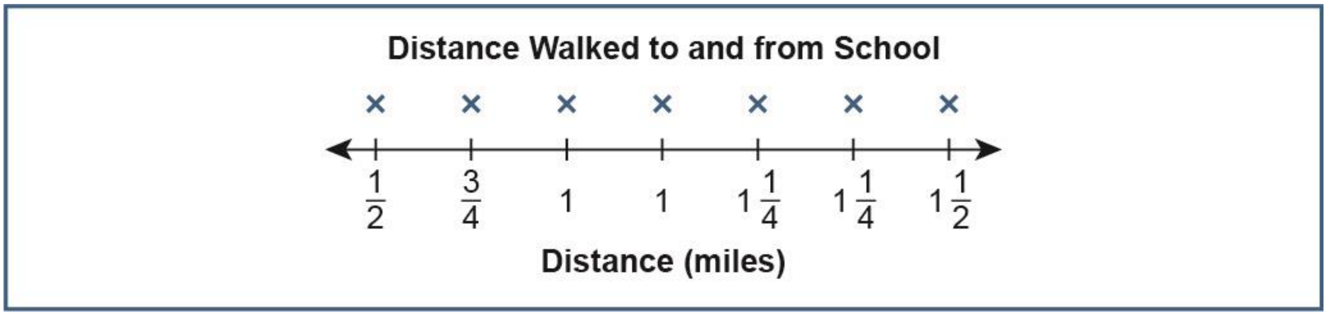
D-M.2.1.2 Solve problems involving addition and subtraction of fractions by using information presented in line plots (line plots must be labeled with common denominators, such as 1/4, 2/4, 3/4).

► Instructional Supports

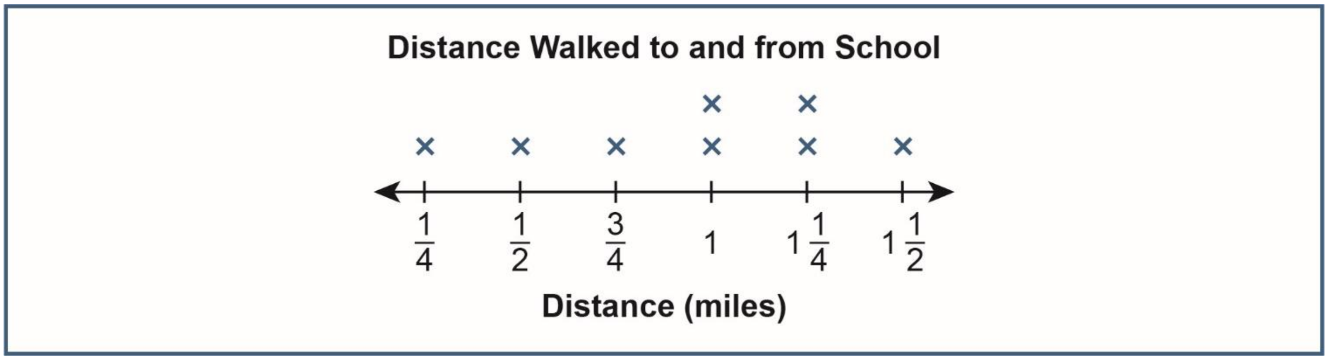
- Ask students to represent measurement data by displaying it in a line plot. For example, “Nick records the shoe sizes of some of his classmates. The shoe sizes are 5, 5, $5\frac{1}{2}$, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, $7\frac{1}{2}$, $7\frac{1}{2}$, and 8. Create a line plot to represent the data.” Students determine that because the smallest shoe size is 5 and the biggest shoe size is 8, the line plot needs to contain values ranging from 5 to 8. In addition, because the shoe sizes are either whole numbers or half-sizes, the scale should be in increments of $\frac{1}{2}$. A possible student response is shown.



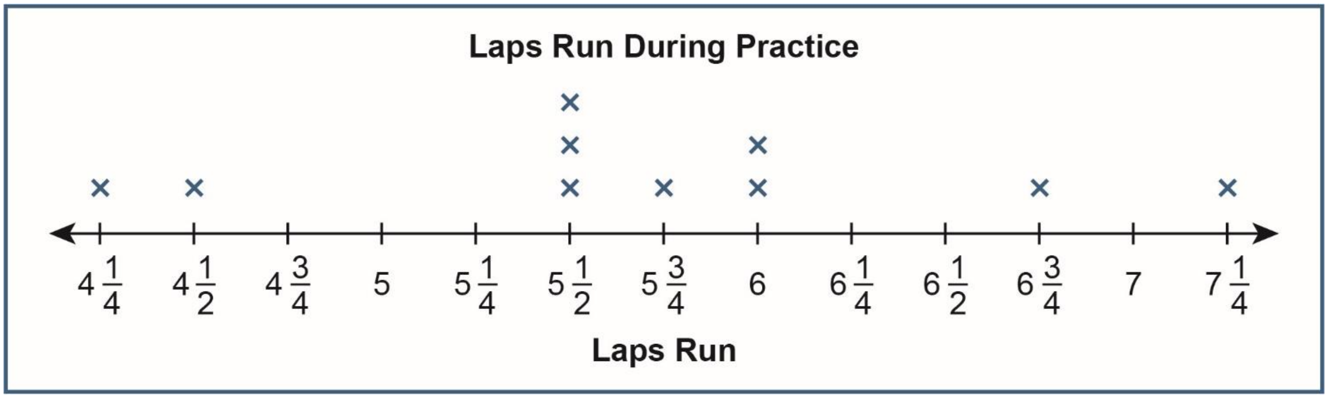
- Ask students to find, analyze, and correct errors in a line plot. For example, give students the following situation: "A teacher asked 8 students how far they walk, in miles, to and from school each day. These were the results: $\frac{1}{2}$, $\frac{3}{4}$, $1\frac{1}{4}$, $1\frac{1}{4}$, $1\frac{1}{2}$, 1, 1, and $\frac{1}{4}$. The teacher created the following line plot but made some errors."



Ask students to explain any possible errors with the line plot and then make a line plot to correctly display the teacher's data. Students should note that a separate hash mark was made on the line plot for each piece of data rather than keeping a consistent scale. Also, only 7 values were plotted, but there are 8 pieces of data; the data for $\frac{1}{4}$ was not included in the line plot. Students should explain that a correct line plot should have a consistent scale and should represent every piece of data collected. A corrected line plot using the same title and label that the teacher used is shown.

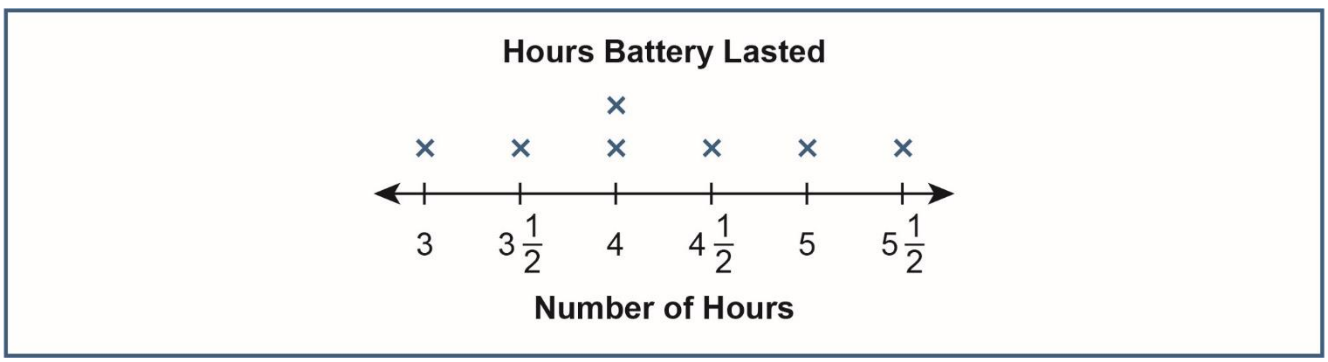


- Ask students to use the data displayed in a line plot to solve a related contextual problem. For example, give students the following line plot showing the number of laps each of 10 runners ran during track practice. Then ask them to determine how many more laps the runner who ran the most laps ran than the runner who ran the fewest laps.



Students determine that the point farthest to the left on the line plot represents the runner who ran the fewest laps. That runner ran $4\frac{1}{4}$ laps. The point farthest to the right on the line plot represents the runner who ran the most laps. That runner ran $7\frac{1}{4}$ laps. Students can solve $7\frac{1}{4} - 4\frac{1}{4}$ to calculate the difference in the number of laps. Therefore, the runner who ran the most laps ran 3 more laps than the runner who ran the fewest laps.

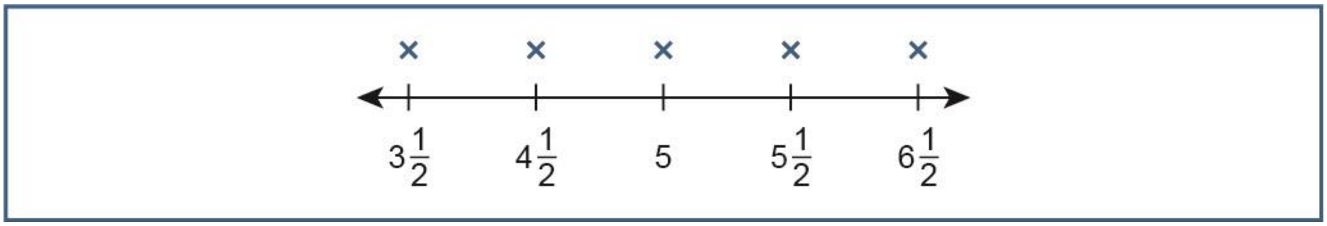
- Ask students to complete an incomplete line plot by using other given information. For example, give students the following situation: "Beth recorded the number of hours each of eight batteries lasted in a flashlight. The eight batteries lasted a total of 34 hours. The line plot shown represents Beth's data, but one data point is missing."



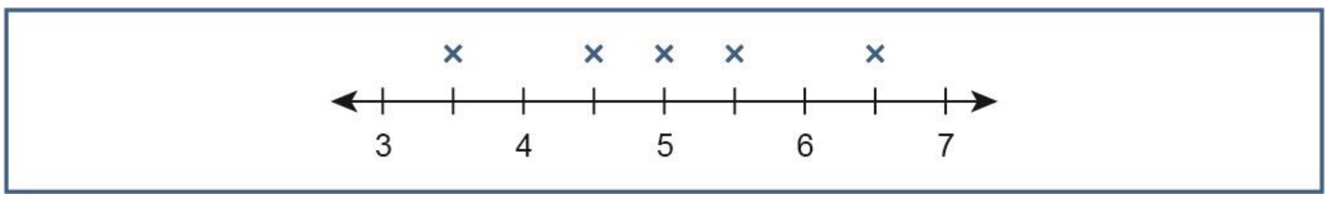
Ask students to complete the line plot to include the data point for the missing battery. The line plot shows that the times of seven of the batteries are 3, $3\frac{1}{2}$, 4, 4, $4\frac{1}{2}$, 5, and $5\frac{1}{2}$. Students calculate the total times of those seven batteries as $29\frac{1}{2}$ hours because $3 + 3\frac{1}{2} + 4 + 4 + 4\frac{1}{2} + 5 + 5\frac{1}{2} = 29\frac{1}{2}$. The time for the missing eighth battery is $4\frac{1}{2}$ hours because $34 - 29\frac{1}{2} = 4\frac{1}{2}$, so the missing data point should be plotted above $4\frac{1}{2}$. Therefore, the completed line plot should show two X's plotted above $4\frac{1}{2}$.

▶ Identifying and Addressing Common Misconceptions

- Some students may omit values on the scale of a line plot for which there are no corresponding data points. Ask students to create a line plot to represent the data $3\frac{1}{2}$, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, and $6\frac{1}{2}$. Some students may create the following plot.



While a line plot with gaps in the scale still shows the data values, it does not properly show spread or clustering of data and distorts comparisons within the data. A corrected plot is shown.



- Some students may repeat values on the scale of a line plot to represent repeated data points. Ask students to create a line plot to represent the data 6, $6\frac{1}{4}$, $6\frac{1}{4}$, $6\frac{1}{2}$, $6\frac{3}{4}$, $6\frac{3}{4}$, and 7. Some students may make a line plot with a scale that includes $6\frac{1}{4}$ and $6\frac{3}{4}$ twice because those values appear twice. Ask students to create a line plot with repeated numbers appearing multiple times and to create a corrected line plot. Ask students to describe the benefits of not repeating values on the scale. Using a proper scale for a line plot gives a line plot a distinct shape, which can show features like shape, spread, and clustered values much more clearly.

- Some students may omit data points when creating a line plot. Ask students to create a line plot given 10 data points. Some students may not include all 10 data points, either due to accidentally omitting points or not plotting repeated values. Choose an X on their plot and ask them to explain what the X indicates. Help students recognize that each X represents one data point and that as a result, there should be one X in the plot for each data point. Encourage students to quickly double-check that they plotted the correct number of pieces of data, but also explain that this double-check does not necessarily guarantee that the values are graphed correctly.

► Enrichment

- Give students two different line plots, each representing a similar situation, and ask students to compare the two data sets. For example, the two line plots could represent the number of hours spent reading a book on 7 consecutive days by Annie and Hassan, respectively. Ask students to determine who read for more total hours and determine the difference in the total number of hours each person read. Encourage students to first cross off any matching values. For example, if Annie read for 3 hours on one day and Hassan read for 3 hours on one day, those values cancel each other out.
- Give students a line plot that displays measurements and ask students to create a line plot to represent the same data in which the measurements are given in a smaller unit in the same system. For example, given a line plot which displays the weights, in pounds, of $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{1}{2}$, 3, and $3\frac{3}{4}$ pounds, students can create a line plot which displays the same weights in ounces: 36, 40, 40, 48, and 60. Students should discuss a reasonable scale and explain why the scale is reasonable. In this example, 4 is a reasonable scale for the second line plot because the first line plot has a scale of $\frac{1}{4}$ pound, which is equivalent to 4 ounces.
- Give students a line plot that is missing more than one value and the total of all the values. Ask students to find possible combinations for the multiple missing values.

► Key Terms

data set, fraction, label, line plot, maximum, measurement, minimum, mixed number, scale, title, visual model

D-M.2.1.1 & D-M.2.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Organize, display, and answer questions based on data.

► Eligible Content

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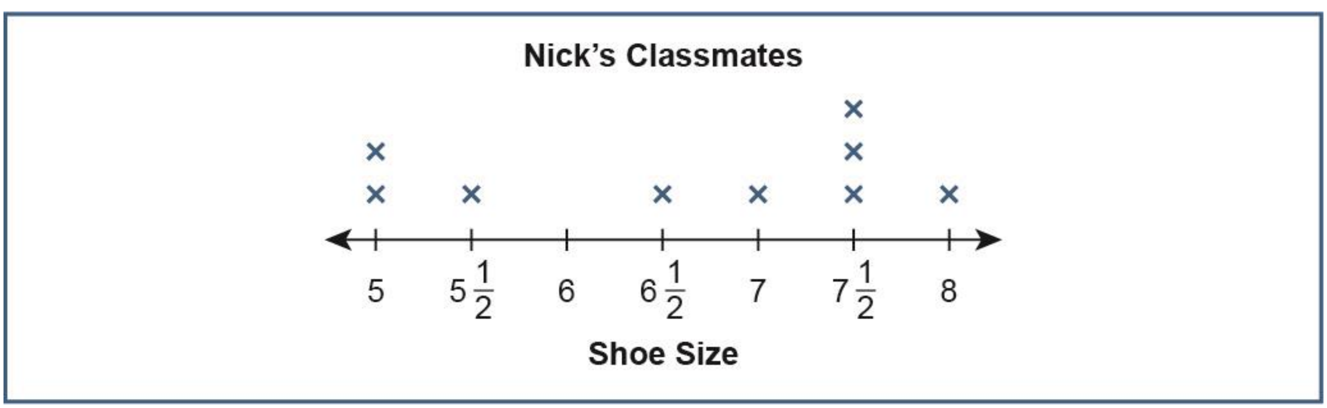
► Instructional Supports

- Ask students to represent measurement data by displaying it in a line plot. For example, "Nick records the shoe sizes of some of his classmates. The shoe sizes are 5, 5, $5\frac{1}{2}$, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, $7\frac{1}{2}$, $7\frac{1}{2}$, and 8. Create a line plot to represent the data." Students determine that because the smallest shoe size is 5 and the biggest shoe size is 8, the line plot needs to contain values ranging from 5 to 8. In addition, because the shoe sizes are either whole numbers or half-sizes, the scale should be in increments of $\frac{1}{2}$. A possible student response is shown.



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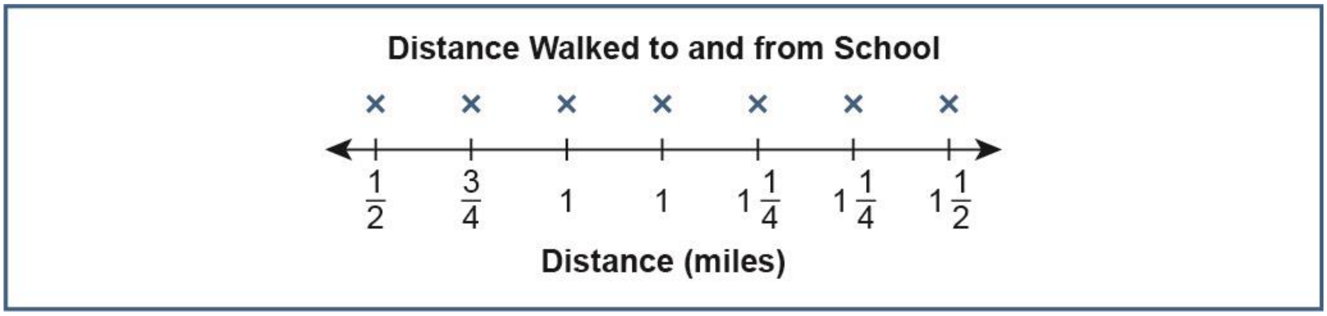

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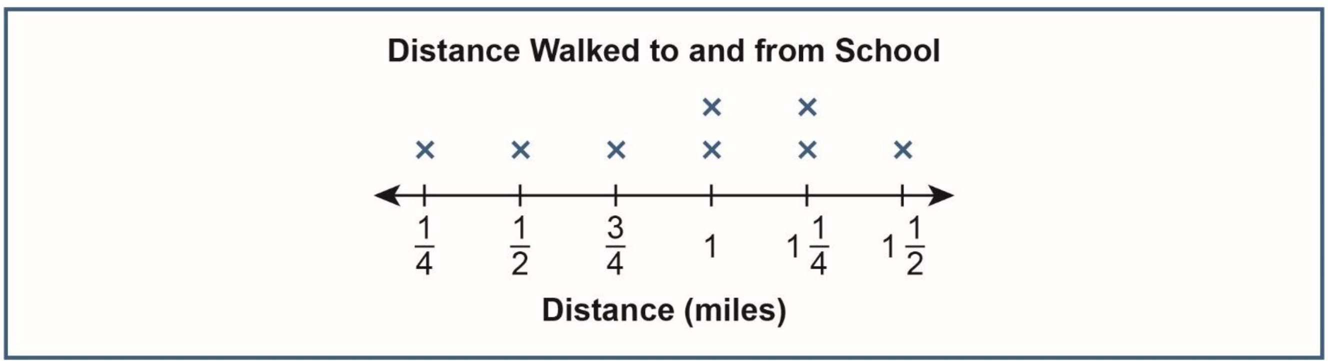

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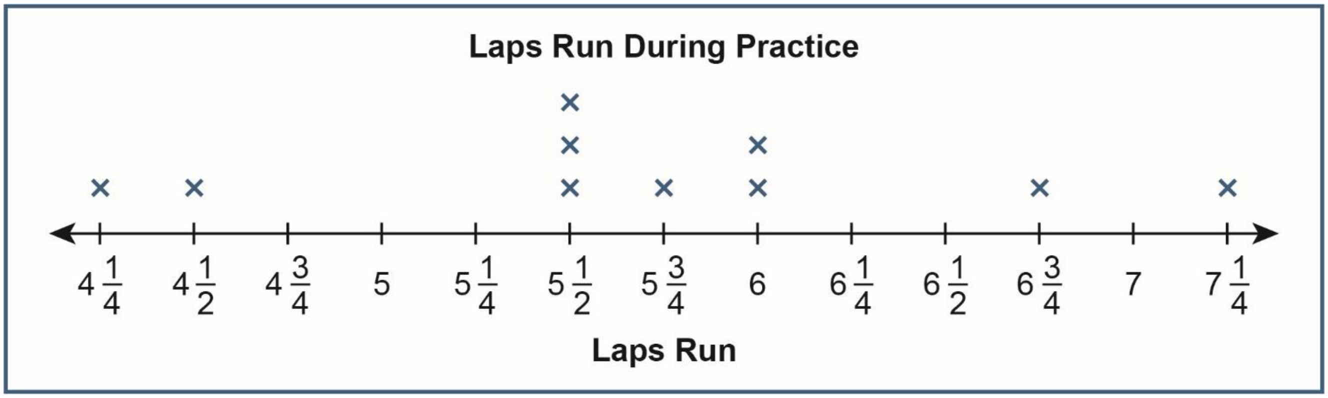

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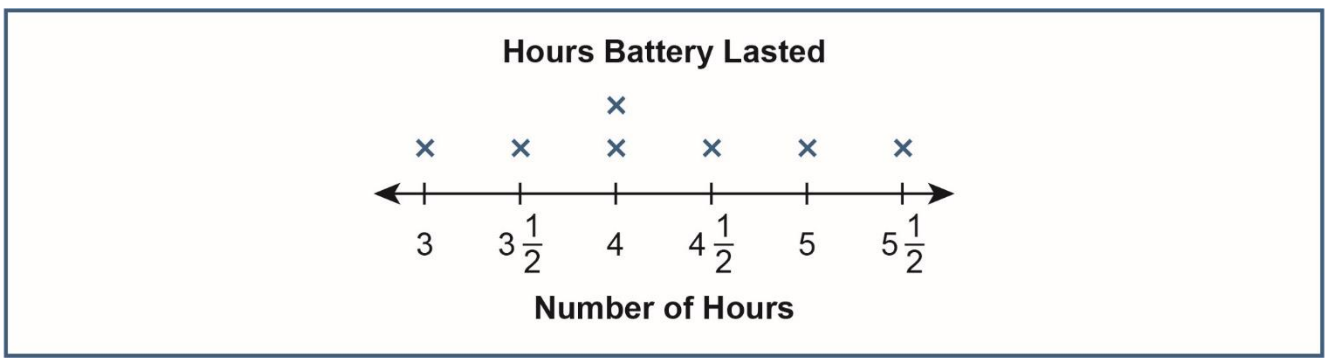

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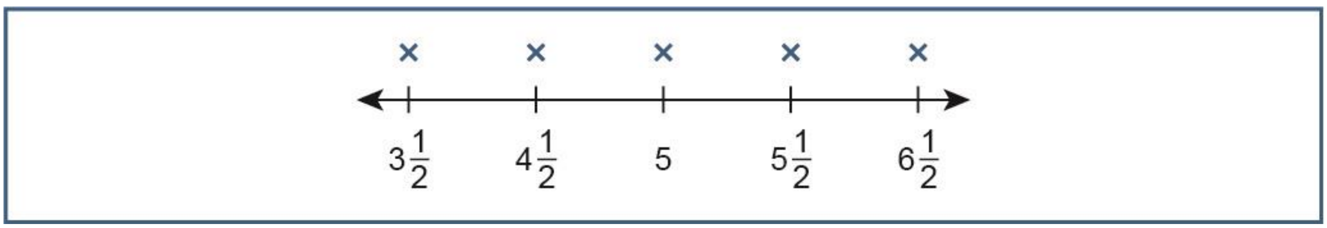

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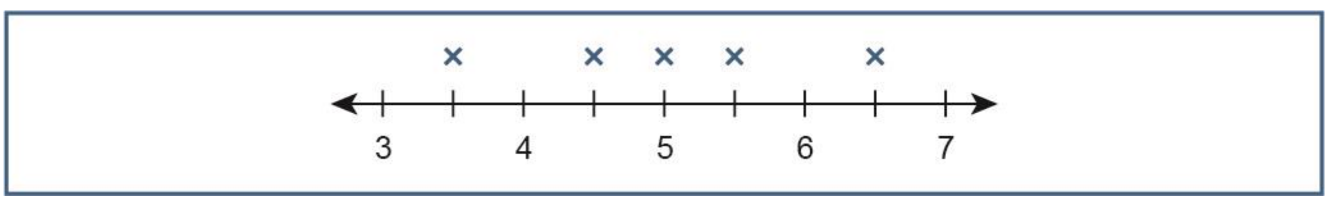

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► Enrichment

- Give students two different line plots, each representing a similar situation, and ask students to compare the two data sets. For example, the two line plots could represent the number of hours spent reading a book on 7 consecutive days by Annie and Hassan, respectively. Ask students to determine who read for more total hours and determine the difference in the total number of hours each person read. Encourage students to first cross off any matching values. For example, if Annie read for 3 hours on one day and Hassan read for 3 hours on one day, those values cancel each other out.
- Give students a line plot that displays measurements and ask students to create a line plot to represent the same data in which the measurements are given in a smaller unit in the same system. For example, given a line plot which displays the weights, in pounds, of $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{1}{2}$, 3, and $3\frac{3}{4}$ pounds, students can create a line plot which displays the same weights in ounces: 36, 40, 40, 48, and 60. Students should discuss a reasonable scale and explain why the scale is reasonable. In this example, 4 is a reasonable scale for the second line plot because the first line plot has a scale of $\frac{1}{4}$ pound, which is equivalent to 4 ounces.
- Give students a line plot that is missing more than one value and the total of all the values. Ask students to find possible combinations for the multiple missing values.

► Key Terms

data set, fraction, label, line plot, maximum, measurement, minimum, mixed number, scale, title, visual model

D-M.3.1.1

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Geometric measurement: understand concepts of angle and measure angles.

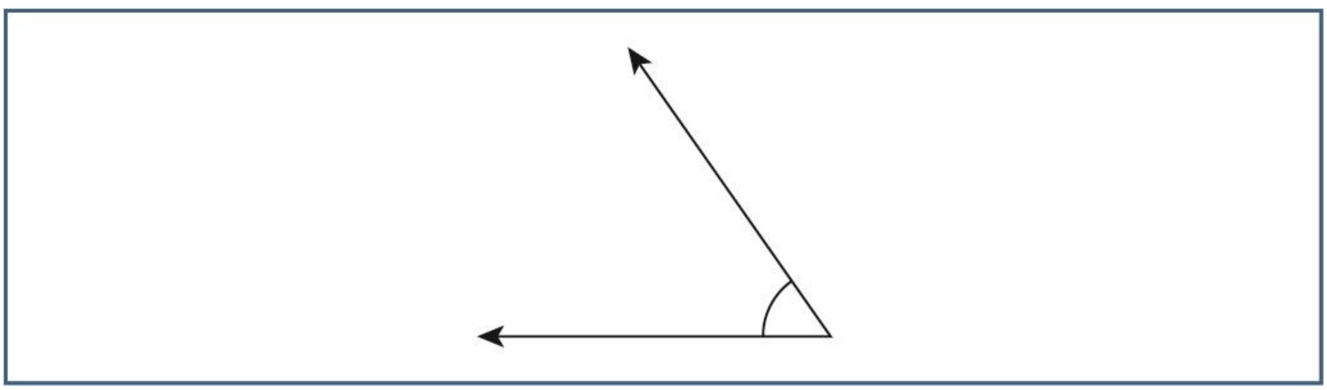
Use appropriate tools and units to sketch an angle and determine angle measurements.

► Eligible Content

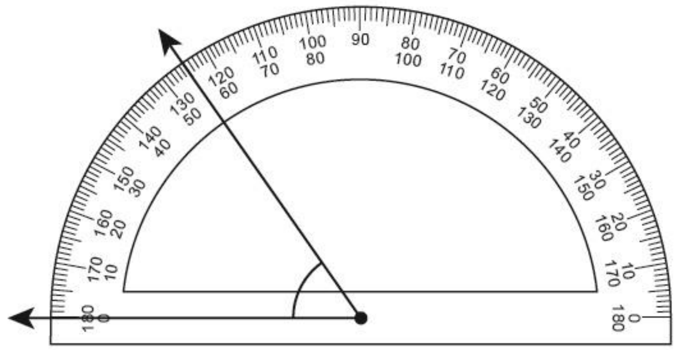
D-M.3.1.1 Measure angles in whole-number degrees using a protractor. With the aid of a protractor, sketch angles of specified measure.

► Instructional Supports

- Ask students to measure an angle using a protractor. For example, give students the angle shown.

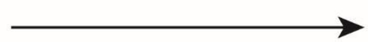


Ask students to align the vertex of the angle with the center point on the protractor and align one of the rays of the angle with one of the 0-degree marks on the protractor.

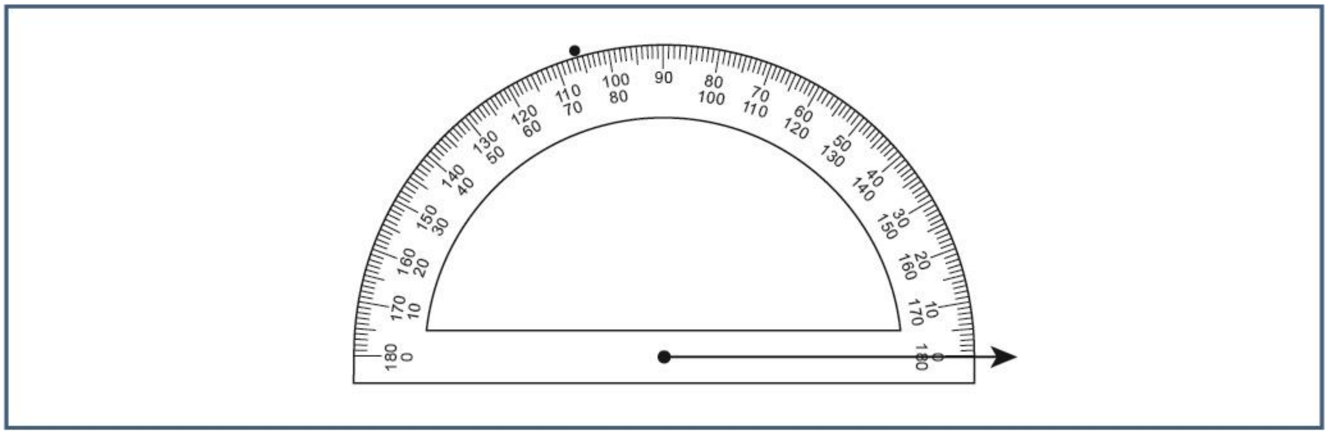


The protractor has two values where the ray intersects it. Because the horizontal ray intersects the 0-degree mark on the inner band of numbers, students determine the angle measurement by looking at the values on the inner band of numbers. Since the ray that is not aligned with the 0-degree mark intersects the protractor exactly halfway between 50 and 60, the angle has a measure of 55 degrees. Additionally, because the ray intersects the protractor at both 55 and 125 but is clearly part of an acute angle, students can conclude that the angle measure must be 55 degrees. To read the angle measure correctly, start where the horizontal ray intersects the 0-degree mark, follow the angle measures on the side in which the measures are increasing, and stop where the other ray intersects the protractor.

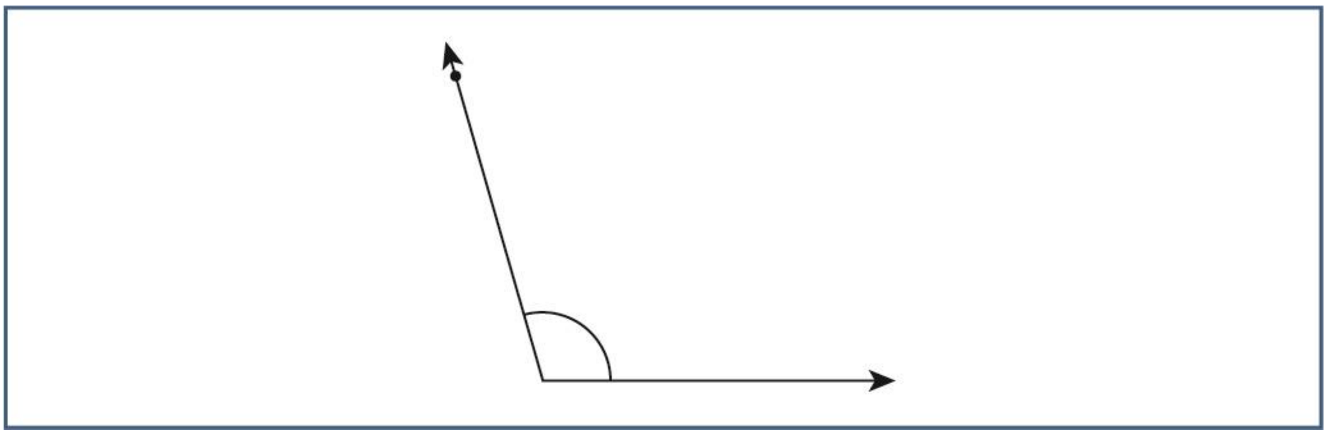
- Ask students to draw an angle of a given measurement using a protractor. For example, ask students to draw a 106-degree angle. Ask students to begin by drawing a ray using a straight edge, making sure that the ray is long enough to intersect the measurement part of the protractor.



Then ask students to align the protractor with the ray such that the vertex of the ray is centered and the ray intersects the 0-degree mark. Since this ray intersects the 0-degree mark of the outer band of numbers, students use the outer band of numbers to identify and mark a point at 106 degrees.

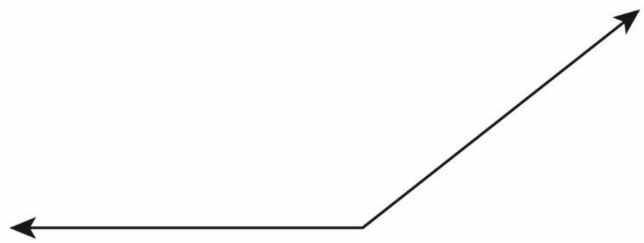


Ask students to remove the protractor, use a straight edge to draw a ray from the endpoint of the first ray through the point drawn, and draw an arc mark to indicate which of the two angles created is the 106-degree angle.

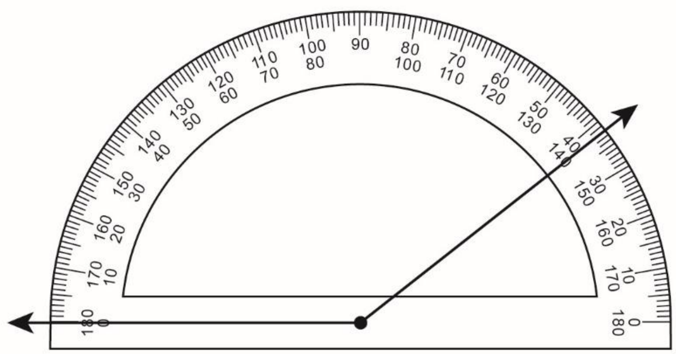


► Identifying and Addressing Common Misconceptions

- Some students may not align one of the two rays of the angle with the 0-degree mark on the protractor prior to measuring the angle. Ask students to measure a given angle. Some students may misalign their protractor with the angle. Ask students to trace an arc along the edge of their protractor from their point of measurement to zero. Then remove the protractor and point out that their arc does not match their angle.
- Some students may read from the wrong band of numbers on the protractor when drawing or measuring angles. Ask students to draw a 38-degree angle using a protractor. Some students may draw an angle like the one shown.

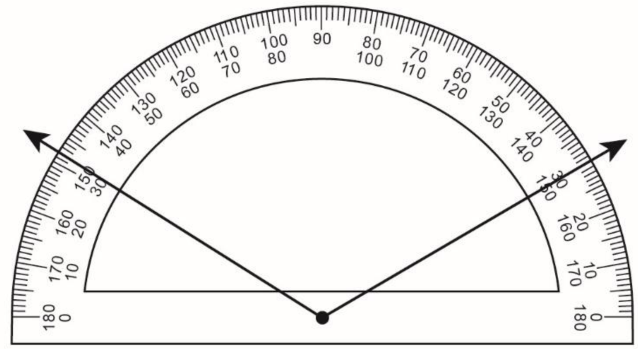


Ask students what it means for an angle to be acute or obtuse and identify whether their drawn angle is acute or obtuse. Help students recognize that each pair of numbers on a protractor (other than 0, 90, or 180) contains one acute measurement and one obtuse measurement.



► Enrichment

- Ask students to measure an angle by aligning the angle so both rays intersect the protractor and calculating the difference between the values of the marks at which the two rays intersect with the protractor. For example, a student measures an angle using the setup shown.



The rays intersect the inner band of numbers at 150 and 32. The angle has a measure of 118 degrees because 150 – 32 = 118. Or, the rays intersect the outer band of numbers at 148 and 30. Reading the outer band, the angle has a measure of 118 degrees because 148 – 30 = 118.

- Ask students to draw two angles that share a common ray. Have students measure each of the three created angles (the two small angles and the larger one) and make observations about the relationships among the measures of the three angles.
- Ask students to determine multiple methods to measure an angle with a measurement that is greater than 180 degrees using a protractor.
- Ask students to play a game in pairs in which each student gives the other student an angle measure between 0 degrees and 180 degrees. The goal is to draw the angle as close to the given angle measure as possible using a straightedge but without a protractor. Then, students measure their angles to see how close they were. They score 1 point for each degree the measure of their angle is away from the given angle measure. Students can play up to a given point total, such as 20, in which the game is over when one person scores 20 points. The winner is the person with fewer points.

► Key Terms

align, angle, degree, endpoint, leg, measure, point, protractor, ray, vertex

D-M.3.1.2

► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Geometric measurement: understand concepts of angle and measure angles.

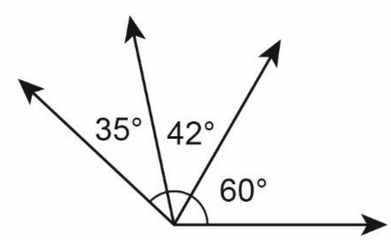
Use appropriate tools and units to sketch an angle and determine angle measurements.

► Eligible Content

D-M.3.1.2 Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems. (Angles must be adjacent and non-overlapping.)

► Instructional Supports

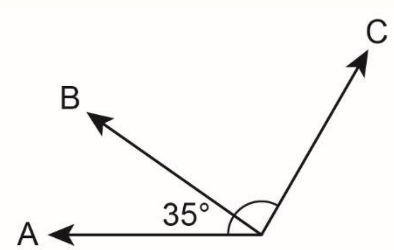
- Ask students to determine the measure of an angle composed of smaller angles. For example, give students the following figure and ask them to find the measure of the large angle in the figure.



  Students determine that the large angle is made up of the three smaller angles. Therefore, the sum of the measures of the smaller angles is equal to the measure of the largest angle. Students add the measures of the three smaller angles to get 60 + 42 + 35 = 137. The largest angle has a measure of 137 degrees.

- Ask students to determine multiple ways in which an angle can be decomposed into smaller angles. For example, give students this problem: “An angle that has a measure of 85 degrees is decomposed into an angle that measures 40 degrees, an angle that measures 14 degrees, and two other angles. What are two examples of possible measures of the two other angles?” First, students determine that the sum of the two given angle measures is 54 degrees. Then, they determine that the sum of the measures of the two unknown angles must be 31 degrees because 85 – 54 = 31. Students need to calculate two angle measures that have a sum of 31 degrees. Some possibilities include 1 degree and 30 degrees, 5 degrees and 26 degrees, or 15 degrees and 16 degrees.

- Ask students to solve a real-world problem by composing or decomposing angles. For example, "A park has three paths that all meet at a single point. Each path is represented by a ray as shown in the diagram. The angle formed between path A and path B has a measure of 35 degrees. The angle formed between path A and path C has a measure of 120 degrees. What is the measure of the angle formed between path B and path C?"

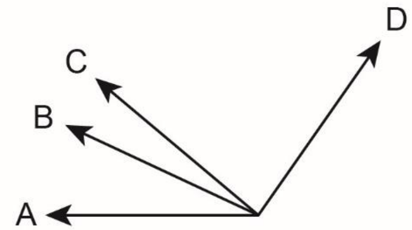


The angle formed between path A and path C is composed of the angle formed between path A and path B as well as the angle formed between path B and path C. The measure of the angle formed between path A and path C is 120 degrees, which is the sum of the two smaller angles, one of which is 35 degrees. Therefore, the degree measure of the angle formed between path B and path C is equal to 120 – 35 = 85.

► Identifying and Addressing Common Misconceptions

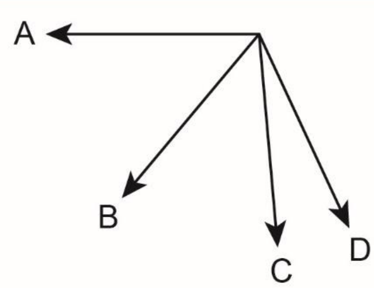
- Some students may be unsure of when to add and when to subtract given angle measures. Give students this example: "Rays A, B, and C all have a common endpoint, and ray B is between ray A and ray C. The measure of the angle formed between ray A and ray B is 64 degrees. The measure of the angle formed between ray A and ray C is 111 degrees. What is the measure of the angle formed between ray B and ray C?" Some students may respond that the angle measure is 175 degrees because 111 + 64 = 175. Ask students to draw a diagram, if necessary, and identify which of the angles formed by rays A, B, and C is the largest. Students should identify the angle formed by ray A and ray C as the largest, and they should determine that they need to use a take-away model with subtraction to determine the missing, smaller angles.

- Some students may try to add measures of angles that overlap. Ask students, “In the following figure, the angle formed between A and C is 40 degrees, and the angle formed between B and D is 100 degrees. What is the measure of the angle formed between A and D?”



Some students may respond that the angle measure between A and D is 140 degrees since 100 + 40 = 140. Ask students to shade the inside of the angle between A and C as well as the inside of the angle between B and D. Help students recognize that because the angles overlap, the sum of their measures includes the overlap twice, which leads to an incorrect measure for the angle formed between A and D.

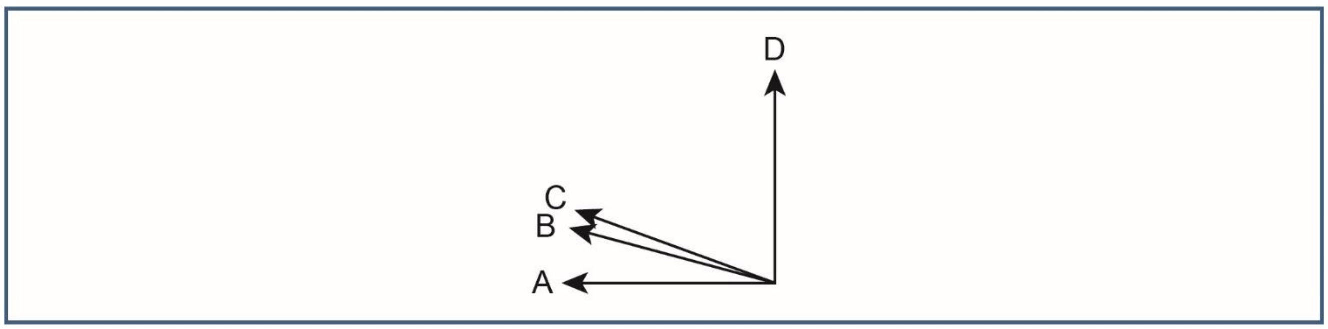
- Some students may try to add angles that are not adjacent. Ask students, “In the following figure, the angle formed between A and B is 50 degrees, and the angle formed between C and D is 20 degrees. What is the measure of the angle formed between A and D?”



Some students may respond that the angle measure between A and D is 70 degrees since 50 + 20 = 70. Ask students to shade the inside of the angle between A and B as well as the inside of the angle between C and D. Help students recognize that because the angles are not adjacent, adding the measures does not give the entire measure of the angle between A and D.

► Enrichment

- Ask students to solve problems in which a large angle is composed of multiple "groups" of angles and all the angles in each group have the same measure. For example, "An angle with a measure of 110 degrees is made up of 5 smaller angles. Of those, 2 angles have the same measure as one another and the other 3 angles have the same measure as one another. What are some possible measures of the 5 angles that make up the 110-degree angle?" One possible solution is 2 angles that each measure 25 degrees and 3 angles that each measure 20 degrees.
- Ask students to solve problems in which the relationships among measures of smaller angles which combine to make a larger angle are given as multiplicative comparisons. For example, ask students to determine the measures of two angles that make a 90-degree angle given that the measure of one angle is 4 times as great as the measure of the other angle.
- Ask students to solve multistep problems involving overlapping angles. For example, give students the following diagram.



Ask students to calculate the measure of the angle formed between B and C given that:

- the measure of the angle formed between A and C is 20 degrees,
- the measure of the angle formed between B and D is 75 degrees, and
- the measure of the angle formed between A and D is 90 degrees.

► Key Terms

addend, addition, additive, angle, angle measure, compose, decompose, diagram, equation, non-overlapping, subtraction, sum