# A-F.1.1.1

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations – Fractions

Develop understanding of fractions as numbers.

Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers.

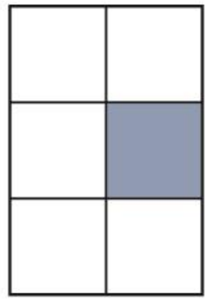
## ► Eligible Content

A-F.1.1.1 Demonstrate that when a whole or set is partitioned into y equal parts, the fraction 1/y represents 1 part of the whole and/or the fraction x/y represents x equal parts of the whole (limit the denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator; no simplification necessary).

## ► Instructional Supports

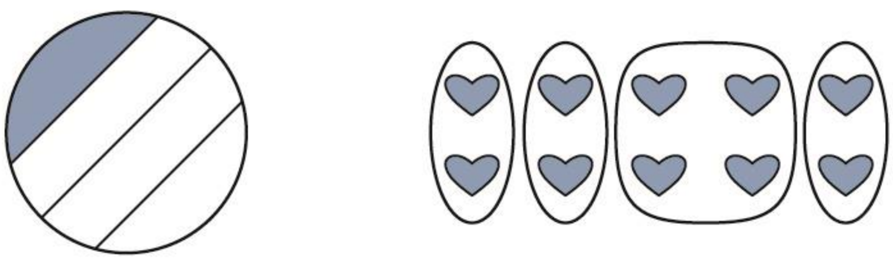
**What fractions mean**

- Ask students to identify, write, and explain the meaning of the numerators and denominators of fractions represented by visual models. Explain that the denominator shows how many equally sized parts make one whole and the numerator shows the number of equally sized parts being considered or counted. For example, the following area model shows 1 shaded square in a rectangle that has been partitioned into 6 equally sized squares. Students should record the fraction $\frac{1}{6}$ and explain that the denominator represents the 6 equally sized parts of the whole and the numerator represents the 1 shaded square being considered or counted.



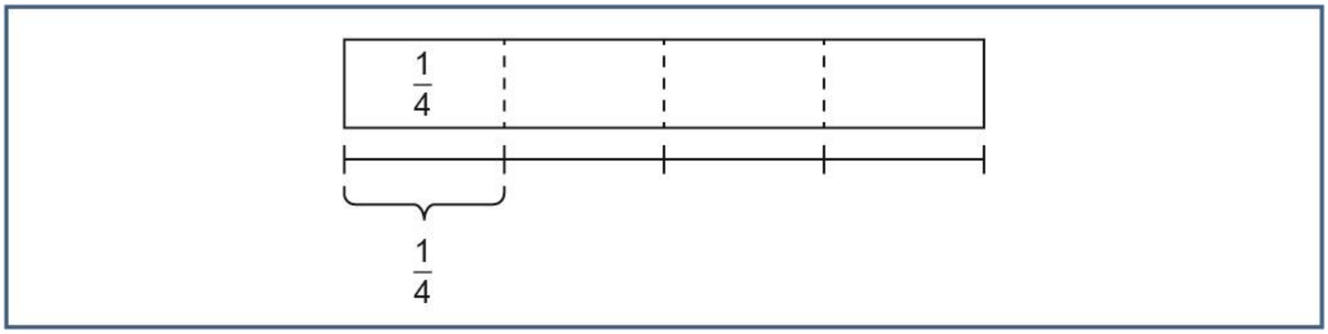
These descriptions can be extended to other visual models including length models, such as a number line, and set models, such as a group of marbles. Ask students to discuss similarities and differences they notice between the models.

- Ask students to identify or create non-examples of fraction models where parts are unequal in size. For example, the following models are divided into parts that are not equally sized. Students should identify and discuss why each part does not accurately represent $\frac{1}{4}$ of either model.



- Ask students to make connections between the composition of whole numbers and the composition of fractions. For example, just as the number 520 is composed of 52 tens, the fraction $\frac{3}{2}$ is composed of three one-halves. Explain that fractions have one value, similar to whole numbers, and that no matter the number of digits in the numerator or denominator, each fraction has a single value. Just as the numeral 520 has 3 digits and represents one number with a single value, the fraction $\frac{3}{2}$ has two digits and represents a number with a single value.

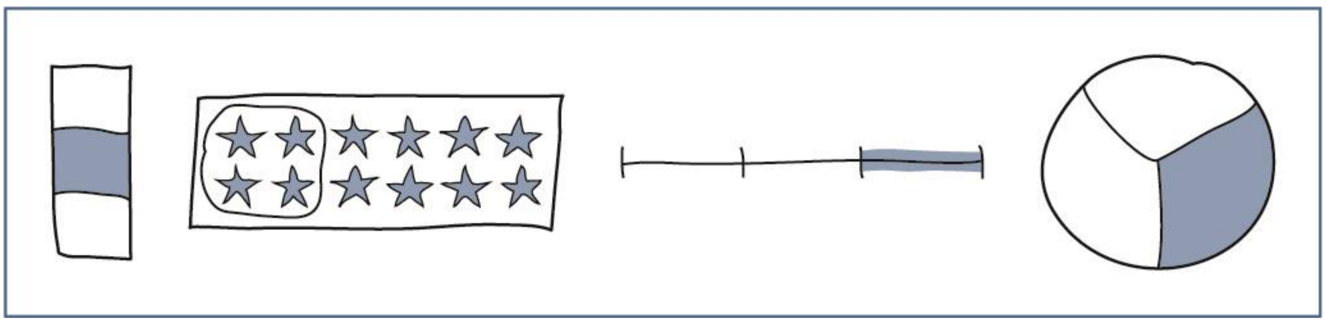
- Help students create area models using paper strips and connect these models to length models. For example, give students a paper strip to fold into 4 equally sized parts. Ask students to place the partitioned paper strip at the top of another piece of paper. Then, have students draw a line segment of the same length below the paper strip. Next, have students use the fold marks on the paper strip to mark the partitions on the line segment as shown in the following diagram.




Ask students to discuss how the area model connects to the length model. Students should see that just as one part of the paper strip is $\frac{1}{4}$ of the paper strip, the length between any two consecutive tick marks on the line segment is $\frac{1}{4}$ the length of the entire line segment.

**Unit fractions**

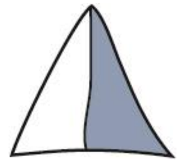
- Ask students to identify and describe unit fractions using visual representations and models. For example, given the fraction $\frac{1}{3}$, students create models that accurately depict $\frac{1}{3}$.



Ask students to trade with partners and identify the unit fraction given the models.

- Ask students to draw representations and write unit fractions in number and word form. For example, students write "$\frac{1}{2}$," draw a model that represents one-half, and write "one-half."

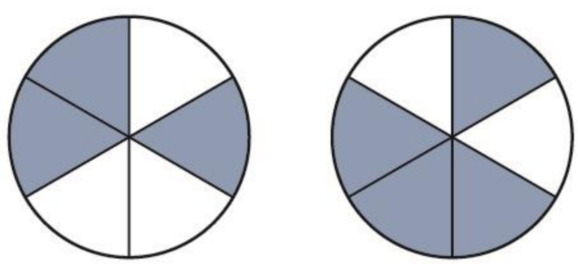
| Fraction | Model | Word Form |
|---|---|---|

| | | |
|---|---|---|
| $\frac{1}{2}$ |  | one-half |

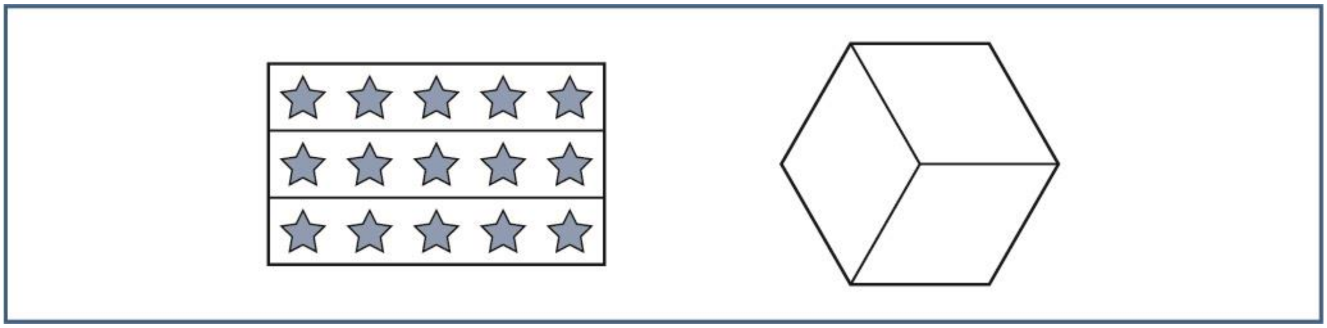
- Ask students to investigate how the sizes of unit fractions change as the denominators change. Have students create representations of unit fractions with denominators 2, 3, 4, 6, and 8. Students then discuss what they notice about how the size of the fraction changes as the denominator changes. Students should see that as the value of the number in the denominator becomes greater, the value of the unit fraction becomes less.

**Other fractions**

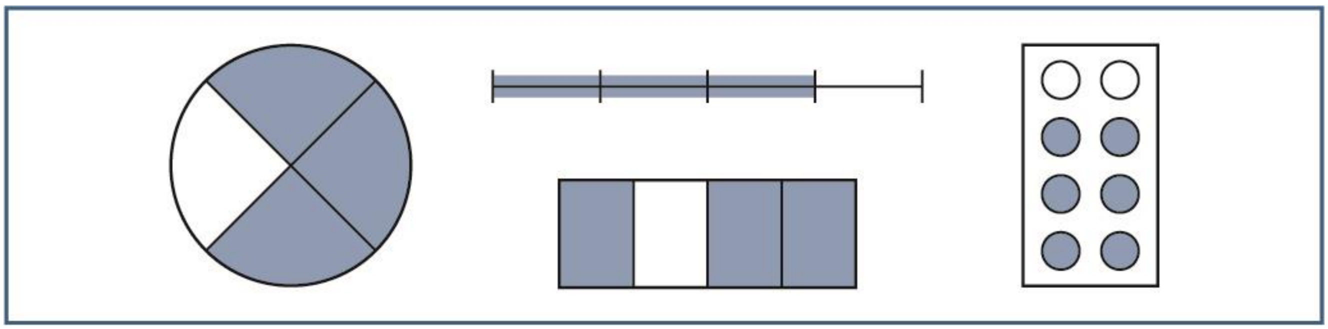
- When given a visual fraction model, ask students to describe a non-unit fraction in terms of the unit fraction. For example, show students the following diagram that represents the fraction $\frac{7}{6}$ and tell them that each circle represents one whole unit. Ask students to name the fraction represented by the diagram and to describe the fraction in terms of the unit fraction $\frac{1}{6}$. Have students explain how they determined their answer to a partner. Students should recognize that the diagram represents seven-sixths, because it is equivalent to seven one-sixths.



- Ask students to identify the given parts and the whole in a variety of fraction models. For example, in the following set model, students should identify the set of 15 stars as the whole and each group of 5 stars as $\frac{1}{3}$ of that whole. In the area model, students should identify the hexagon as the whole and each partition as $\frac{1}{3}$ of that whole.



- Ask students to create visual models to represent the same non-unit fraction in multiple ways. For example, students might represent $\frac{3}{4}$ in any of the four following ways.



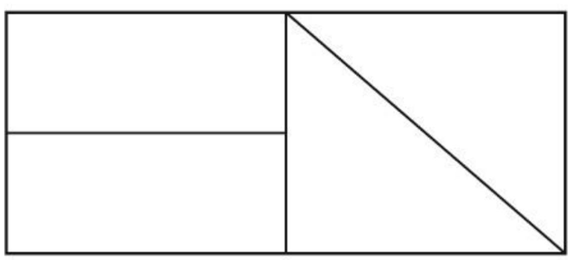
## ► Identifying and Addressing Common Misconceptions

- Some students believe that any object divided into parts, regardless of the size or area of those parts, is partitioned correctly into fractional parts. Show students some figures that have been correctly partitioned into fractional parts and some figures that have been divided into parts of unequal sizes. Ask students to explain which figures are correctly partitioned into fractional parts. Some students may think a figure that has been divided into two parts of unequal size has been divided into halves because there are two parts. Use the idea of fair shares (from division) to reinforce that the whole is divided into equal shares or parts. Emphasize that the number of equally sized parts that the whole is partitioned into determines the fractional amount. Use the vocabulary of fractional parts to describe each part in order to further emphasize that the parts are equally sized.
- When given a diagram of a figure that has been partitioned and partially shaded to represent a fraction, some students believe that just as the numerator is represented by the number of shaded parts, the denominator is represented by the number of unshaded parts. Show students the

following area model and ask them what fraction the model represents and why. Some students may think the model is shaded to represent $\frac{1}{3}$ because one part is shaded and three parts are unshaded.

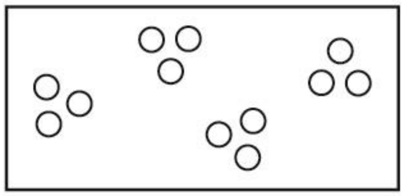
Use visual fraction models to emphasize that the denominator represents the number of equally sized parts that make one whole. Use models that represent fractions with the same denominator and different numerators. For example, show that models representing $\frac{1}{4}$ and $\frac{3}{4}$ are both partitioned into four equally sized parts.

- Some students believe that equally sized parts must be the same shape when a figure is correctly partitioned into fractional parts. Show students the following figure, which has been divided into equally sized parts that are different shapes, and ask the students to explain whether they think the figure is or is not partitioned correctly. Some students may think the figure is not partitioned correctly because the shapes of the parts are not the same.



Use grid and/or dot paper to focus on the equivalent areas of the parts. Ask students to use the grid or dot paper to divide shapes into fractional parts in multiple ways. Include a variety of shapes other than rectangles (e.g., triangles) in order to develop more flexible thinking about fractions.

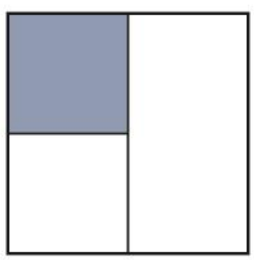
- When partitioning set models, some students believe the number of objects in a share indicates the fractional name of the share. Ask students to determine the fractional name of each share represented in the following model while explaining their thinking. Some students may think the 12 circles are partitioned into thirds because there are three circles in each share.



Use visual models to show that the number of equal shares indicates the fractional name, again focusing on the fact that each share is equal. Ask students to practice partitioning sets of objects that are not already grouped into equal shares and determining the fractional name of each share.

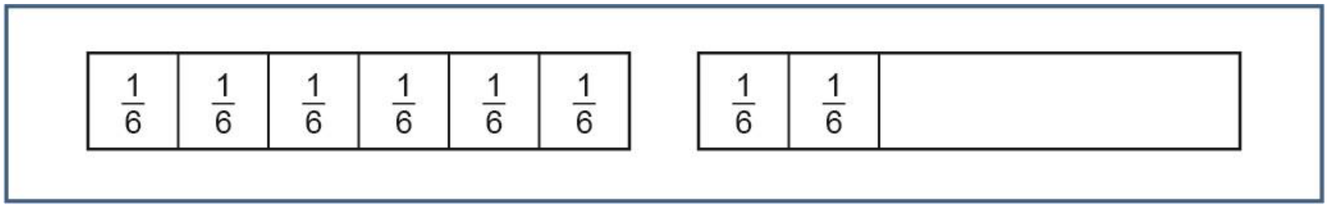
## ► Enrichment

- Ask students to determine the fraction that is represented by part of a shape that shows only some partition marks. For example, ask students to identify the fraction represented by the shaded square in the following model while explaining their thinking.



- Ask students to choose an appropriate model to represent fractional quantities and to explain why they chose that model. For example, students may choose to use area models such as fraction circles when partitioning a whole into equal parts, while they may choose fraction bars or rods when iterating a fractional part to create a whole.

- Ask students to informally explore equivalent fractions by describing the same fractional amounts using different-sized fractional parts. For example, given the fraction $\frac{8}{6}$, students create a representation using 8 unit fraction parts, each representing $\frac{1}{6}$, as shown in the following diagram.




Students can also compose $\frac{8}{6}$ using 4 unit fraction pieces, each representing $\frac{1}{3}$, or another combination of a variety of unit fractions.

- Ask students to explore how the sizes of the parts change as the size of the whole changes. Challenge students to think of scenarios where $\frac{1}{2}$ of one object is bigger or smaller than $\frac{1}{2}$ of a different object. Further, challenge students to think of scenarios where $\frac{1}{3}$ of one object may be bigger than $\frac{1}{2}$ of another object.

## ► Key Terms:

denominator, divide, equally sized parts, fraction, numerator, part, partition, unit fraction, whole

# A-F.1.1.2

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations – Fractions

Develop understanding of fractions as numbers.

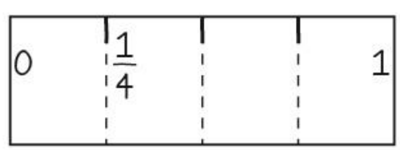
Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers.

## ▶ Eligible Content

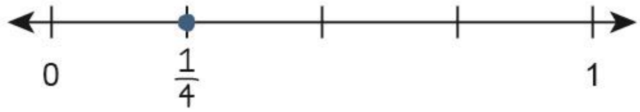
A-F.1.1.2 Represent fractions on a number line (limit the denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator; no simplification necessary).

## ▶ Instructional Supports

- Ask students to construct number lines and rulers in 2, 3, 4, 6, and 8 equally sized parts by folding paper strips. In the example shown, students can use the fold marks to help count the parts and determine that the denominator of the fractions represented by the parts is 4. Then, students should note that each section is $\frac{1}{4}$ of the strip. Ask students to mark and identify the unit fraction $\frac{1}{4}$ on the paper strip.




- Ask students to identify and label unit fractions with denominators of 2, 3, 4, 6, and 8 on number lines. Given the following number line, students determine that there are 4 parts between 0 and 1, so the denominator is 4. The first tick mark represents 1 part, so the fraction is $\frac{1}{4}$.




- Ask students to partition number lines by hand using approximate locations. Give students a number line with a minimum of two labeled tick marks to define the whole. Then ask students to draw the missing tick marks to partition the whole into equally sized parts. For example, students are given a number line labeled with 0, 1, $\frac{1}{8}$, and an unlabeled tick mark at $\frac{1}{2}$. Students then draw tick marks for the other $\frac{1}{8}$ parts.

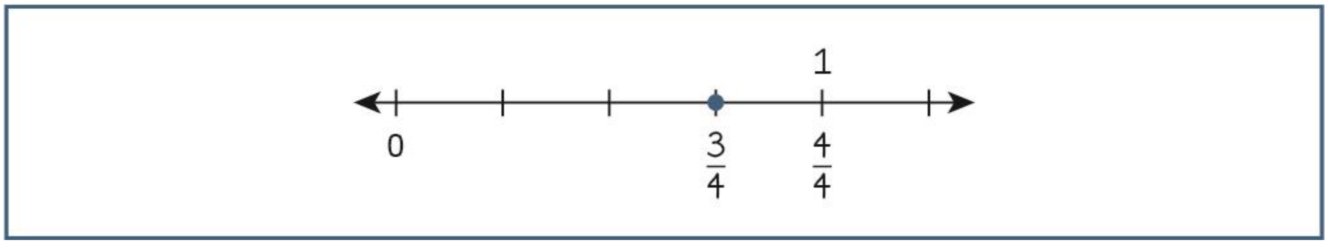

As students become more familiar with this, they should practice partitioning and labeling number lines without any guiding tick marks or labels.

- Given a number line that includes some labels and some tick marks, ask students to estimate the location of a given unit fraction. Students can further partition the number line as a strategy to best estimate the location of the unit fraction. For example, give students the following number line and ask them to estimate the location of $\frac{1}{6}$. Ask students to explain their thinking as they use the number line to determine their estimated location.

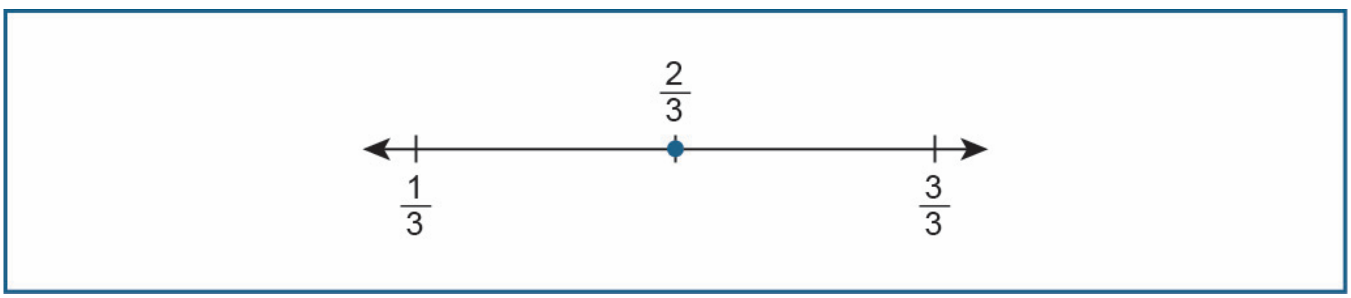

- Ask students to identify unit fractions on number lines that extend beyond 1 to help make sense of the whole. For example, give students the portion of a number line shown and ask them to plot a point at $\frac{1}{3}$ and explain their thinking.


**Determine the whole and partition the number line**

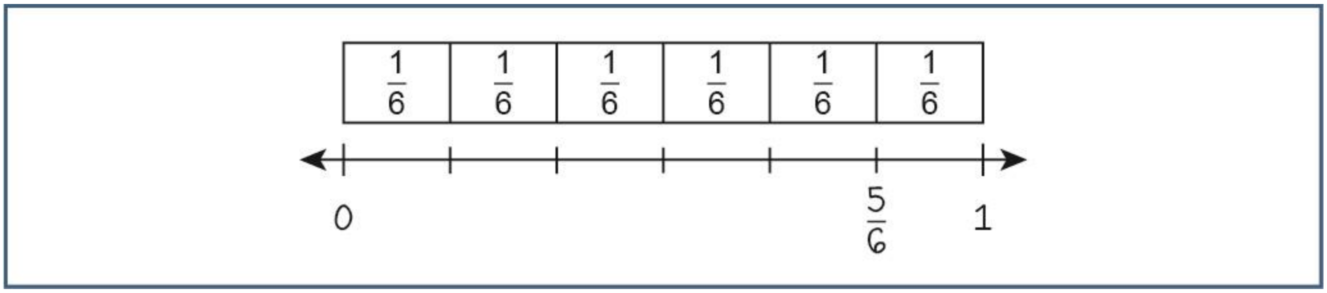
- Ask students to determine the locations of fractions on partitioned number lines. For example, given a number line with six tick marks and only the zero labeled, ask students to plot a point at the fraction $\frac{3}{4}$ by first designating the whole and then explaining that $\frac{3}{4}$ is located at the third tick mark when counting by fourths from 0.



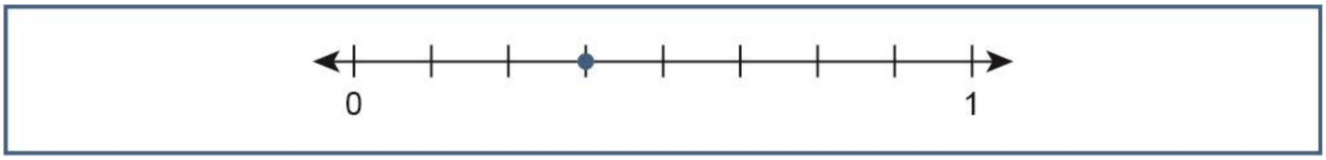

Also, ask students to determine the locations of fractions on number lines that do not begin at zero. For example, students plot a point at the fraction $\frac{2}{3}$ on the following number line and explain how they determined their answer.



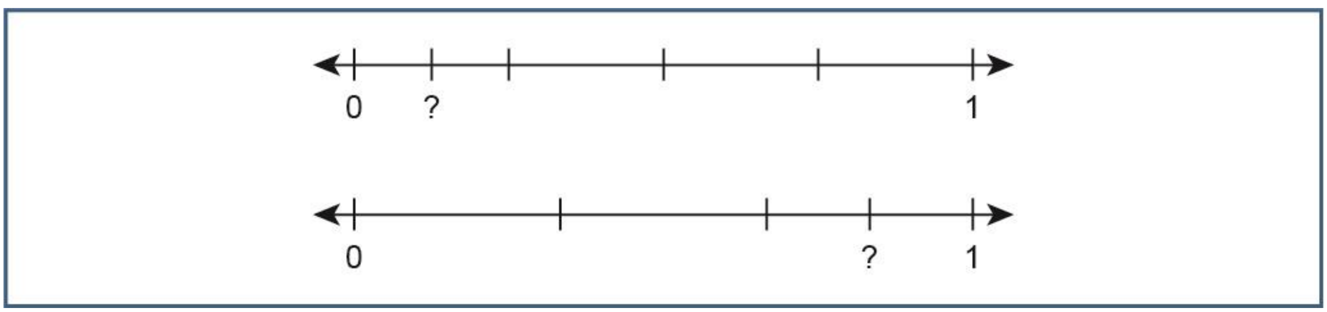

- Ask students to partition blank number lines and place fractions at the appropriate locations. For example, give students a blank number line and ask them to determine the location of the fraction $\frac{5}{6}$. Students may partition the number line into sixths by iterating a $\frac{1}{6}$-section 6 times, marking each $\frac{1}{6}$-section with a tick mark, then marking the end of the fifth section with a point labeled $\frac{5}{6}$. Students may also use tools such as grid paper to assist with even partitioning.




- Ask students to name the fraction represented by a point on a number line. For example, given the following point plotted on the number line, students may use their knowledge of fraction concepts such as denominator and numerator, identifying the whole, and counting the number of unit fractions to determine that the point on the number line represents $\frac{3}{8}$.



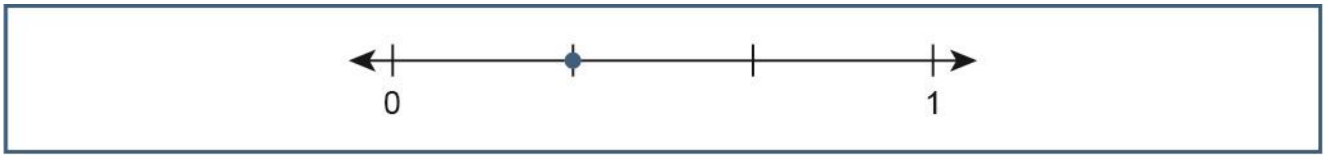

- Ask students to find values on number lines that show only some partition marks. For example, ask students to find the missing values on the following number lines while explaining their thinking. Students can measure and/or fold the paper to help find the values or show their thinking.



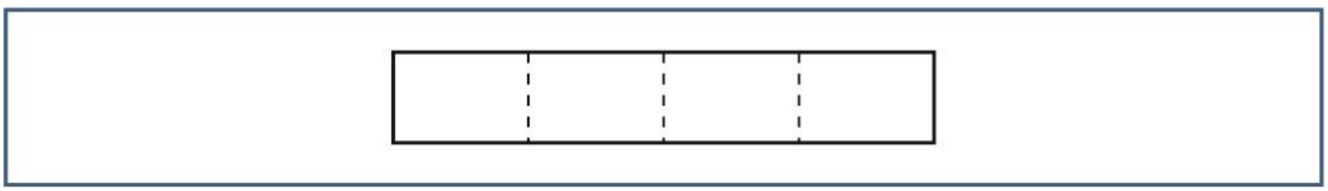

Folding the top line reveals that three of the marks divide the interval from 0 to 1 into fourths and the question mark further divides the interval into eighths. Folding the bottom line reveals that two of the marks divide the interval from 0 to 1 into thirds and the question mark further divides the interval into sixths. Therefore, the question marks indicate the fractions $\frac{1}{8}$ and $\frac{5}{6}$.

## ▶ Identifying and Addressing Common Misconceptions

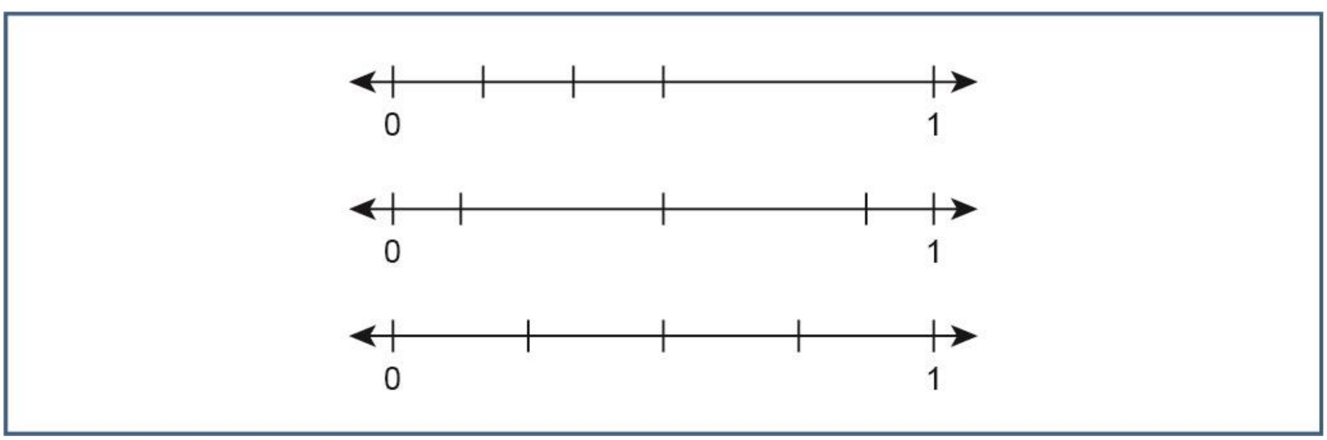
- When determining the location of a fraction on a number line, some students may identify an incorrect denominator because they count the number of tick marks instead of the number of parts. Show students the following number line and ask them to determine the value represented by the point. Some students may think the tick marks indicate fourths because there are four tick marks or halves because there are two interior tick marks.




Connect area models to the number line in order to emphasize that students should count the number of parts. Ask students to describe the differences between the previous number line and an area model of a rectangle that is partitioned into fourths.



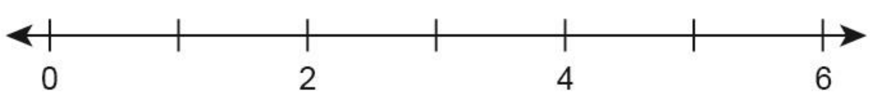
- Some students believe that on a number line, a whole that has been divided into parts, regardless of the sizes of the parts, is partitioned correctly into fractional parts. Show students the following number lines and ask them to explain which number line shows a whole that is correctly partitioned into fourths.



Some students may think each number line shows fourths because there are four parts in all of them. Connect the correct number line to an area model to emphasize that the size of each part is the same. Ask students to practice iterating unit fractions to create a whole to help recognize that each part is the same size.

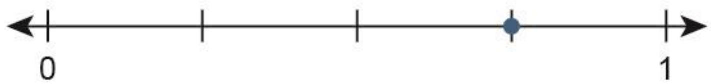
- Some students believe the numerator and denominator of a fraction represent two separate whole numbers. Show students a number line with unlabeled tick marks and ask them to explain how to plot the fraction $\frac{1}{2}$. Some students will label and plot a point at 1 and label and plot a point at 2 to represent $\frac{1}{2}$. Reinforce the fact that the fraction $\frac{1}{2}$ is one number with a single value. Review the fact that the denominator tells the number of equal parts while the numerator tells the number of parts counted. Ask students to practice partitioning wholes on number lines into equally sized parts and counting the unit parts using the correct fraction names "one-half," "one-third," "one-fourth," etc.
- Some students confuse fraction properties with whole number properties when iterating and counting unit fractions on a number line. Ask students to create a number line and show the locations of given unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Some students may think that counting up by unit fractions involves changing the denominator rather than the numerator and will label the tick marks in the following way.


Some students may also use the denominator of a unit fraction to count and label the parts, again confusing fraction counting and whole number counting. For example, some students may label their number lines in this way to show halves.



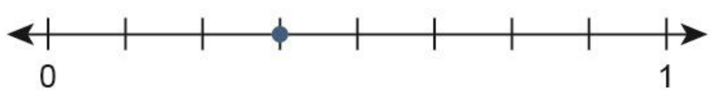
Connect the number line to an area model. Ask students to describe the whole in each model. Then ask students to describe the equally sized parts. Students should connect that one of two equal parts of a figure can be represented by one of two equal lengths of a whole that is partitioned on a number line.

- When determining the location of a fraction on a number line, some students count the number of tick marks instead of the number of parts. Show students the following number line and ask them to determine the value represented by the point.




Some students may think the point is plotted at $\frac{4}{5}$ because there are a total of 5 tick marks from 0 to 1 and the point lies on the fourth tick mark. Some students may think the point is plotted at $\frac{4}{4}$, recognizing that the whole is partitioned into four equally sized parts but still counting the tick mark at 0 to determine the fraction shown. Ask students to practice labeling tick marks on number lines, starting at 0. Students determine the denominator based on the number of equally sized parts and then count from 0, labeling each tick mark accordingly. Ask students to explain how the tick mark at 0 is different from the tick mark at the unit fraction.

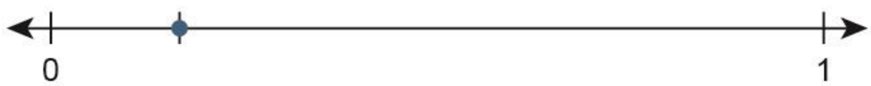
- When given a partitioned number line with a point plotted to represent a fraction, some students believe that the numerator is represented by the number of counted parts and the denominator is represented by the number of remaining parts of the whole. Show students the following number line and ask them what fraction is represented by the point and why. Some students may think the point represents $\frac{3}{5}$ because there are three counted parts to the left of the point and five remaining parts of the whole to the right of the point.




Connect area models to the number line to emphasize that the denominator represents the number of equally sized parts that make one whole. Use number lines that represent fractions with the same denominator and different numerators. For example, show that number lines with points plotted to represent $\frac{1}{6}$ and $\frac{5}{6}$ are both partitioned into six equally sized parts.

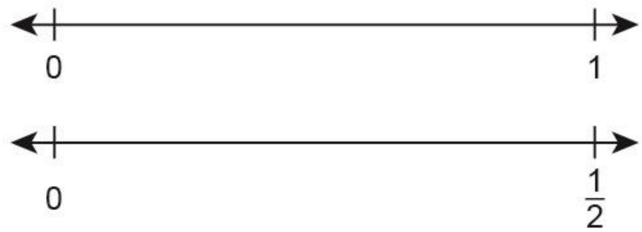
## ▶ Enrichment

- Ask students to determine the value of a unit fraction on a number line where the whole or denominator is not readily apparent. For example, give students the following number line.

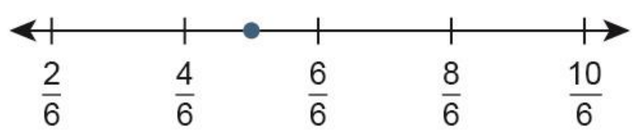



Students may need to add more partition marks to help determine that the value represented by the point is $\frac{1}{6}$.

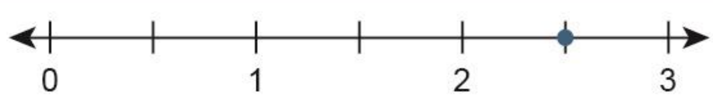

- Ask students to perform tasks that involve iterating unit fractions to find the location of 1 on a number line. For example, have students begin with a strip of paper and tell them it is $\frac{1}{3}$ of the whole. Students should begin at 0 and iterate the strip of paper to help locate and label 1 on the number line while explaining their thinking during the process.
- Help students explain the effect of different scales. Show students the following number lines and ask them to draw and label tick marks at $\frac{1}{4}$ on each number line while explaining their thinking. Ask students to explain why it appears that $\frac{1}{4}$ is located at a different place on each number line.




- Ask students to perform tasks that involve iterating lengths. For example, have students begin with a strip of paper and tell them it is $\frac{2}{3}$ of a whole. Ask them to make strips that represent $\frac{1}{3}$ of the whole, $1\frac{1}{3}$ of the whole, and 3 wholes. Students should partition the strip representing $\frac{2}{3}$ of the whole into two equally sized parts, each representing $\frac{1}{3}$ of the whole. Then, students should iterate the strip representing $\frac{1}{3}$ of the whole to find the other quantities. Repeat with a strip of paper representing $\frac{3}{4}$ of a whole and ask students to find representations of the whole such as $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, and $2\frac{1}{4}$ while explaining their thinking during the process.
- Ask students to name the fraction represented by a point on a number line that begins at a value other than 0 or 1 and has missing values or tick marks. For example, given the following point plotted on the number line, students should determine that the point represents $\frac{5}{6}$.




- Ask students to find different ways to decompose a fraction using a number line. This helps reinforce the concept that although fractions are numbers with a specific value, that value can be represented in different ways. For example, show students the point on the following number line and ask students to use parts to describe how the fraction is composed.



Students may say "five halves." Ask students for another way to say five halves. Some answers may include: two wholes and one more half, one whole and three more halves, three halves and two more halves, etc. Encourage students to describe the fraction in as many ways as possible. Answers may be recorded on a class chart.

## ► Key Terms:

denominator, divide, equal intervals, equal parts, fraction, number line, numerator, partition, tick mark, unit fraction, value, whole

# A-F.1.1.3

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations – Fractions

Develop understanding of fractions as numbers.

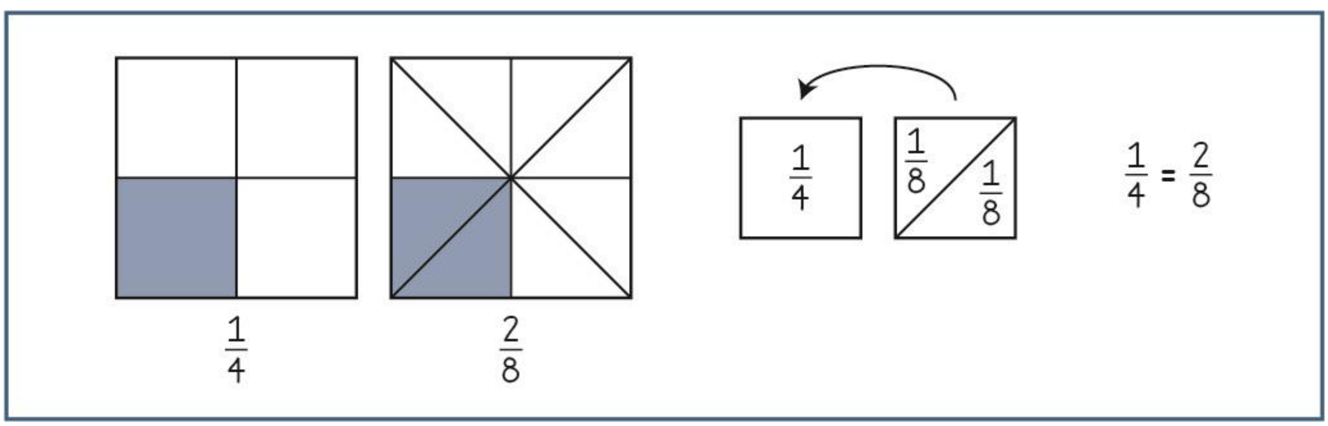
Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers.

## ▶ Eligible Content

A-F.1.1.3 Recognize and generate simple equivalent fractions (limit the denominators to 1, 2, 3, 4, 6, and 8; limit numerators to whole numbers less than the denominator).

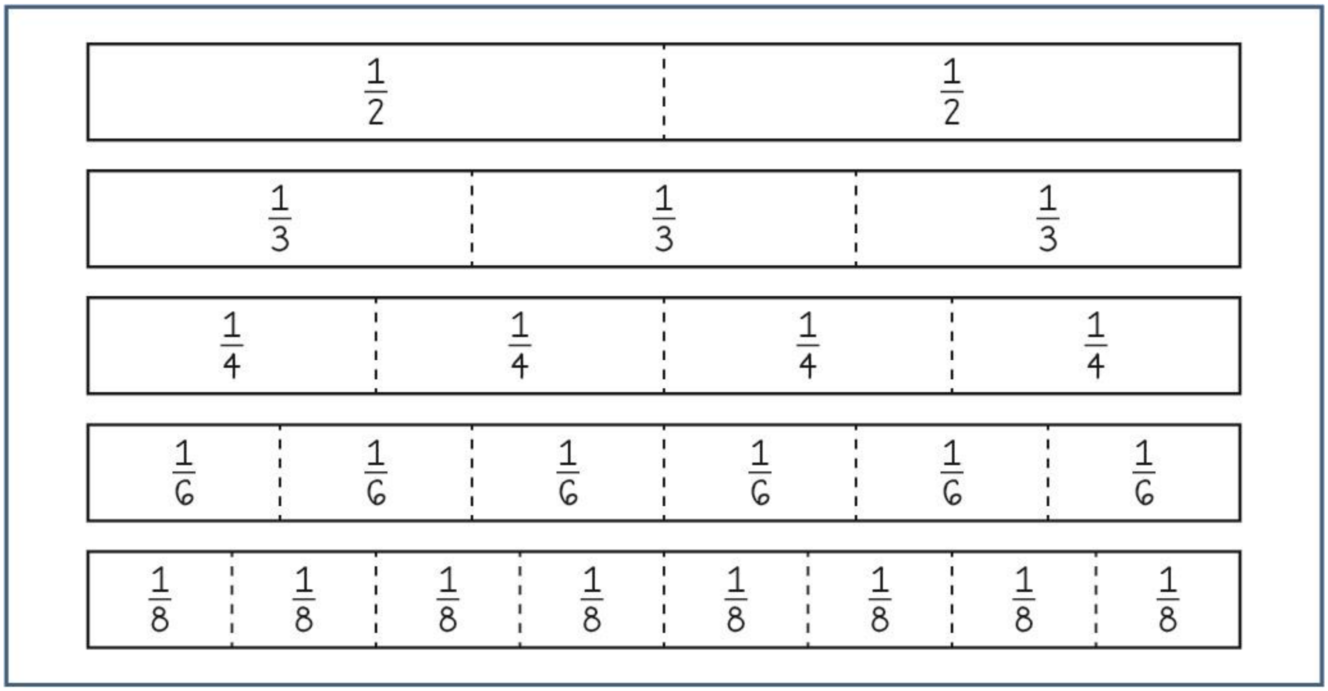
## ▶ Instructional Supports

- Ask students to use physical or visual models to explain why two fractions are equivalent. For example, ask students to show that $\frac{1}{4}$ is equivalent to $\frac{2}{8}$. Students can fold one square piece of paper into fourths and another square piece of paper into eighths. Ask students to cut out one $\frac{1}{4}$ piece and two $\frac{1}{8}$ pieces and use them to show that two $\frac{1}{8}$ pieces cover up the $\frac{1}{4}$ piece exactly.

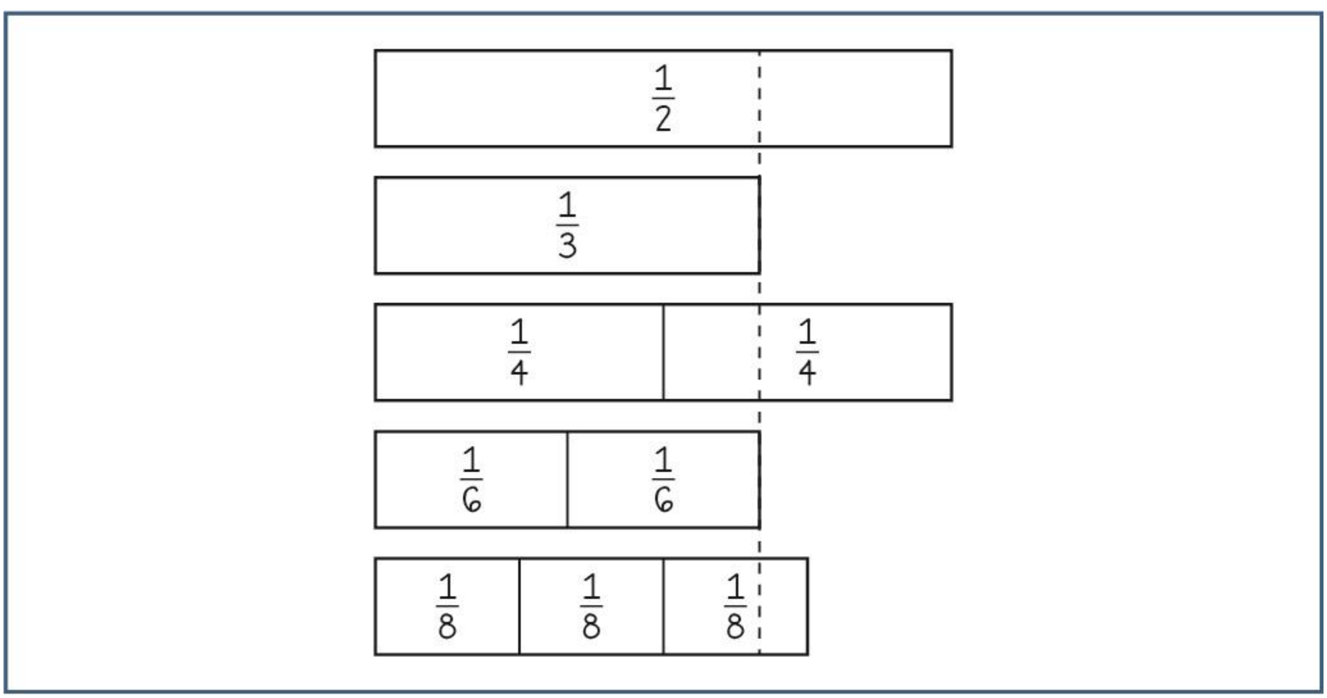



Ask students to discuss the connections between the models and how they show that the fractions are equivalent.

- Ask students to use manipulatives to generate equivalent fractions. For example, ask students to use fraction strips to find fractions that are equivalent to $\frac{1}{3}$. Ask students to create fraction strips by folding and labeling strips of paper of the same length, as shown.




Then, ask students to use the strips to identify fractions that are equivalent to $\frac{1}{3}$.



Students explain that $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent because the fraction strips representing them have the same length.

- Ask students to fill in a missing part of a given equation expressing fraction equivalence. For example, give students the following equation.
  - $\frac{4}{6} = \frac{?}{3}$

  Students should get an answer of 2. Ask students to show and explain how they determined the missing number.

- Ask students to explore multiplication patterns in equivalent fractions without formally developing the algorithm. For example, ask students to fold a piece of paper in half, unfold it, and shade one of the halves. Have students record $\frac{1}{2}$. Next, have students fold the shaded paper in half twice, unfold it, and record the equivalent fraction created. Ask students to repeat this process, folding the paper in half three times, and to record the equivalent fraction created. Provide students time to look at their papers and the fractions they recorded, and then have them share with a partner or within a small group. For students needing more support, teachers can model their thinking, making statements such as, "When I folded my fraction strip (showing $\frac{1}{2}$) in half again, the total number of parts (denominator) doubled to 4, and the number of shaded parts (numerator) doubled to 2. Even though the numerator and denominator both doubled, the area of $\frac{1}{2}$ of the paper is the same as the area of $\frac{2}{4}$ of the paper, so the fractions are equivalent." Help students make the connection between multiplication and fraction equivalence to encourage them to move beyond simply counting the parts.

## ► Identifying and Addressing Common Misconceptions

- Some students may think the size of a fraction is relative to the digits in it. Ask students whether $\frac{1}{3}$ or $\frac{2}{6}$ is greater. Some students may think $\frac{2}{6}$ is greater because 2 > 1 and 6 > 3. Help students draw fraction models with same-sized wholes partitioned into thirds and sixths and recognize that two $\frac{1}{6}$ parts are the same size as one $\frac{1}{3}$ part.

- Some students may confuse multiplication and addition when generating equivalent fractions. Ask students whether $\frac{2}{4}$ is equivalent to $\frac{4}{6}$. Some students may think the fractions are equivalent because 2 + 2 = 4 and 4 + 2 = 6. Have students create and compare fraction models for $\frac{2}{4}$ and $\frac{4}{6}$ using same-sized wholes to help them see that the fractions are different sizes. Also, ask students to create a fraction model for $\frac{4}{8}$ and help them understand that the sizes of the parts are half as large as in $\frac{2}{4}$ but the number of parts are multiplied by 2 rather than increased by 2.
- Some students may think that fractions that have the same number of unit parts, regardless of their size, must be equivalent. Ask students to use fraction pieces of $\frac{1}{3}$ and $\frac{1}{4}$ to create equivalent fractions. Some students may say that two $\frac{1}{3}$ pieces, or $\frac{2}{3}$, is equivalent to two $\frac{1}{4}$ pieces, or $\frac{2}{4}$, because there are two parts counted in each case. Have students place two $\frac{1}{3}$ pieces on top of two $\frac{1}{4}$ pieces to show that they are not the same size.

## ► Enrichment

- Ask students to informally identify fractions written in simplest terms. For example, give students a fraction and ask them to find as many equivalent fractions as they can. Then, ask them to identify the equivalent fraction that uses the smallest numbers.
- Give students a fraction and ask them whether it is possible to generate an equivalent fraction with a given denominator. For example, give students the fraction $\frac{5}{6}$ and ask if they can create a fraction that is equivalent and has a denominator of 8.
- Give students two fractions and ask them to find an equivalent fraction for each, sharing the same numerator or denominator. For example, give students the fractions $\frac{1}{3}$ and $\frac{3}{4}$ and ask them to find one fraction that is equivalent to $\frac{1}{3}$ and one fraction that is equivalent to $\frac{3}{4}$ with both fractions having the same denominator.
- Ask students to explain why, in any pair of equivalent fractions, the fraction with the greater numerator must also have the greater denominator.

## ► Key Terms:

equivalent fractions, fraction strip, model

# A-F.1.1.4

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations – Fractions

Develop understanding of fractions as numbers.

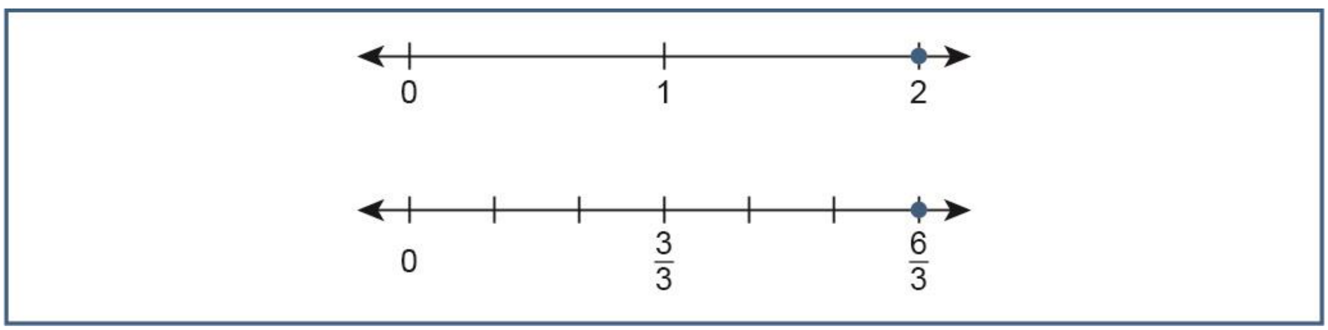
Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers.

## ▶ Eligible Content

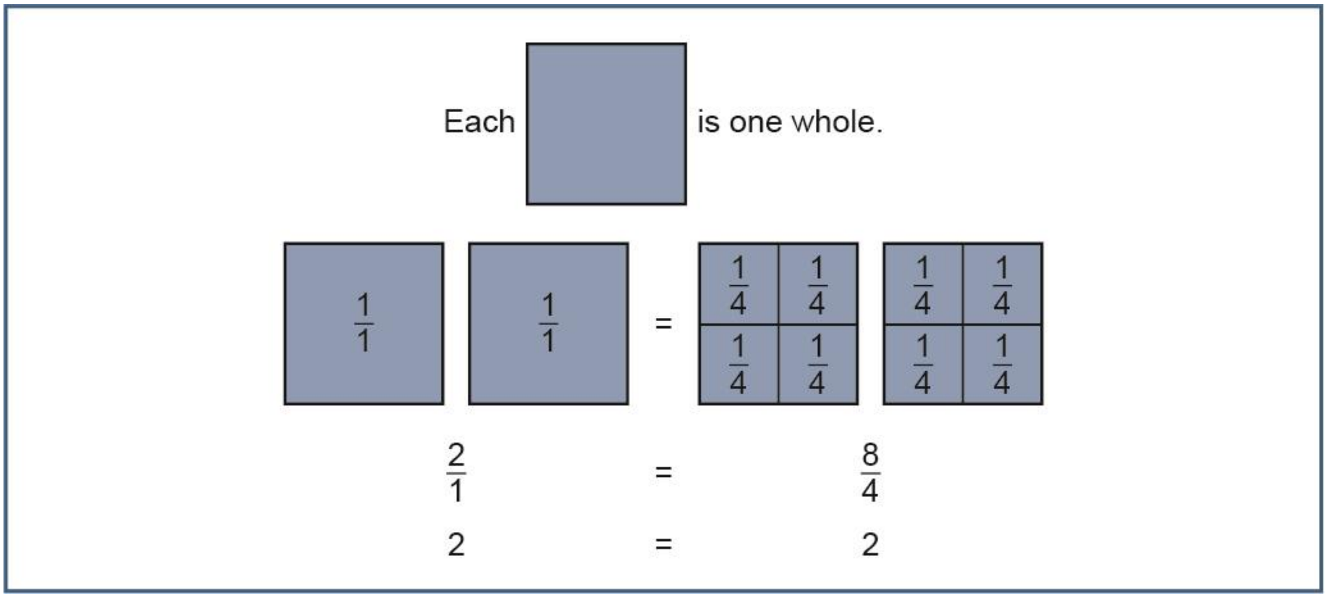
A-F.1.1.4 Express whole numbers as fractions, and/or generate fractions that are equivalent to whole numbers (limit the denominators to 1, 2, 3, 4, 6, and 8).

## ▶ Instructional Supports

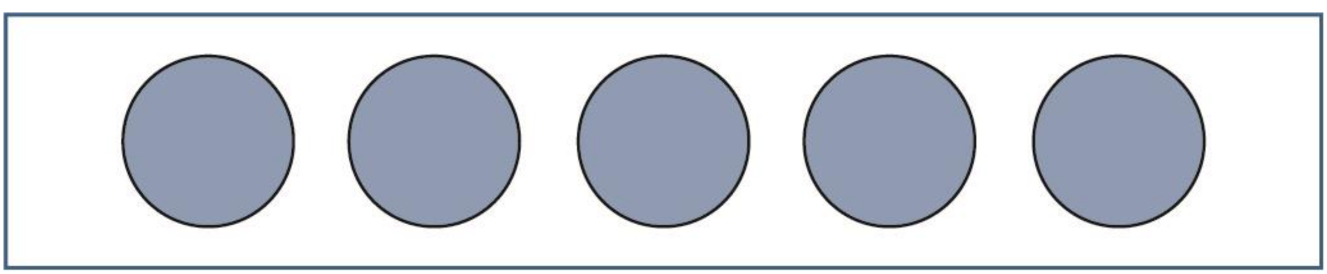
- Ask students to show equivalence between fractions and whole numbers using area models and number lines.
    - Ask students to explain why 2 and $\frac{6}{3}$ are equivalent. The following number lines show that 2 and $\frac{6}{3}$ are equivalent because they are at the same location on each number line.




    - Ask students to use partitions of squares to show that $\frac{2}{1}$, 2, and $\frac{8}{4}$ are all equivalent when one square is one whole.

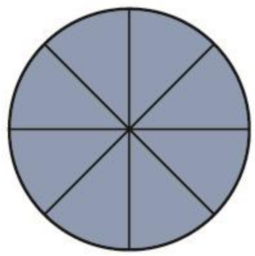



- Ask students to use the language of fractions when working with a denominator that has a value of 1. For example, the fractions $\frac{4}{1}$, $\frac{10}{1}$, and $\frac{9}{1}$ are read as "four-wholes," "ten-wholes," and "nine-wholes."
- Ask students to make conclusions, formal or informal, about the equivalence of fractions and whole numbers. After repeated experience, students may make connections between fractions and division. Students may state, "I noticed that $\frac{10}{2}$ is equal to 5, and $10 \div 2 = 5$. Ten-halves is the same as half of 10."
- Ask students to use models (e.g., fraction circles, fraction strips, or area models) to create fractions that are equal to a given whole number, with special emphasis on fractions that are equal to 1.
    - Ask students to create a fraction that is equal to 5.



When considering each of the 5 circles as a whole, each whole is divided into 1 equal part. In total there are 5 wholes. Therefore the 5 circles can be represented by the fraction $\frac{5}{1}$.

- Ask students to create a fraction that is equal to 1. In the diagram shown, a student creates a circle to show that a whole, which is 1, is equal to $\frac{8}{8}$, which is eight equal parts with eight parts shaded.



- Ask students to explain with words and models why a fraction is equal to 1 when the numerator and denominator are the same nonzero digit. For example, ask students to explain why $\frac{4}{4}$ is equal to 1. A student may state, "When slicing a pizza into four equal parts, there are four fourths that create one whole, so $\frac{4}{4}$ is the same as 1 whole."

## ► Identifying and Addressing Common Misconceptions

- If the value of the numerator and denominator in a fraction are the same nonzero number, a student may conclude that the fraction has a value of that nonzero number rather than a value of 1. Ask students the value of $\frac{3}{3}$. Some students may give an answer of 3. Ask students to create an area model representing 1 whole and partition the model into equal parts so that the collection of the equal parts, shown in both numerator and denominator, have a value of 1.
- Some students may multiply when converting a fraction into a whole number. Ask students to write the whole number that is equivalent to $\frac{8}{2}$. Some students may give an answer of 16. Ask the student to create a model showing the collection of 8 halves to determine the whole number equivalent.

- Some students may think that whole numbers cannot be fractions. Ask students to write 3 as a fraction. Some students may think that this is not possible. Ask students to draw 3 circles, each divided into two equal parts. Ask how many parts are in each circle and how many parts there are in total, then help students explain why this shows that $3 = \frac{6}{2}$. Then ask students to draw 3 circles that are not divided into parts and ask the same two questions in order to help them conclude that $3 = \frac{3}{1}$. Help students generalize that all whole numbers can be written as fractions.

## ► Enrichment

- Ask students to create a model for a fraction that is not a whole number and that has a value greater than 1. Then, ask students to decompose the fraction into parts that can be represented by a mixed number. For example, give students the fraction $\frac{7}{2}$ and ask them to create an area model using four circles partitioned into halves with 7 halves shaded. The students can then interpret the $\frac{7}{2}$ model as a $\frac{1}{2}$ portion and a $\frac{6}{2}$ portion, which is equivalent to $3\frac{1}{2}$.
- Ask students to create multiple models representing the same whole number in fraction form and use the models to create equivalent fractions. For example, two circles may be partitioned into 4 equal parts each, representing $\frac{8}{4}$, and those two circles can also be partitioned into 6 equal parts each, representing $\frac{12}{6}$. The students conclude that $\frac{8}{4}$ and $\frac{12}{6}$ have the same value.
- Ask students to observe and explain patterns on a number line with fractions that are whole numbers. For example, ask students to partition a number line to represent thirds and denote each whole number equivalent, such as $\frac{3}{3}$, $\frac{6}{3}$, and $\frac{9}{3}$. Then ask them to partition the same number line into sixths, denoting each whole number equivalent, such as $\frac{6}{6}$, $\frac{12}{6}$, and $\frac{18}{6}$. Students may then say, “When we made sixths on the number line, we were partitioning each third into halves, and the result was that each numerator in the pattern doubled.”

## ► Key Terms:

denominator, equivalent, fractions, nonzero, number line, numerator, region, whole

# A-F.1.1.5

## ► Standard

Numbers and Operations – Fractions

Develop understanding of fractions as numbers.

Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers.

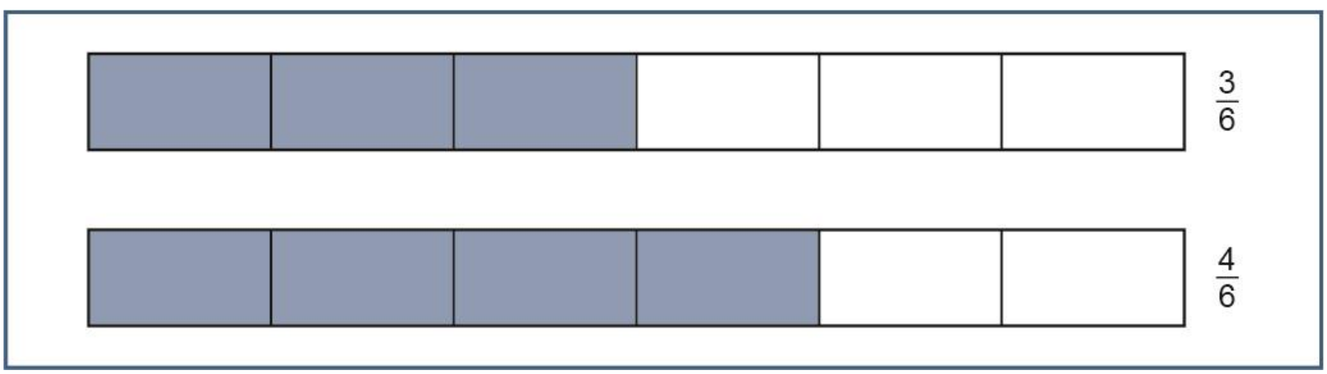
## ► Eligible Content

A-F.1.1.5 Compare two fractions with the same denominator (limit the denominators to 1, 2, 3, 4, 6, and 8), using the symbols >, =, or <, and/or justify the conclusions.

## ► Instructional Supports

**Same denominator**

- Ask students to compare fractions with the same denominator but different numerators using models like number lines, area models, and fraction circles. Using the bar model shown, students conclude that $\frac{3}{6}$ is less than $\frac{4}{6}$ because there is one less $\frac{1}{6}$ piece in $\frac{3}{6}$.




- Ask students to reason and deduce, informally, when comparing fractions that have the same denominator. After several experiences comparing fractions with like denominators, students should be able to reason that the fraction with the larger number in the numerator has a greater value because it represents a greater collection of parts that are the same size.

- Ask students to use the symbols <, >, and = in fraction comparisons. For example, ask students to generate a fraction to complete the sentence $\square > \frac{1}{4}$. Students can use visual models to determine that $\frac{1}{3}$ and $\frac{1}{2}$ are two possible solutions.

## ► Identifying and Addressing Common Misconceptions

- Some students may mix up the meaning of the inequality symbols. Ask students to compare $\frac{4}{6} \square \frac{5}{6}$ by writing the correct inequality symbol in the box and explain their choice. Some students may choose to use the ">" symbol, which is incorrect. Remind students that the "pointed side" of the symbol "points" to the lesser number. Another way to remember is to notice that the symbol "<" resembles the letter "L" for "less than." The students may also use the index finger and thumb of their left hand to form an "L" to remind them of the less than symbol.
- Some students may not consider context when comparing fractions. Ask students whether there is more water in half a bottle or half a liter. Some students may not consider that the answer cannot be determined without knowing the actual size of the bottle. Ask students to draw half an apple and half a watermelon and then compare the sizes of those parts in their drawing. Help the students conclude that equating halves is not always accurate because the wholes may represent very different amounts or sizes.

## ► Enrichment

- Ask students to devise a context in which comparing two equal fractions of two different things is a valid comparison. For example, $\frac{2}{3}$ cup of flour has the same volume as $\frac{2}{3}$ cup of milk.
- Ask students to roll two number cubes, one to represent the numerator of a fraction and the other to represent the denominator of the fraction. After rolling and generating the random fraction, ask students to predict whether rerolling for the numerator or rerolling for the denominator is more likely to result in a greater fraction. Help students check their guess by rerolling for the numerator 10 times and counting the number of times that the reroll resulted in a greater fraction.

Then, reroll for the denominator 10 times and count the number of times that the reroll resulted in a greater fraction.

- Ask students to compare the value of a fraction to the value of that fraction when the numerator and denominator are switched. Students may use physical models or diagrams to justify their conclusions.

## ▶ Key Terms:

compare, denominator, equal (=), fraction models, fractions, greater than (>), less than (<), numerator, part, whole

# A-T.1.1.1

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Apply place-value strategies to solve problems.

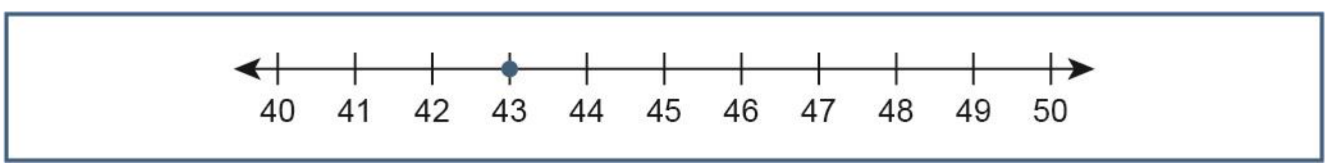
## ▶ Eligible Content

A-T.1.1.1 Round two- and three-digit whole numbers to the nearest ten or hundred, respectively.

## ▶ Instructional Supports

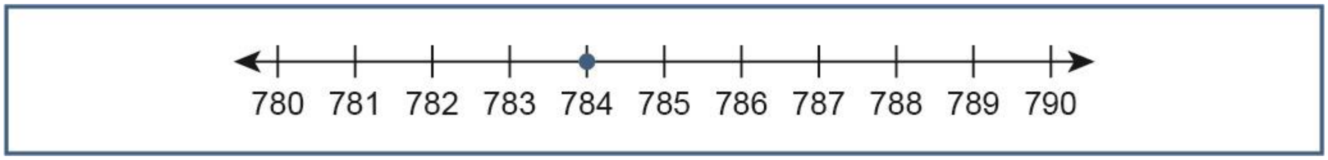
**Nearest ten**

- Ask students to recognize tens and identify the tens that are closest to a given number. For example, ask students which tens are closest to 237. While most students readily think of numbers like 60, 70, and 80 as tens, some students may not necessarily think of 230 and 240 as tens.
- Using a number line, ask students to round a two-digit number to the nearest 10. For example, give students the number 43. Students should focus on the part of the number line containing 43 and the multiples of 10 immediately before and immediately after 43. The part of the number line from 40 to 50 is shown with a point plotted at 43.




The distance from 43 to 40 is 3 units, and the distance from 43 to 50 is 7 units. The number 43 is closer to 40 than it is to 50. Therefore, 43 rounded to the nearest 10 is 40.

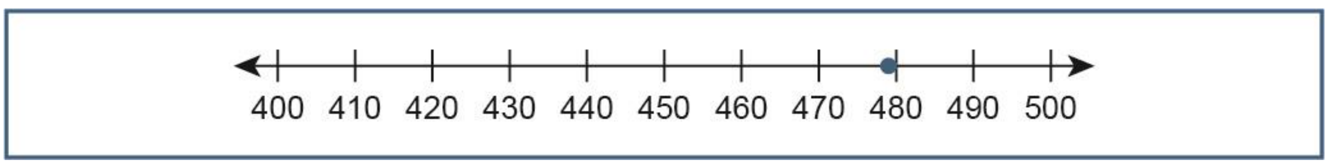
- Using a number line, ask students to round a three-digit number to the nearest 10. For example, give students a number such as 784. Have students focus on the part of the number line containing 784 and the multiples of 10 immediately before and immediately after 784. The part of the number line from 780 to 790 is shown with a point plotted at 784.




The distance from 784 to 780 is 4, and the distance from 784 to 790 is 6. Because 780 is closer to 784 than 790 is, when 784 is rounded to the nearest 10, the value is 780.

**Nearest hundred**

- Using a number line, ask students to round a three-digit number to the nearest 100. For example, give students a number such as 479. Have students focus on the part of the number line containing 479 and the multiples of 100 immediately before and immediately after 479. The numbers 400 and 500 are the multiples of 100 immediately before and immediately after 479. The part of the number line from 400 to 500 is shown with a point plotted at 479.




The distance from 479 to 400 is 79, which is slightly less than 8 tens (80 units), and the distance from 479 to 500 is 21, which is slightly more than 2 tens (20 units). The number 479 is closer to 500 than it is to 400. When 479 is rounded to the nearest 100, the value is 500.

- Ask students to round numbers to the nearest 100 when the numbers contain a 5 in the tens place. For example, have students round 450 to the nearest 100. The closest multiples of 100 are 400 and 500. Observe that both 400 and 500 are 50 units away from 450. When both of the closest hundreds are equally close, the convention is to round up.

**General rounding**

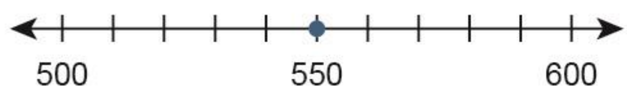
- Ask students to explain, in general, when to round up and when to round down. The general rule is for students to round the digit in question down when the digit in the next place value to the right is 0, 1, 2, 3, or 4. Students round the digit in question up when the digit in the next place value to the right is 5, 6, 7, 8, or 9.

## ▶ Identifying and Addressing Common Misconceptions

- Students may think that when a number is rounded to the nearest 10, either of the nearest consecutive multiples of 10 will work, instead of being a clear choice of one or the other. Ask students to explain how to round each of the numbers 46 and 43 to the nearest ten. Some students may think that both numbers can be rounded to either 40 or 50 because those tens are close to the given numbers. Ask students to list other tens that are close to the given numbers and then compare them to determine which ten is closest to 43 and which ten is closest to 46. Emphasize that only the closest ten is correct for each number.
- Students may be confused as to which digit determines whether a number gets rounded up or down. Ask students to explain how to round 38 to the nearest ten. Students may think that 38 would be rounded to 30 because 3 is less than 5. Rather than trying to memorize a rule, help students focus on the idea of rounding to the **nearest** ten. Ask students to consider whether 30 or 40 is closer to 38 on a number line. Discuss how the number in the ones place determines whether 38 is closer to 30 or 40.
- Students may think that when a two-digit number is rounded to the nearest ten, the digit in the ones place can simply be made a zero. Ask students to explain how to round 26 to the nearest ten. They may answer with 20, making the 6 into a zero. Students may have this misconception if they have done rounding problems where the number almost exclusively rounds down. Remind students that they are not merely changing 26 into a ten but into the **nearest** ten. If necessary, ask students to list out several tens and then determine which of the tens is closest to 26.
- Students may attempt to round a number by only changing the specified place value. Ask students to round 58 to the nearest ten. Some students may round a number such as 58 by rounding the 5 up to a 6 without changing the value of the digit in the ones place, resulting in a rounded value of 68. Students may be so focused on changing the tens digit that they forget to change the ones digit. Ask them to list several examples of tens, then ask them if their value is a multiple of ten.

## ► Enrichment

- Help students explore the convention that the presence of a 5 in the place value one place to the right indicates that a number should be rounded up. Ask students to discuss whether it is more reasonable to round 550 down to 500 or to round it up to 600. Because both 500 and 600 are 50 units from 550, neither one is closer. Therefore, either rounding up to 600 or down to 500 is reasonable.




  Then ask students to consider how to round 551, 552, . . ., 559 to the nearest hundred. All these numbers are closer to 600 than to 500. Because all other numbers with a 5 in the tens place round to 600, it makes sense to use the convention that 550 also rounds up to 600, even though 500 is just as close.

- Ask students to work backward to find all whole numbers that round to a given target value. For example, ask students to find the interval of whole numbers that are rounded to 60 when rounding to the nearest ten, the interval of whole numbers that are rounded to 270 when rounding to the nearest ten, or the interval of whole numbers that are rounded to 500 when rounding to the nearest hundred.

- Ask students to round numbers for which rounding creates additional digits. These special cases occur for numbers less than but near powers of 10. For example, ask students to round 995 to the nearest 10.

## ► Key Terms:

multi-digit, nearest 10, nearest 100, place value, round

# A-T.1.1.2

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

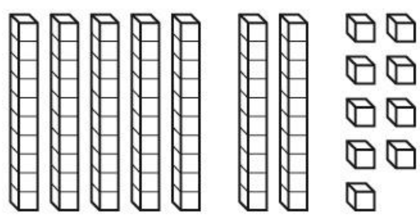
Apply place-value strategies to solve problems.

## ▶ Eligible Content

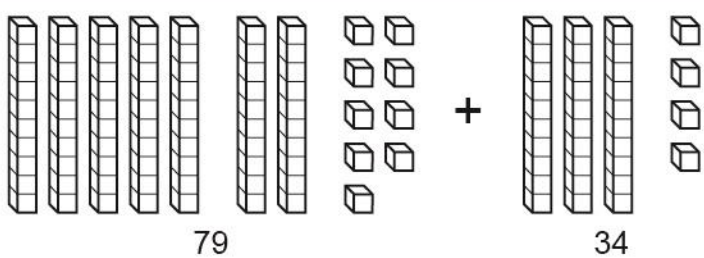
A-T.1.1.2 Add two- and three-digit whole numbers (limit sums from 100 through 1,000), and/or subtract two-and three-digit numbers from three-digit whole numbers.

## ▶ Instructional Supports

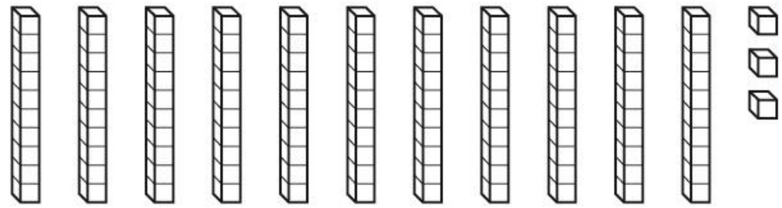
- Ask students to use base-ten blocks to show addition. For example, give students the sum 79 + 34. The first addend, 79, is represented with base-ten blocks as shown.



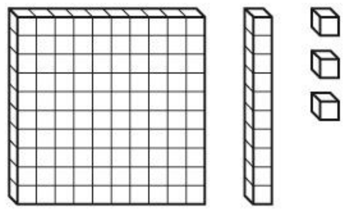
Students then add 3 ten-blocks (rods) and 4 one-blocks (units) to the figure to represent adding 34.



Since there are thirteen one-blocks, students should replace 10 of the one-blocks with a single ten-block. After the replacement, three one-blocks remain. Students should then count the base-ten blocks to determine the sum.

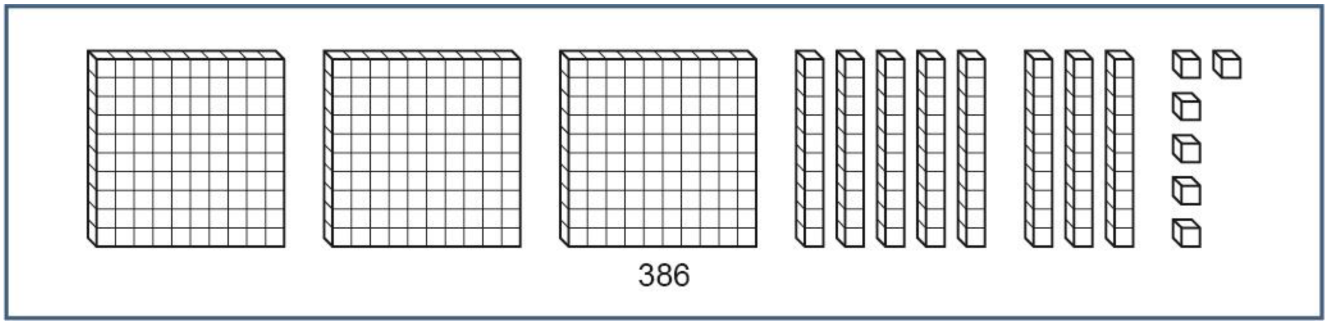


Students may note that 10 ten-blocks can be replaced with 1 hundred-block (flat).

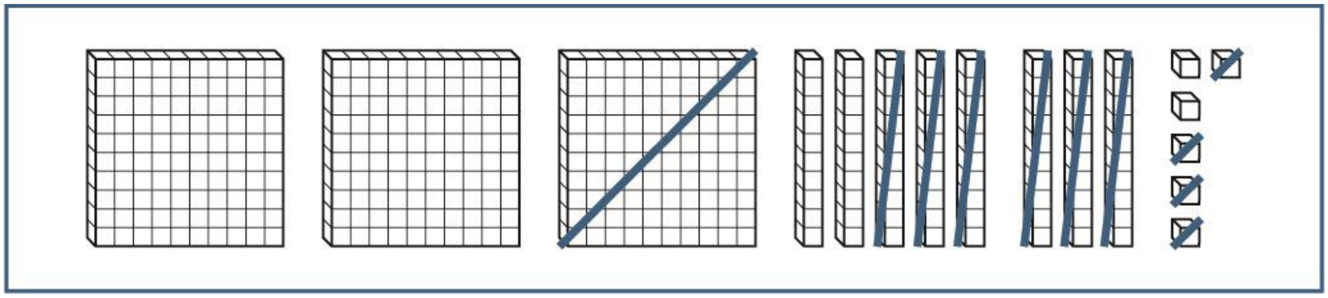


The sum of 79 and 34 is 113 since the students will have 1 hundred-block, 1 ten-block, and 3 one-blocks.

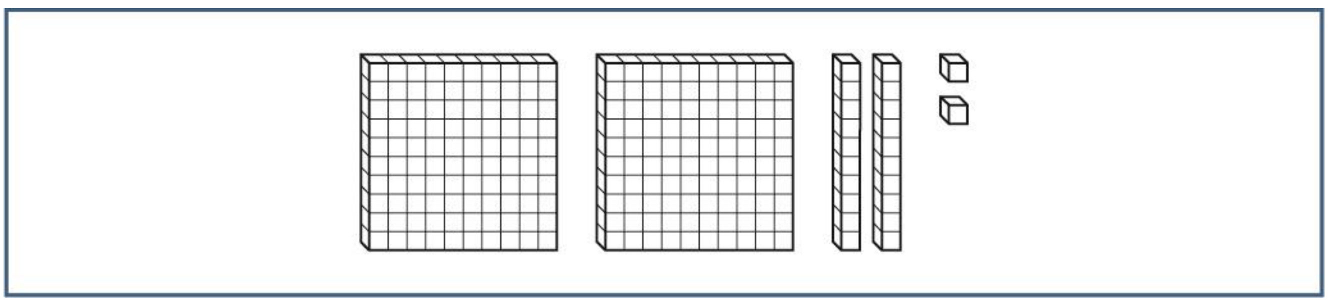
- Ask students to add or subtract two numbers by rewriting one of the numbers.
  - Calculate 649 + 47 by changing the first addend of 649 to 650 – 1. Then, rewrite the addition problem as 650 + 47 – 1. Since 47 – 1 = 46, the values of 650 + 46 and 649 + 47 are both 696. Another example is adding 402 and 86 by changing 402 to 400 + 2 and rewriting the addition problem as 400 + 86 + 2. The value of 402 + 86 is 488 because 86 + 2 = 88.
  - Calculate 847 – 99 by changing 99 to 100 and then rewriting the subtraction problem as 847 – 100 + 1. The value of 847 – 99 is 748, since 847 – 100 = 747 and 747 + 1 = 748.
- Ask students to use base-ten blocks to model subtraction. For example, give students the problem 386 – 164, with 386 represented with base-ten blocks as shown.




Students should remove 1 hundred-block (flat), 6 ten-blocks (rods), and 4 one-blocks (units) to represent 164 as shown by the crossed-off figures.



Students count the remaining base-ten blocks. With 2 hundreds, 2 tens, and 2 ones remaining, students conclude that the value of 386 – 164 is 222.



- Ask students to rewrite a subtraction problem as an addition problem with an unknown addend. For example, the subtraction problem 863 – 275 can be rewritten as 275 + ___ = 863.

## ► Identifying and Addressing Common Misconceptions

- Students may think that there is only one correct method to add or subtract. Ask students to calculate 653 – 97 as many ways as possible. If students get stuck, suggest strategies such as thinking of subtracting 97 as the same as subtracting 100 and adding 3, or adding 3 to each number to make the subtraction problem 656 – 100.

- Students may think that there is only one way to compose or decompose a number. Ask students to explain how 53 tens and 8 ones relate to the number 538. A number such as 538 may be recognized as 5 hundreds, 3 tens, and 8 ones, but students may not immediately recognize that 538 can also be decomposed as 53 tens and 8 ones. Help students illustrate this with base-ten blocks.
- Students may rewrite numbers incorrectly when regrouping in a subtraction problem while using the standard algorithm. Ask students to calculate 1000 – 239, showing all work. Students may correctly regroup a value from one digit and give the value to the digit to the right, but they may continue to do so in an automatic manner, even doing so in the ones place, making the 0 a 10 and then a 9. When dealing with whole numbers, there is no value past the ones place, so the 0 in the ones place changed to a 10 must stay a 10. Check student understanding by asking students to explain the reason for each change from a 0 to a 10 and a 10 to a 9 when regrouping. Reinforce these ideas with the use of manipulatives for a more concrete understanding.
- Students may regroup incorrectly when having to regroup across a 0 while using the standard algorithm. Ask students to calculate 803 – 457, showing all work. Some students will regroup from the hundreds place by regrouping 803 into 7 hundreds and 13 ones because they skip the tens place. Have students represent 803 using base 10 blocks. Show them how regrouping a hundred-block creates 10 ten-blocks, and only one of the ten-blocks will have to be regrouped into one-blocks, leaving 7 hundred-blocks, 9 ten-blocks, and 13 one-blocks.

## ► Enrichment

- Ask students to use multiple methods for addition or subtraction. Ask students to solve 86 + 27 using as many methods as possible. Ask students to explain similarities, differences, advantages, and disadvantages of each of the methods.
- Challenge students to find all numbers that can be subtracted from 1000 that result in a 5 in the tens place. Ask students to explain how they found their answers.
- Ask students to investigate which digits of a sum can be calculated without looking at any other place values. For example, when adding 283 and 579, the ones digit can be determined by adding

only 3 and 9, but the hundreds digit cannot be determined by adding 2 and 5. Discuss which digits in sums can be determined in this manner.

## ► Key Terms:

algorithm, base-ten blocks, difference, multi-digit, place value, properties of operations, sum

# A-T.1.1.3

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Apply place-value strategies to solve problems.

## ▶ Eligible Content

A-T.1.1.3 Multiply one-digit whole numbers by two-digit multiples of 10 (from 10 through 90).

## ▶ Instructional Supports

- Extend multiplication of one-digit numbers to multiplication of a one-digit number by a multiple of 10 using an understanding of place value and properties of operations. For example, ask students to identify the number of tens in the product of a one-digit number and a multiple of 10. In the example $4 \times 70$, students recognize that $4 \times 70$ is equivalent to $4 \times 7$ tens. Since $4 \times 7 = 28$, the product of $4 \times 70$ is equivalent to 28 tens, or 280.
- Ask students to rewrite multiplication problems containing a multiple of 10 using properties of multiplication. For example, give students the problem $7 \times 80$.

$$\begin{aligned} & 7 \times 80 & \\ &= 7 \times (8 \times 10) & \leftarrow 8 \times 10 = 80 \\ &= (7 \times 8) \times 10 & \leftarrow \text{Associative Property} \\ &= 56 \times 10 & \leftarrow 7 \times 8 = 56 \\ &= 560 & \end{aligned}$$

The problem can be rewritten as $7 \times (8 \times 10)$, since $8 \times 10 = 80$. This expression can be rewritten as $(7 \times 8) \times 10$ using the associative property. Since $7 \times 8 = 56$, the original expression is equivalent to $56 \times 10$, which is equal to 560.

- Ask students to find the product of a one-digit number and a multiple of 10 by finding the product of two one-digit numbers and then using place value reasoning. For example, give students the problem 4 × 90. The problem can be rewritten as (4 × 9) × 10. The first product is 4 × 9 = 36, which consists of 3 tens and 6 ones. The 3 tens and 6 ones each then get multiplied by 10, changing the place values from tens to hundreds and from ones to tens. The final product will have 3 hundreds, 6 tens, and no ones, which is equal to 360.

## ► Identifying and Addressing Common Misconceptions

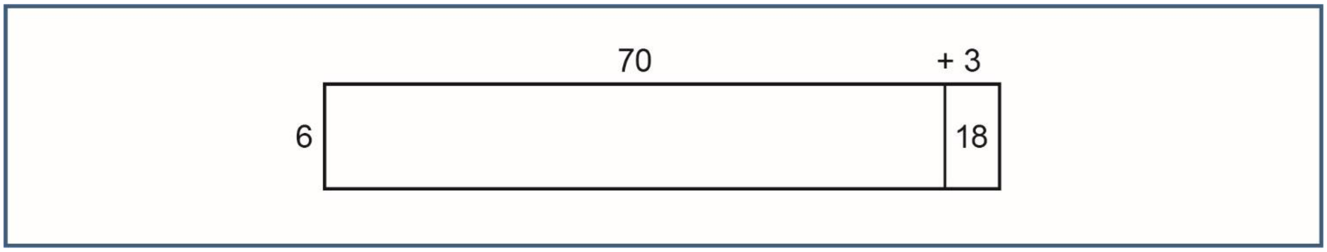
- Students may use the method of multiplying the tens digit in a multiple of ten by the one-digit factor and then forget to write the zero after the initial product to get the final product. Ask students to multiply 4 by 60. Remind students that when using this method, this becomes a two-step problem. Have students check their work to ensure that they have completed both steps, first multiplying of the front digits (4 × 6) and then multiplying that result by 10 for the final product of 240.
- Students may think that the product of a one-digit number and a two-digit number is always a three-digit number. Ask students to find the product of 4 × 20. For students who think the product is 800, emphasize that when calculating 4 × 20, the 2 tens are multiplied by 4, which results in 8 tens, not 8 hundreds. Eight tens is less than 10 tens, so 8 tens must be less than 100.

## ► Enrichment

- Ask students to find the missing digits in a multiplication problem. For example, give students the following problem.

$$\begin{array}{r} \square 0 \\ \times \quad 4 \\ \hline 28\square \end{array}$$

- Ask students to extend this standard in order to multiply a one-digit number and a two-digit number. For example, ask students to create an area model to multiply 6 by 73. The number 73 can be decomposed to 70 + 3 and used to create the area model shown.




Help students use the shortcuts from this standard to find the area of the left portion of the model and then combine the parts to compute the product.

- Ask students to extend this standard in order to multiply two multiples of ten.

## ► Key Terms:

multiple of 10, multiply, one-digit, place value, properties of operations

# B-O.1.1.1

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

Understand various meanings of multiplication and division.

## ► Eligible Content

B-O.1.1.1 Interpret and/or describe products of whole numbers (up to and including $10 \times 10$).

## ► Instructional Supports

**Interpreting multiplication as repeated addition**

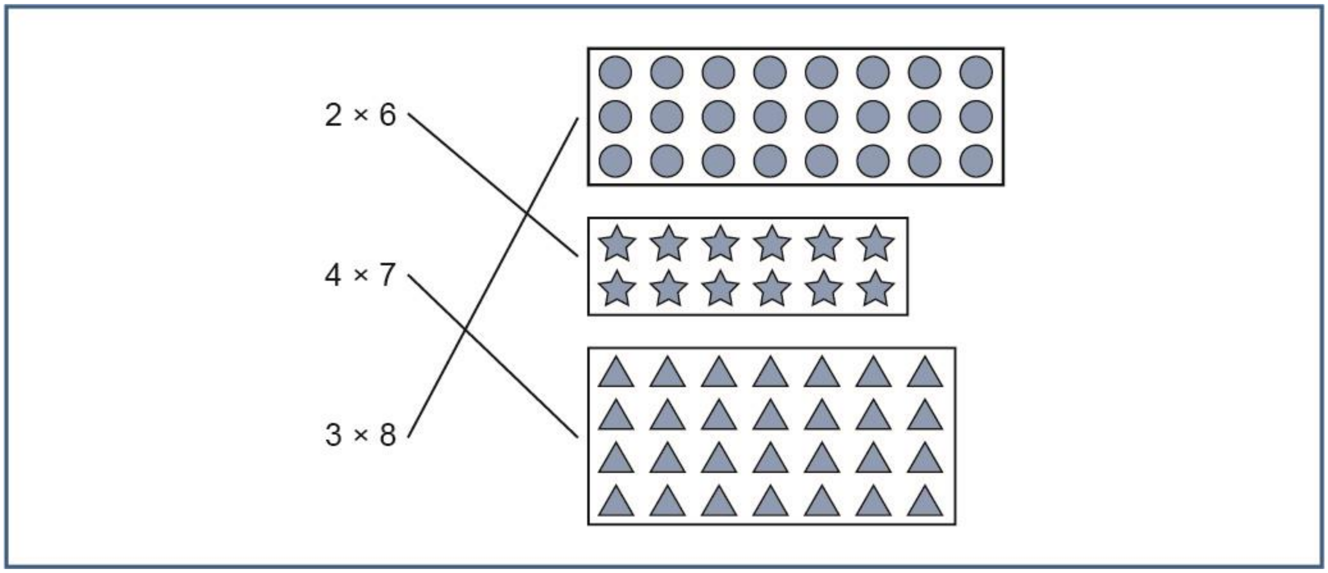
- Ask students to rewrite a given multiplication problem as an addition problem. For example, give students the following multiplication problems and have them write the related addition problems as shown.
    - $2 \times 7$

      $7 + 7$
    - $5 \times 6$

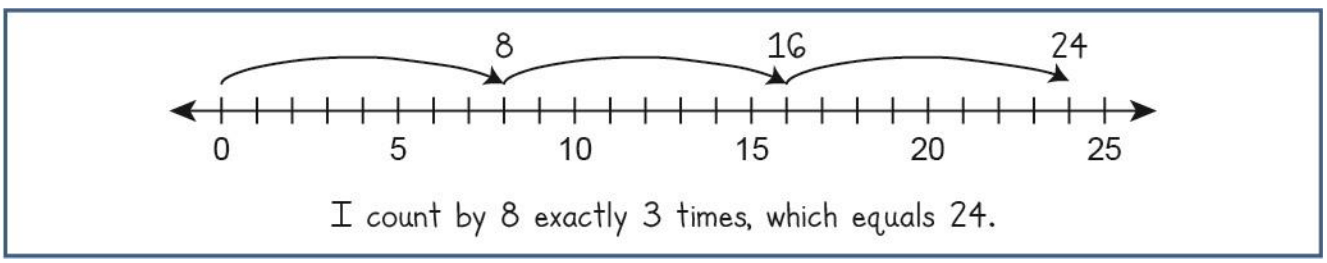
      $6 + 6 + 6 + 6 + 6$
- Ask students to provide a brief explanation of how skip-counting can be used to interpret a multiplication problem. For example, give students the problem $5 \times 7$. Students may interpret it as counting by 5 a total of 7 times (5, 10, 15, 20, 25, 30, 35) or as counting by 7 a total of 5 times (7, 14, 21, 28, 35).

**Interpreting multiplication visually**

- Ask students to match an array to a multiplication problem it could represent. For example, give students the following multiplication statements and arrays and ask them to match each multiplication statement with the correct array.

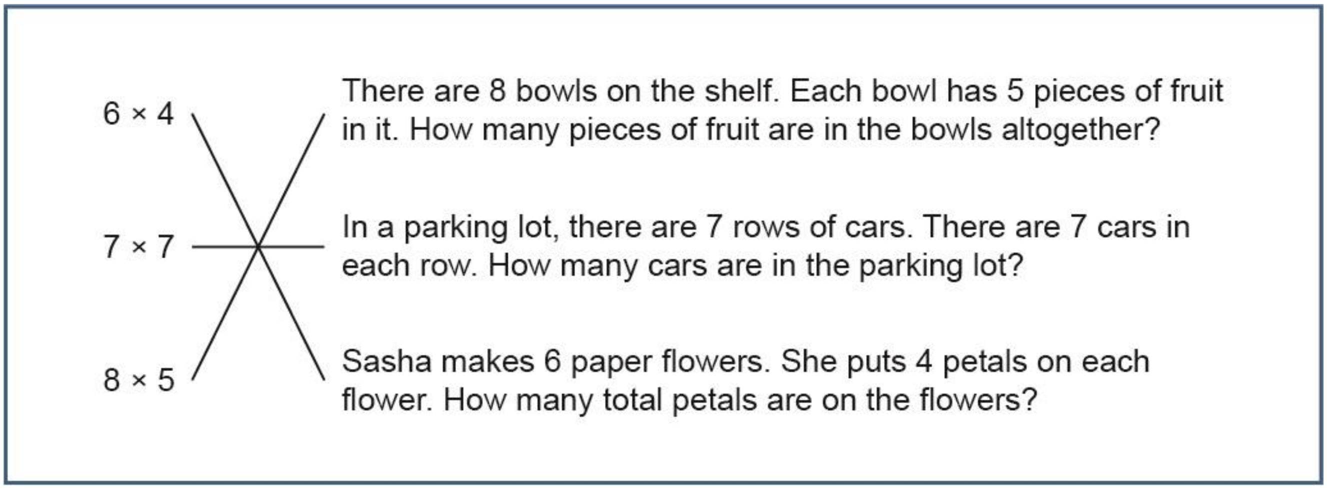



- Ask students to create a visual model (e.g., an array or a number line) that could be used to represent a given multiplication problem. Then, ask students to explain how to use the model with skip-counting to solve the multiplication problem. For example, ask students to create a visual model to illustrate the problem 3 × 8. A possible student response is shown.

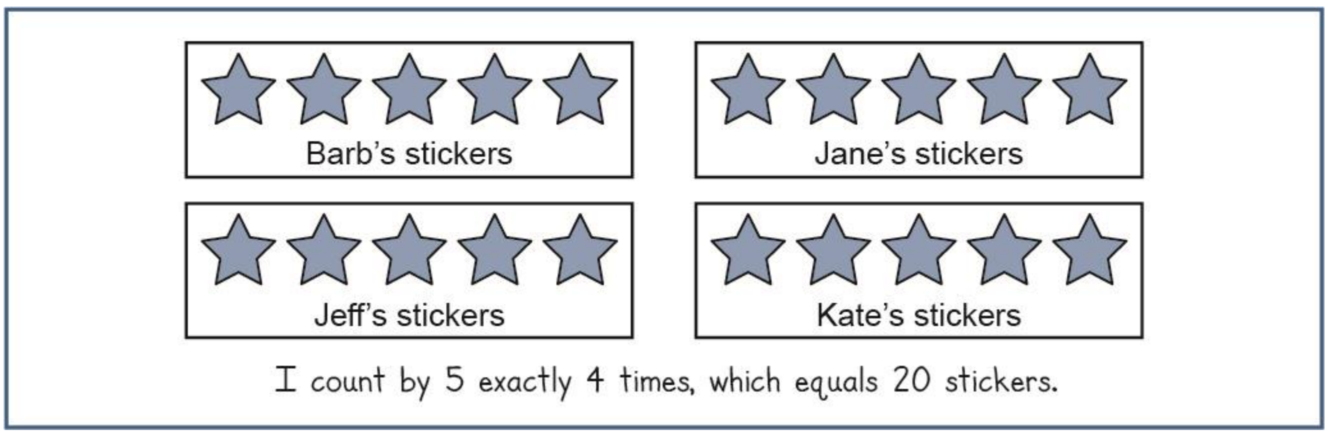



**Interpreting multiplication in context**

- Ask students to match a multiplication expression to a scenario of *x* groups of *y* objects that could be used to interpret a word problem. For example, give students the following multiplication expressions and word problems and ask them to match each word problem with the appropriate expression.

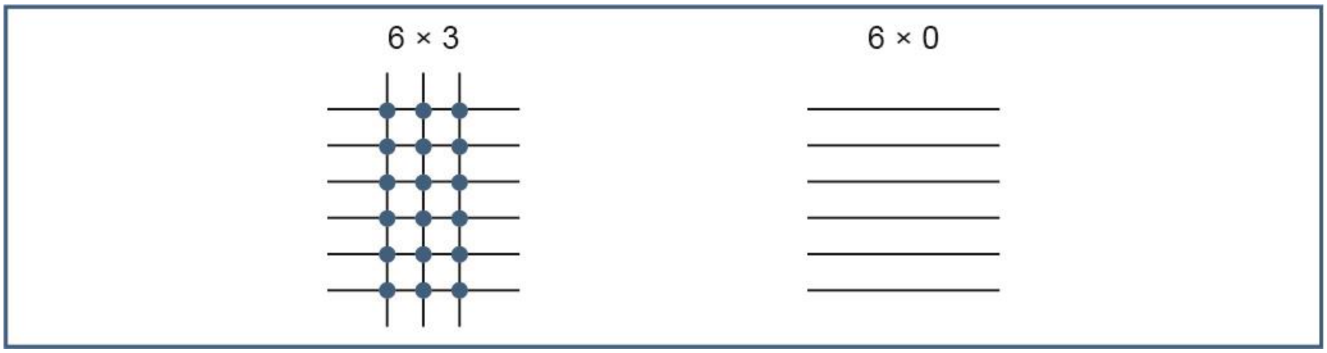



- Ask students to create a visual model that could be used to represent a situation of *x* objects in *y* groups. Then ask students to explain how to use skip-counting to solve the multiplication problem. For example, Steve gives 5 star-shaped stickers to each of his 4 friends. How many star stickers does Steve give to his friends altogether? A possible student response is shown.




## ► Identifying and Addressing Common Misconceptions

- Students may think that the product of a number and 0 is equal to the number. Use a grid diagram to compare multiplication with zero with multiplication with a nonzero number. For example, give students the following model for 6 × 3 and ask students to explain the model. Then give students the model for 6 × 0 and help them conclude that because there are no vertical lines, there are no dots, so the product is 0.




- Some students may not realize that multiplication is only equivalent to skip-counting when starting at zero. For example, ask the students to determine which of the following sequences use skip-counting to show multiplication.

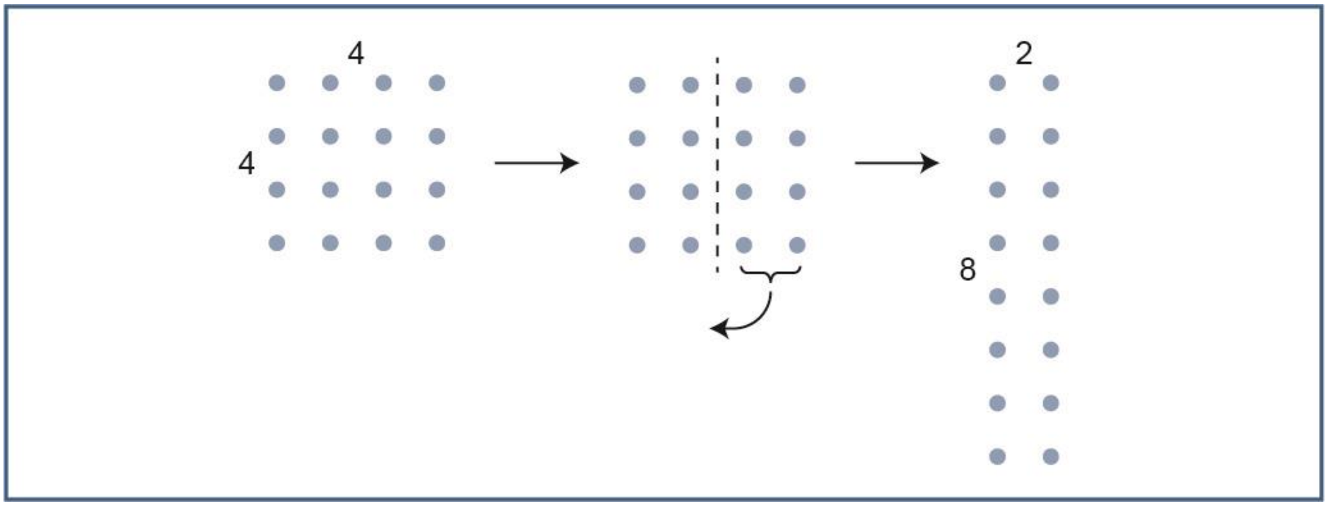
  0, 3, 6, 9, 12 . . .

  1, 4, 7, 10, 13 . . .

  Some students may think the second sequence can be used to show multiplication because the numbers have a difference of 3. However, the numbers in the second sequence are not multiples of 3. Demonstrate that the numbers in the first sequence represent the number of dots in an array, with a total of 3 dots in the first row ($1 \times 3$), a total of 6 dots in the first two rows ($2 \times 3$), a total of 9 dots in the first 3 rows ($3 \times 9$), etc., with the total number of dots always being a multiple of 3.

## ► Enrichment

- Ask students to use arrays to illustrate the commutative property of multiplication. Ask students to make an array of circles on a piece of paper that shows $3 \times 5$. Turn the visual model sideways and ask the student what expression that same visual model now shows. Use the visual model to show that $3 \times 5 = 5 \times 3$.
- Ask students to use visual models to create new multiplication expressions with the same value. For example, given an array model for $4 \times 4$, students rearrange the shapes to show that $4 \times 4$ is equivalent to $2 \times 8$.




## ► Key Terms:

array, column, commutative property, counting on, groups of equal size, number line, product, row, skip-count

# B-O.1.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

Understand various meanings of multiplication and division.

## ► Eligible Content

B-O.1.1.2 Interpret and/or describe whole-number quotients of whole numbers (limit dividends through 50, and limit divisors and quotients through 10).

## ► Instructional Supports

- Ask students to write a division statement to solve a given real-world situation. For example, give students the following problems and ask them to write the related division equations as shown.
  - A store has 42 dolls on its shelves. Each shelf holds 7 dolls. How many shelves of dolls are in the store?

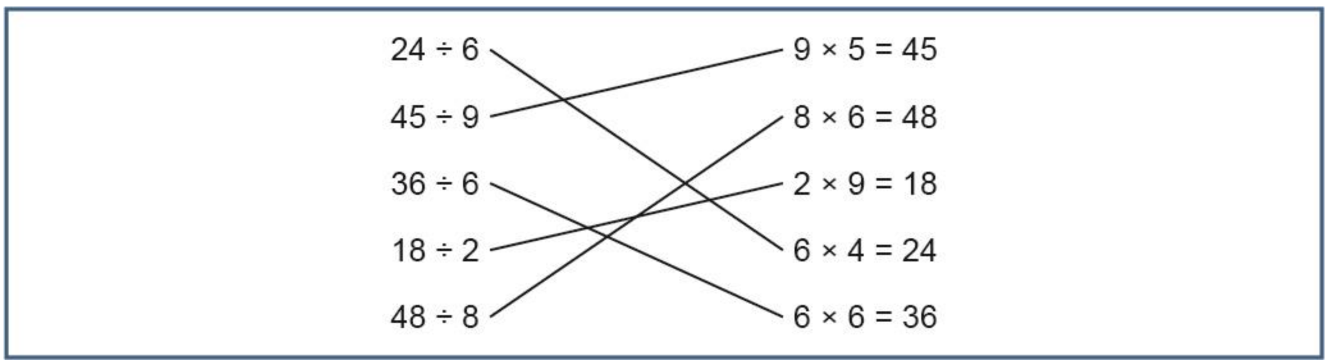
    $42 \div 7 = 6$
  - Zack has 32 sheets of construction paper. He has the same number of sheets of 4 different colors. How many sheets of each color does Zack have?

    $32 \div 4 = 8$
- Ask students to write two situations that could be modeled by the same division problem. Ask them to write the first situation so the quotient is the number of objects in each group and the second situation so the quotient is the number of groups. For example, give students the division problem 28 ÷ 4 = 7. The students could create the following situations.

*Situation 1—Jonah has 28 mini-trucks in his collection. He lines up all the trucks into 4 equal rows on his dresser. How many mini-trucks does Jonah have in each row?*

*Situation 2—There are 28 fourth-grade students going on a field trip. The students will ride in vans that each seat 4 students. How many vans are needed to take all the students on the field trip?*

- Ask students to match division problems to related multiplication equations. For example, give students the following division problems and multiplication equations.




- Ask students to write a multiplication equation that could be used to help solve a division problem. For example, give students the following division problems. The students could write the related multiplication equations as shown.
  - $49 \div 7$

    *$7 \times 7 = 49$*

  - $27 \div 3$

    *$3 \times 9 = 27$*

- Ask students to complete a multiplication statement that can be used to solve a given division problem. For example, give students the following division problems and incomplete multiplication statements.
  - $63 \div 9$

    “9 times ☐ is 63”

    Answer: 7
  - $18 \div 6$

    “6 times ☐ is 18”

    Answer: 3

## ► Identifying and Addressing Common Misconceptions

- Some students may always multiply the numbers provided in a word problem involving identical groups. Ask students to write an expression that can be used to solve this problem: “Sarah has some cans of tennis balls with 2 tennis balls in each can. Sarah has 8 tennis balls in total. How many cans of tennis balls does Sarah have?” Some students may write the expression $2 \times 8$. Ask students whether they are trying to find the size of a whole, a number of groups, or the size of a group. Division is applied to determine the number of groups or the size of a group. To use multiplication, a missing factor equation is needed rather than an expression.
- Students may think a number divided by 1 is 1 because they remember “anything divided by 1 keeps the number the same” and they interpret it as keeping the 1 the same. Ask students to solve $10 \div 1$ and explain their answer. Remind students that 10 objects divided into groups of size 1 would result in 10 groups and that 10 objects divided into 1 group would result in a group of size 10.

## ► Enrichment

- Ask students to make a number line and an array for a division problem. Then ask students to compare or discuss the parallels of the models as well as the strengths and weaknesses of each model.
- Ask students to use division facts to solve a division problem with greater numbers. For example, Joe knows that 36 divided by 9 is 4. How can he use that fact to determine the answer to 360 divided by 90?
- Ask students to determine the number of ways that a quantity can be divided equally. For example, how many ways can 24 coins be divided into equal piles?

## ► Key Terms:

division, equal share, expression, multiplication, partition, quotient

# B-O.1.2.1

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

Solve mathematical and real-world problems using multiplication and division, including determining the missing number in a multiplication and/or division equation.

## ▶ Eligible Content

B-O.1.2.1 Use multiplication (up to and including 10 × 10) and/or division (limit dividends through 50, and limit divisors and quotients through 10) to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.

## ▶ Instructional Supports

- Ask students to write and solve a division equation and a multiplication equation that could both be used to represent a given word problem. For example, give students this problem: "There are 3 vases in which the florist puts 27 roses. Each vase must have the same number of roses in it. How many roses must be in each vase?" A possible student response is shown.

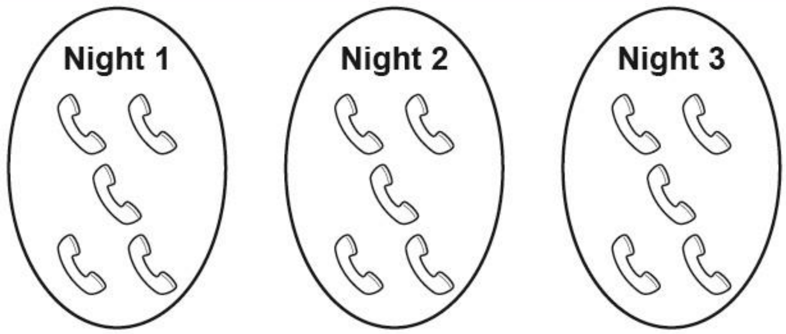
  27 ÷ 3 = ☐, and 3 × ☐ = 27

  ☐ = 9

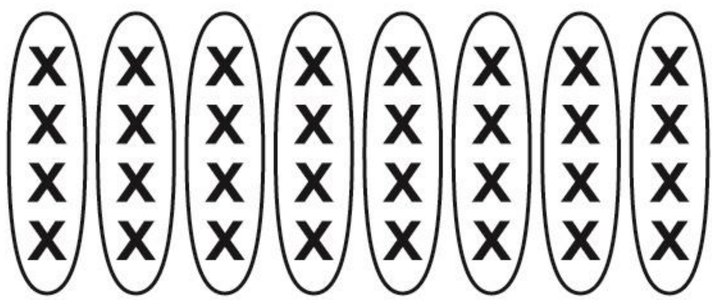
- Ask students to use a multiplication equation and a related division equation to write a word problem that could be represented by both equations. For example, give students the equations ☐ × 6 = 42 and 42 ÷ ☐ = 6. A possible student response is shown.

  *Mr. Sims worked for a total of 42 hours over 6 days. He worked the same number of hours each day. How many hours did Mr. Sims work each day?*

- Ask students to model real-world multiplication problems using an equation, an array, or a set of objects. For example, give students this problem: "Juli spends 5 minutes each night on the phone with her aunt for 3 nights. How many total minutes does Juli spend on the phone with her aunt?" A possible student response is shown.




- Ask students to model real-world division problems using an equation, an array, or a set of objects. For example, give students this problem: "Mari has 32 cherries and puts them onto 8 different plates. Each plate has the same number of cherries. How many cherries are on each plate?" A possible student response is shown.



## ► Identifying and Addressing Common Misconceptions

- Some students may think that certain types of problems can be modeled using only multiplication or only division. Ask students to write as many possible equations (fact families) with unknowns as they can for this problem: "Duncan has 4 boxes of toys, and each box contains 6 toys. How many toys does Duncan have in the boxes?" Some students may use only one operation for their equations. Remind students that the unknown does not always have to be the result of the operation. The unknown can instead be one of the components of the operation, i.e., a factor, dividend, or divisor.

- Some students may think that the divisor in a division problem can only represent the number of groups. Ask students to create two division equations to solve this problem: “Hakeem has 36 strawberries and puts them into bags that can hold 4 strawberries each. How many bags does he need?” Some students may only write the equation $36 \div 4 = \square$ and not $36 \div \square = 4$. Help students create an array model and recognize that rotating the array so that rows and columns are switched shows that both equation setups are equivalent and will always yield the same answer. Observe that in a division statement, either the divisor or the quotient could represent the number of groups and either could represent the number of objects in each group.

## ► Enrichment

- Ask students to solve several pairs of related multiplication and division equations, identify which equation was easier to solve in each pair, and explain why. For example, give students the following pairs of equations:
  - $3 \times \square = 18$ and $18 \div 3 = \square$
  - $5 \times 6 = \square$ and $\square \div 6 = 5$
  - $\square \times 8 = 56$ and $56 \div \square = 8$

  Some students may find that when the unknown is the product or quotient, that equation is the easier equation to solve. Some may always think multiplication is easier to solve than division. Yet others may find that the difficulty in solving equations depends on the numbers used in each individual problem, regardless of the operation used.

- Give students numbers to insert into a real-world statement and ask them to determine ways of inserting the numbers that would make sense. For example, ask students to fill in the blanks in the following statement using the numbers 4, 12, and one other number:

  “Jerry has ___ toy blocks that he keeps in ___ containers with exactly ___ blocks in each container.”

Ask students to explain which ways of inserting the numbers into the sentence make sense and whether each way results in a multiplication problem or a division problem when students solve for the remaining blank.

## ► Key Terms:

array, column, division, equal groups, equation, factor, measurement quantities, multiplication, product, represent, row, unknown

# B-O.1.2.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

Solve mathematical and real-world problems using multiplication and division, including determining the missing number in a multiplication and/or division equation.

## ► Eligible Content

B-O.1.2.2 Determine the unknown whole number in a multiplication (up to and including 10 × 10) or division (limit dividends through 50, and limit divisors and quotients through 10) equation relating three whole numbers.

## ► Instructional Supports

- Ask students to use the meaning of multiplication or division to solve a problem. For example, when given 7 × 3, students may draw 7 sets of 3 dots and count the total number of dots. As a further example, when given 18 ÷ 6, students may sort 18 cards into 6 stacks and count the number of cards in each stack.
- Ask students to identify the unknown number in an equation by using a related fact of the opposite operation that makes a multiplication or division equation true.
  - $4 \times \square = 36$

    $\square = 9$ *because* $36 \div 4 = 9$
  - $56 \div \square = 7$

    $\square = 8$ *because* $7 \times 8 = 56$
  - $\square \times 6 = 54$

☐ = 9 because $54 \div 9 = 6$

- ☐ $\div 5 = 7$

  ☐ = 35 because $7 \times 5 = 35$

- Ask students to identify the unknown number that makes a multiplication or a division equation true and then use that to write the three other equations that are related to the given equation.
  - $9 \times 2 =$ ☐

    ☐ = 18

    $2 \times 9 = 18$

    $18 \div 2 = 9$

    $18 \div 9 = 2$
  - $24 \div$ ☐ $= 3$

    ☐ = 8

    $24 \div 3 = 8$

    $3 \times 8 = 24$

    $8 \times 3 = 24$
- Ask students to solve multiplication and division problems where the operation may be on either the left or right side of the equation. For example, give students the following riddle and have

them find the answer by solving the equations from the table and inserting the corresponding letters into the blanks to find the answer.

What is full of holes but still holds water?

__    __ __ __ __ __ __

8    7 18 6 3 24 40

| | |
|---|---|
| $2 \times 7 =$ ___ | R |
| $40 = 5 \times$ ___ | A |
| $6 =$ ___ $\div 3$ | P |
| $24 \div 6 =$ ___ | K |
| $27 = 3 \times$ ___ | B |
| ___ $\div 8 = 5$ | E |

| | |
|---|---|
| $24 = 4 \times$ ___ | O |
| ___ $\times 2 = 14$ | S |
| $40 = 8 \times$ ___ | D |
| ___ $\div 7 = 2$ | L |
| $6 \times 4 =$ ___ | G |
| $6 = 18 \div$ ___ | N |

## ▶ Identifying and Addressing Common Misconceptions

- Students may solve an equation based only on the numbers and the operation involved without paying attention to the structure of the equation. Ask students to solve the equation $3 \times ? = 6$. Some students may arrive at the answer of 18, seeing the operation $3 \times 6$. Ask students to verbalize the equation out loud by saying “three times what equals 6?” Ask students how to find the “what” in their question. Discuss how to use the given numbers to create the division equation $6 \div 3 = ?$ to help solve for the missing “what.”
- Some students may incorrectly write the related division equations to a given multiplicative expression. Ask students to write the related equations to $8 \times 2 = ?$. Some students will write the equations $8 \div ? = 2$ or $2 \div ? = 8$, recognizing there are division equations related to the given multiplication equation but not understanding that the product of a multiplication equation is the dividend of the related division equation. Ask students to determine the number that would make each of their equations true and discuss why the ? must represent the same number in all their related equations.

## ▶ Enrichment

- Discuss with students the meaning of the equal sign in an equation. For example, ask students to fill in the blank for the equation $5 \times 8 =$ ___ $\times 4$. Many students give 40 as the answer. Discuss how the equal sign is like a balance scale where the values on the left side of the equal sign must be equal to the values on the right side of the equal sign. Remind students that the equal sign is not to be considered an “enter” button on a calculator that means “write the answer.” Provide several other equations for the student, such as: $2 \times 8 = \_\_\_ \times 4$; $6 \times 3 = \_\_\_ \times 9$; etc.

## ► Key Terms:

division, equation, multiplication, product, unknown

# D-M.2.1.1 & D-M.2.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Represent and interpret data.

Organize, display, and answer questions based on data.

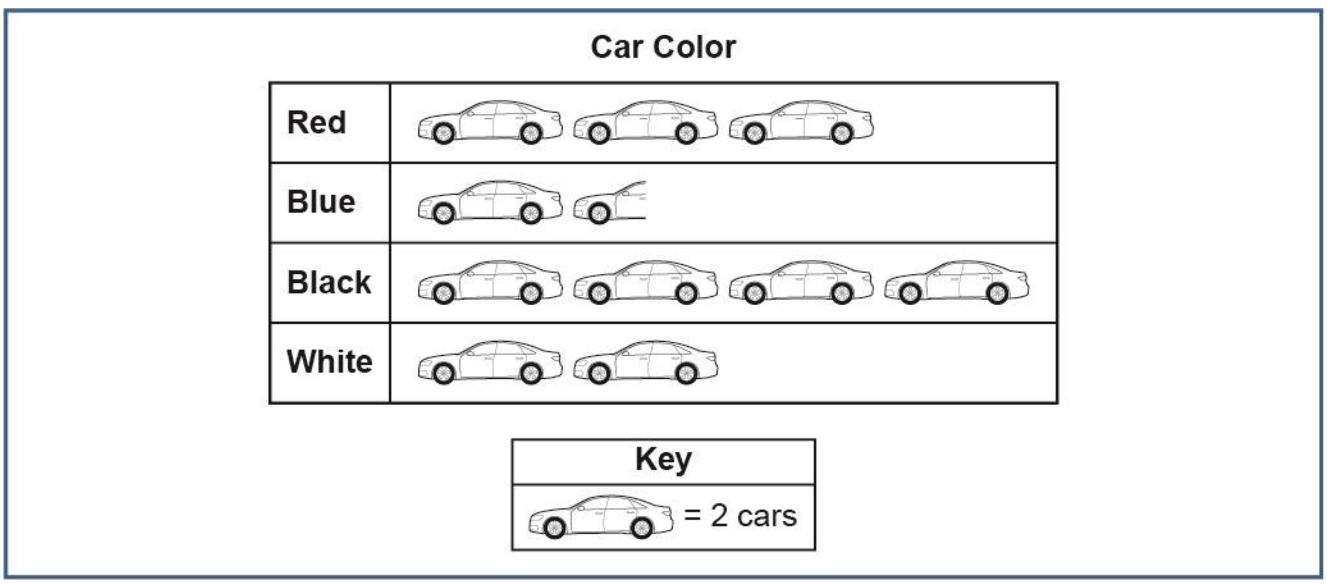
## ► Eligible Content

D-M.2.1.1 Complete a scaled pictograph and/or a scaled bar graph to represent a data set with several categories (scales limited to 1, 2, 5, and 10).

D-M.2.1.2 Solve one- and/or two-step problems using information to interpret data presented in scaled pictographs and/or scaled bar graphs (scales limited to 1, 2, 5, and 10).

## ► Instructional Supports

- Ask students to read information from a given picture graph and discuss the importance of looking at the key before interpreting any of the information in the picture graph. For example, give students the following picture graph representing the colors of cars in a parking lot.

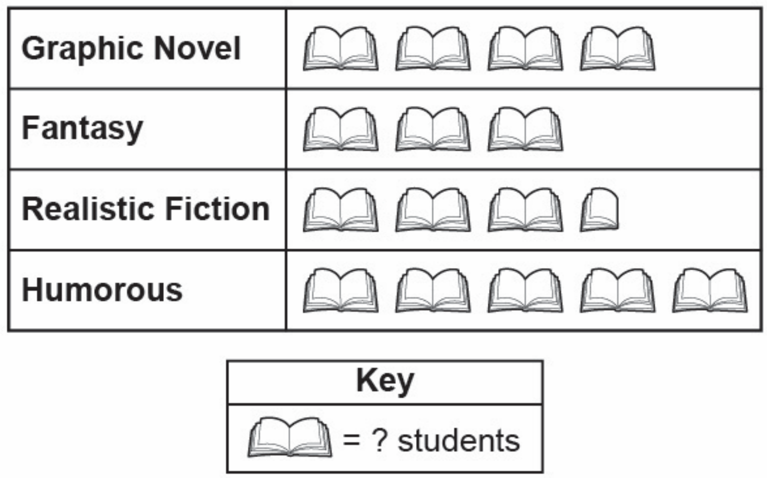


Students identify that the parking lot contains 6 red cars, 3 blue cars, 8 black cars, and 4 white cars. Discuss with students the importance of the key within the context and why the correct response is 6, 3, 8, 4 rather than 3, 1.5, 4, 2.

- Ask students to determine the scale of a picture graph that corresponds to information found in a table of values. For example, give students the following table of Favorite Book Genres and the picture graph containing the same information.

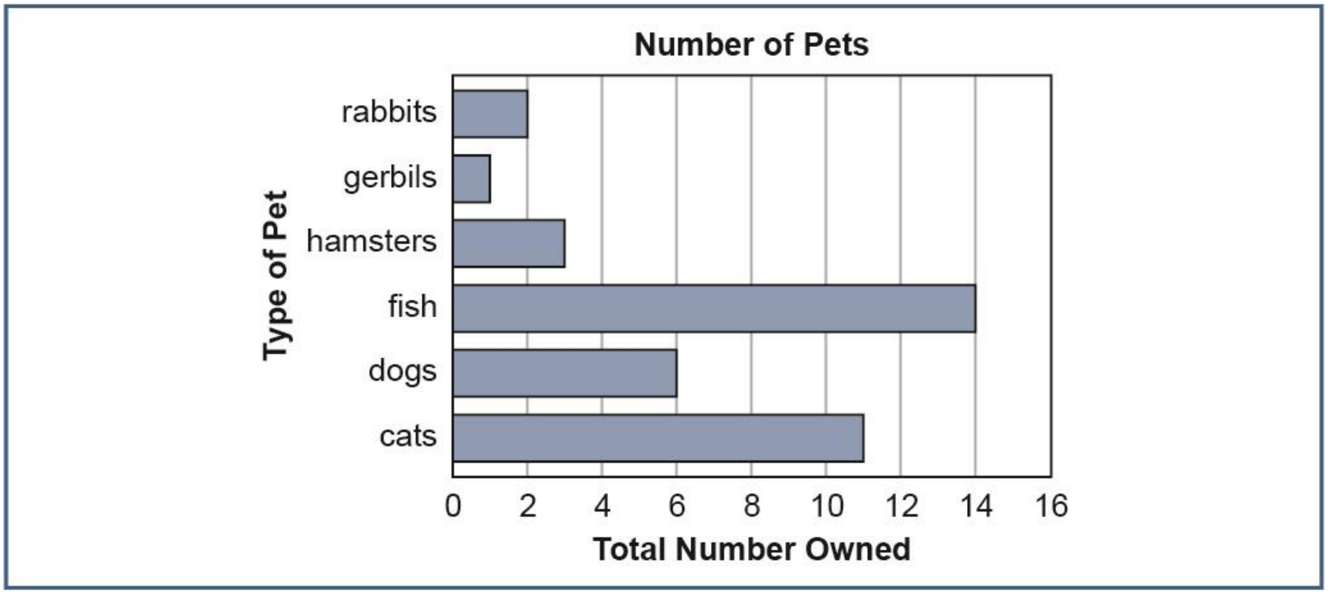
| | |
|---|---|
| Graphic Novel | 8 |
| Fantasy | 6 |
| Realistic Fiction | 7 |
| Humorous | 10 |

**Favorite Book Genres**



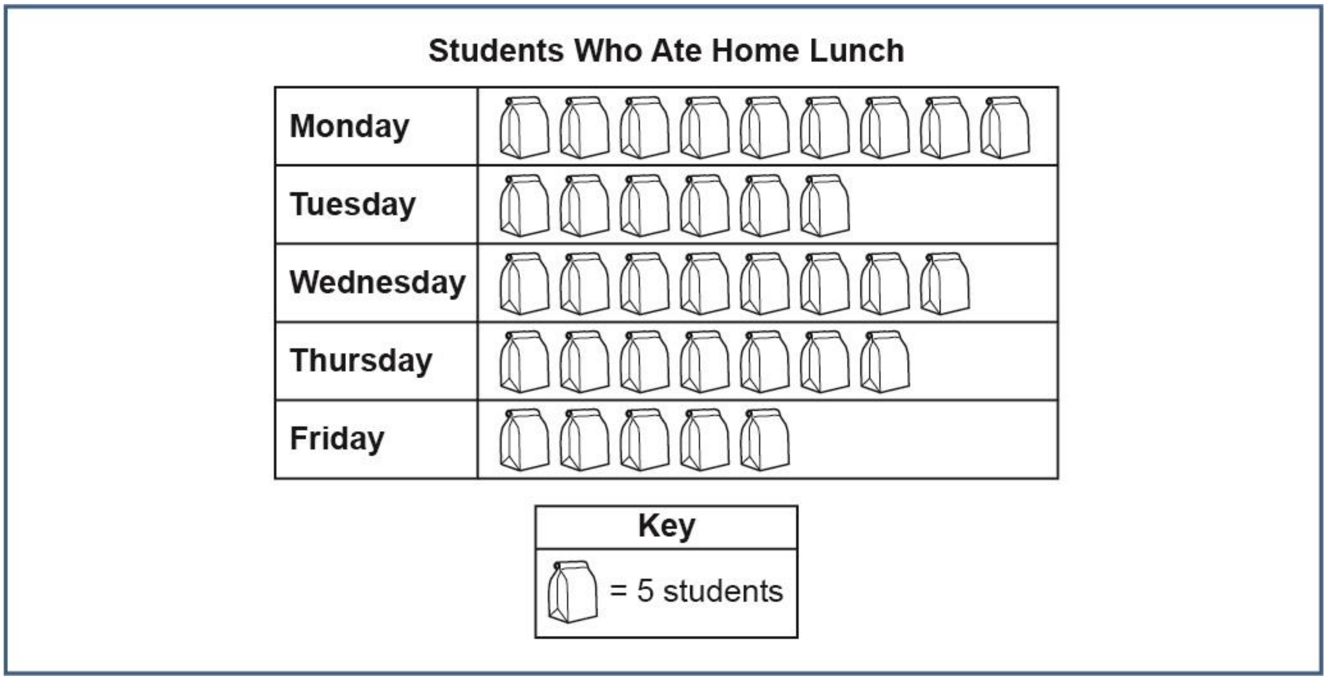

Using number sense to relate the table to the picture graph, students determine the scale of the picture graph is that 1 book represents 2 students.

- Ask students to compare information from two or more bars of a bar graph. For example, give students the following bar graph showing the number of pets owned by students in third grade. Ask them to determine how many more pet cats there are than pet dogs.




Students determine that there are 5 more pet cats than there are pet dogs based on the information in the bar graph.

- Ask students to compare information given in a picture graph. For example, give students the following picture graph showing the number of students who ate a lunch brought from home each day and ask them to determine how many fewer students ate a lunch from home on Friday than on Monday.




The key states that each paper bag represents 5 students, and there are 4 fewer paper bags shown on Friday than there are on Monday. The 4 fewer paper bags represent a total of 20 students. Therefore, there were 20 fewer students who brought lunch from home on Friday than on Monday.

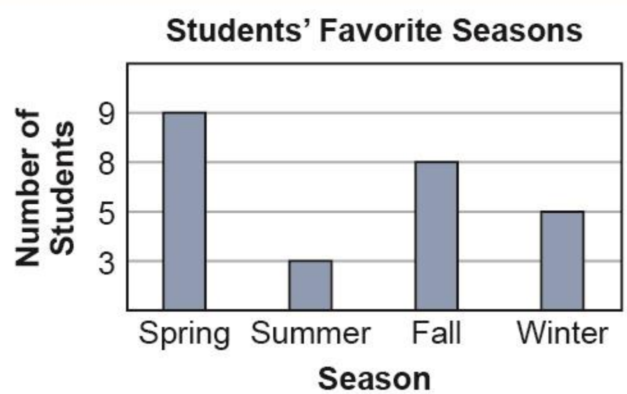
## ► Identifying and Addressing Common Misconceptions

- Some students may not pay attention to the scale of a picture graph. Give students a picture graph and ask what each image in the picture graph represents. Ask students to think of situations where one unit represents multiple objects, like a 6-pack of juice or a carton of 12 eggs. Help students draw parallels between those situations and pictures in a picture graph.
- Some students may create a bar graph that does not use correct scaling. Ask students to create a bar graph using the information in the following table.

**Students' Favorite Seasons**

| **Season** | **Number of Students** |
|---|---|
| Spring | 9 |
| Summer | 3 |
| Fall | 8 |
| Winter | 5 |

Some students may create the following bar graph.




Ask students to draw bars of heights 1, 2, 3, 4, 5, 6, 7, 8, and 9 on the graph. Discuss reasons why having bars with inconsistent height increments is problematic (e.g., a bar of height 4 might not be twice as tall as a bar of height 2). Help students create a new bar graph with consistent height increments.

## ► Enrichment

- Discuss the relationships between representing data in a picture graph and representing the same data in a bar graph. Ask students to explain why one option might sometimes be better than the other.
- Ask students to create a bar graph that displays the same data shown on a picture graph or to create a picture graph that displays the same data show on a bar graph.

## ► Key Terms:

bar graph, data, horizontal axis, pictorial key, picture graph, scale, vertical axis

# B-O.2.1.1 & B-O.2.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Understand properties of multiplication and the relationship between multiplication and division.

Use properties to simplify and solve multiplication problems.

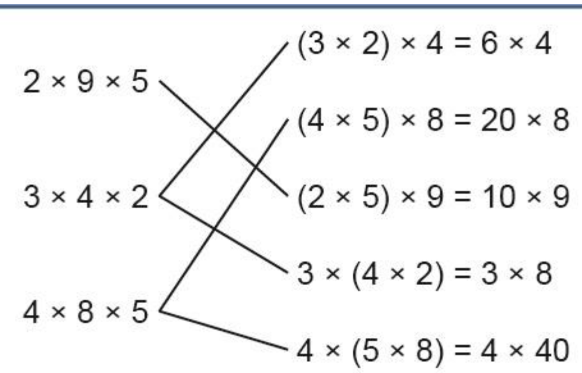
## ► Eligible Content

B-O.2.1.1 Apply the commutative property of multiplication (not identification or definition of the property).

B-O.2.1.2 Apply the associative property of multiplication (not identification or definition of the property).

## ► Instructional Supports

- Ask students to match equivalent multiplication expressions. For example, give the students the expressions listed below on the left. Have students match each expression to the equation(s) on the right that show the numbers in the expressions reorganized or grouped, making the expressions easier to solve. Students draw lines to match the expressions to their equivalent equation(s) as shown.




- Ask students to group known facts to solve multiplication problems.
    - $8 \times 5 \times 2$

$8 \times (5 \times 2) = 8 \times 10 = 80$

- $3 \times 7 \times 3$

  $(3 \times 3) \times 7 = 9 \times 7 = 63$

- Ask students to use an array to illustrate the commutative property of multiplication. For example, ask students to illustrate why 3 × 5 is the same as 5 × 3. A possible student response is shown.

5 × 3 = 15 3 × 5 = 15

- Ask students to determine whether statements involving the commutative property are true or false. For example, give students the statements in the following table.

| | |
|---|---|
| Since 3 × 6 = 18, 6 × 3 = 18. | True |
| Since 72 ÷ 9 = 8, 8 ÷ 72 = 9. | False |
| Since 4 + 8 = 12, 8 + 4 = 12. | True |
| Since 7 – 2 = 5, 2 – 7 = 5. | False |
| 16 ÷ 4 ÷ 2 = 4 ÷ 2 ÷ 16 | False |
| 5 × 7 × 2 = 5 × 2 × 7 | True |

## ► Identifying and Addressing Common Misconceptions

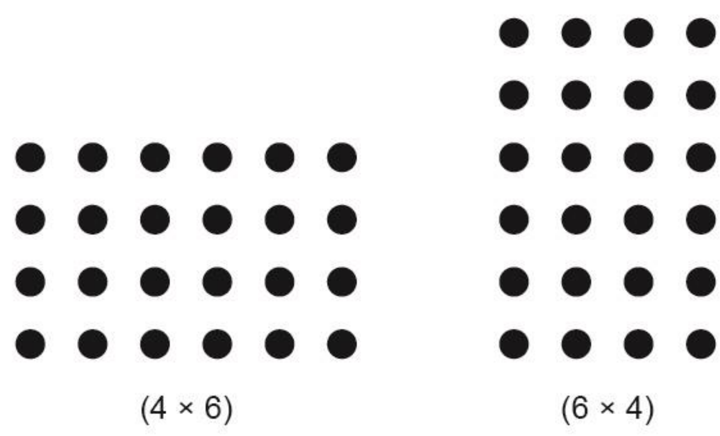
- Students may think that the commutative property applies to division. Ask students to divide 3 ÷ 21. Some students may think that it is equal to 7 because 21 ÷ 3 = 7. Discuss what it means to divide two numbers. One representation of 21 ÷ 3 = 7 is that sharing 21 granola bars among 3 people results in each person getting 7 granola bars. Similarly, 3 ÷ 21 would mean sharing 3 granola bars among 21 people, which would not give the result of 7 granola bars for each person.
- Students may think that division is associative. Ask students if the following equation is true or false without solving.

  $$(36 \div 6) \div 2 = 36 \div (6 \div 2)$$

  Some students may think that both sides are equal. Help students simplify each side of the equation. Explain that although the operation is the same on each side of the equation, the two numbers that are divided first greatly affect the result.

## ► Enrichment

- Ask students to explain why reordering works for multiplication expressions but not division expressions.




A multiplication problem like 4 × 6 can be represented with an array of 4 rows with 6 dots in each row. Reordering the digits to 6 × 4 reverses the numbers, creating an array of 6 rows with

4 dots in each row. The two sets have the same number of dots because one array turned on its side is identical to the other array. A division problem like $12 \div 3$ can be represented by arranging 12 dots into 3 rows of equal dots. Reordering the numbers to $3 \div 12$ would require arranging 3 dots into 12 rows of equal dots, which cannot be done without cutting up the dots.

- Ask students to use the commutative and associative properties to create partial products ending in zero. Provide examples where students can reorder factors to create products involving multiples of 10.
  - $(42 \times 4) \times 25$

    $42 \times (4 \times 25) = 42 \times 100 = 4{,}200$
  - $5 \times 16 \times 2$

    $16 \times (5 \times 2) = 16 \times 10 = 160$
  - $4 \times 7 \times 20$

    $7 \times (4 \times 20) = 7 \times 80 = 560$

## ► Key Terms:

area model, array model, decompose, multiplication expression, product, property of operations, sum

# B-O.2.2.1

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Understand properties of multiplication and the relationship between multiplication and division.

Relate division to a missing number multiplication equation.

## ► Eligible Content

B-O.2.2.1 Interpret and/or model division as a multiplication equation with an unknown factor.

## ► Instructional Supports

- Ask students to find the missing number that makes a division equation and its related multiplication equation both true. For example, give students the following pairs of equations.
    - ☐ ÷ 7 = 6 and 6 × 7 = ☐

      *The missing number is 42.*
    - 18 ÷ ☐ = 2 and 2 × ☐ = 18

      *The missing number is 9.*
    - 36 ÷ 9 = ☐ and ☐ × 9 = 36

      *The missing number is 4.*
- Ask students to find an unknown-factor multiplication equation that is equivalent to a given division equation and then use that to solve the division equation. For example, give students the problem 21 ÷ 3 = ☐. One possible student response may be to rewrite the equation as ☐ × 3 = 21 and arrive at the solution of ☐ = 7.

- Ask students to determine whether a given statement is true or false. For example, give students the following statements.

| | |
|---|---|
| Since $8 \times 8 = 64$, $8 \div 64 = 8$. | False |
| Since $48 \div 8 = 6$, $8 \times 6 = 48$. | True |

- Ask students to write one multiplication equation and one division equation that are equivalent to the given statement.
    - There are 9 groups of 6 in 54.

      $9 \times 6 = 54$

      $54 \div 9 = 6$

    - Multiplying 8 times 3 equals 24.

      $8 \times 3 = 24$

      $24 \div 8 = 3$

## ► Identifying and Addressing Common Misconceptions

- Some students may not correctly interpret an unknown-factor problem. Ask students to explain how to fill in the missing number from $2 \times ? = 6$. Some students may multiply the given numbers any time they see a multiplication symbol. Ask them to replace the ? with the product (12) and help them check the validity of their statement. Demonstrate that the solution is 3 because 2 sets of 3 is 6. Observe that this is the same answer that would be attained by dividing the two numbers. Remind them that solving a division problem and solving an unknown-factor problem are equivalent.
- Some students think that the unknown must always be the first factor or must always be the second factor when rewriting a division problem as multiplication. Ask students to write the problem $32 \div 8 = ?$ as an unknown-factor problem in two different ways. Some students may

only be able to come up with one. Ask them to arrange 32 blocks into 8 rows, and then ask them to arrange those 32 blocks into 8 columns. Use this visual model to demonstrate that both $8 \times ? = 32$ and $? \times 8 = 32$ are equivalent ways of rewriting $32 \div 8 = ?$.

## ▶ Enrichment

- Ask students to use arrays, manipulatives, or other visual models to explain why dividing is the same as solving an unknown-factor problem.
- Ask students to rewrite a missing factor problem with three factors as a division problem in multiple ways. For example, given the expression $2 \times 3 \times ? = 30$, students can rewrite the equation as UPD

## ▶ Key Terms:

division, equivalent, multiplication, product, quotient, unknown factor

# B-O.3.1.1, B-O.3.1.2, B-O.3.1.3, & B-O.3.1.4

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Use operations, patterns, and estimation strategies to solve problems (may include word problems).

## ► Eligible Content

B-O.3.1.1 Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.2 Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.3 Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers.

B-O.3.1.4 Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols).

## ► Instructional Supports

- Ask students to study a two-step word problem and the associated equation and identify what the letter represents in each problem. For example, give students the equation $3 \times 8 \div f = 6$ and this problem: “Julie has 3 bunches of flowers with 8 flowers in each bunch. She puts the flowers into 6 different vases. She puts the same number of flowers in each vase. How many flowers are in each vase?” The variable $f$ represents the number of flowers in each vase.
- Ask students to match an equation with the two-step word problem that it represents. For example, ask students to match the following three word problems and equations.

| | |
|---|---|
| Jody makes a total of 36 charms in 4 days. She makes 8 charms on each of the first 3 days. How many charms does Jody make on the fourth day? | $36 \times 3 - c = 8$ |
| Mick buys 3 packages of cookies that each have 36 cookies. After each student in his grade eats one cookie, there are 8 cookies left. How many cookies did the students eat? | $36 - (8 + c) = 3$ |
| Mr. Bing has 36 dollars to spend at a cheese store. He buys some cheese and a slicer and has 3 dollars left. The slicer costs 8 dollars. How much does the cheese cost? | $8 \times 3 + c = 36$ |

- Ask students to write an equation with a letter as a variable to represent a two-step word problem. For example, give students this problem: “Nick sells candy bars to raise money for his club. He has a total of 72 candy bars to sell. He sells 16 candy bars on Monday and 23 candy bars on Tuesday. How many candy bars does Nick have remaining?” Two possible equations include $72 = 16 + 23 + c$ and $72 – 16 – 23 = c$.

- Ask students to evaluate expressions using the order of operations. For example, give students the following expressions.
  - $3 \times 9 + 7$

    34

  - $18 – 3 \times 5$

    3

  - $48 \div 8 + 6$

    12

- Ask students to write and solve a two-step equation that could be used to model a given situation. For example, give students the following problems.
  - Tyler scores 7 points in each of the first 3 quarters of his basketball game and 4 points in the last quarter. How many total points does he score in the game?

    $$7 \times 3 + 4 = p$$

    The order of operations indicates that multiplication should be done before addition, so start with 7 × 3, which is 21. Then, add 21 + 4 to get 25. The solution is $p$ = 25 points.

  - Maureen collects 38 rocks while on vacation. She gives 8 rocks to her friend and then divides the rest of the rocks equally into 6 buckets. How many rocks are in each bucket?

    $$(38 - 8) \div 6 = r$$

    The parentheses must be evaluated first. Since 38 – 8 = 30, the equation is now 30 ÷ 6 = $r$. This can be rewritten as a related fact using multiplication, giving the equation 30 = 6 × $r$. The number that multiplies by 6 to get 30 is 5. The solution is 5 rocks in each bucket.

- Ask students to match a word problem to an estimation equation that would help them estimate the answer.

| Word problem | Equation |
| --- | --- |
| Joni gives her dog 3 snacks and 2 meals every day. How many total times does Joni feed her dog in 30 days? | 30 ÷ 2 – 3 = 12 |
| Harry has a total of 30 marbles. He gives 2 marbles to his sister and 3 marbles to his brother. How many marbles does Harry have left? | 30 – (2 + 3) = 25 |
| Malia has 30 colored pencils that she divides equally into a pencil holder and a pencil box. She then takes 3 pencils out of the pencil box. How many pencils are left in the pencil box? | (3 + 2) × 30 = 150 |

- Ask students to use estimation to determine whether the answer provided for a word problem is most likely correct or incorrect. Then ask students to solve the problem to find the actual correct answer. For example, give students this problem: "It takes Jack 9 minutes to walk to the bus stop. He walked to the bus stop 4 days last week. On the fifth day, Jack walked all the way to school, which took 32 minutes. What is the total number of minutes Jack spent walking to get to school last week?" Give students the provided answer of 68 minutes. A possible student response is shown.

To estimate, round 9 minutes to 10 and 32 minutes to 30.

$10 \times 4 + 30 = 70$

The answer is most likely correct since 68 is close to 70.

## ► Identifying and Addressing Common Misconceptions

- Some students think that the letter that is used as a variable makes a difference in an equation. Ask students whether either of the equations $16 \times f + 3 = 35$ or $16 \times g + 3 = 35$ can be used to represent this situation: "Amir has 16 small marbles that all have the same mass and a larger marble that has a mass of 3 grams. All his marbles have a total mass of 35 grams. What is the mass of each small marble?" Some students may say that only the second equation is correct because $g$ represents grams. Others may say that the variable needs to be an $m$ for marble or an $s$ for small marble. Remind students of problems they have solved in which a box, circle, or question mark was used to represent the unknown. Discuss why using a different symbol does not change the validity of the equation and help students become comfortable with using many different symbols to indicate unknowns. Discuss why using $s$, $m$, or $g$ may be more helpful than using $f$ but not any more valid.
- Some students may think that the value of a variable is connected to the order of the letters in the alphabet. Give students the information that the value of $c$ is 5 and ask them what, if anything, can be concluded about $d$. Some students may think that $d = 6$ since $d$ is one letter after $c$ or that

*d* must be greater than *c* because it is later in the alphabet. Give students the same questions using an empty box and a circle to represent the unknowns and help them conclude that the second unknown could be any number regardless of what is known about the first variable. Help students understand that using letters as unknowns works in the same manner.

## ▶ Enrichment

- Introduce students to the idea of a variable as an unknown that can represent multiple values rather than just one value. For example, give students a paper ruler with the first three inches cut off and the expression $m - 3$. Help students measure various objects using the shortened ruler and recognize that 3 inches can be subtracted from any measurement shown on the ruler to find the true length of the object. Discuss how the expression fits the situation and how the use of the variable is different from solving equations.

- Ask students to solve simple equations with one or two unknowns. It may be helpful to use shapes instead of letters as the unknowns if students are more accustomed to writing their answer in a box, circle, or some other shape. For example, ask students to find the missing numbers in the following equation.

  $$\Delta + \Delta + \Delta = 21$$

  Emphasize that all the triangles must have the same value.

  For the equation below with two unknowns, ask students to find several pairs of solutions that solve the equation. In this situation, the numbers represented by the triangle and square can change. However, if one number changes, the other number must also change.

  $$(\Delta - \square) + 5 = 8$$

- Ask students to determine upper and lower bounds on non-whole number solutions to two-step problems. For example, Sandra stacks five identical blocks on top of a 1-inch book to a total height of 13 inches. Ask students how tall the blocks are. While students may not have enough experience with fractions and decimals to calculate the exact answer, guide them to recognize that five 2-inch blocks would make a height of 11 inches because $10 + 1 = 11$, so the blocks Sandra uses must be greater than 2 inches tall. However, five 3-inch blocks would make a height of 16 inches because 15 + 1 = 16. Therefore, the blocks Sandra uses must be greater than 2 inches and less than 3 inches tall.

## ► Key Terms:

equation, estimation, expression, mental computation, order of operations, reasonableness, rounding, two-step problem, unknown quantity, variable

# B-O.3.1.1, B-O.3.1.2, B-O.3.1.3, & B-O.3.1.4

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Use operations, patterns, and estimation strategies to solve problems (may include word problems).

## ► Eligible Content

B-O.3.1.1 Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.2 Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.3 Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers.

B-O.3.1.4 Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols).

## ► Instructional Supports

- Ask students to study a two-step word problem and the associated equation and identify what the letter represents in each problem. For example, give students the equation $3 \times 8 \div f = 6$ and this problem: "Julie has 3 bunches of flowers with 8 flowers in each bunch. She puts the flowers into 6 different vases. She puts the same number of flowers in each vase. How many flowers are in each vase?" The variable *f* represents the number of flowers in each vase.
- Ask students to match an equation with the two-step word problem that it represents. For example, ask students to match the following three word problems and equations.

| | |
|---|---|
| Jody makes a total of 36 charms in 4 days. She makes 8 charms on each of the first 3 days. How many charms does Jody make on the fourth day? | $36 \times 3 - c = 8$ |
| Mick buys 3 packages of cookies that each have 36 cookies. After each student in his grade eats one cookie, there are 8 cookies left. How many cookies did the students eat? | $36 - (8 + c) = 3$ |
| Mr. Bing has 36 dollars to spend at a cheese store. He buys some cheese and a slicer and has 3 dollars left. The slicer costs 8 dollars. How much does the cheese cost? | $8 \times 3 + c = 36$ |

- Ask students to write an equation with a letter as a variable to represent a two-step word problem. For example, give students this problem: "Nick sells candy bars to raise money for his club. He has a total of 72 candy bars to sell. He sells 16 candy bars on Monday and 23 candy bars on Tuesday. How many candy bars does Nick have remaining?" Two possible equations include $72 = 16 + 23 + c$ and $72 - 16 - 23 = c$.

- Ask students to evaluate expressions using the order of operations. For example, give students the following expressions.
  - $3 \times 9 + 7$

    34
  - $18 - 3 \times 5$

    3
  - $48 \div 8 + 6$

    12

- Ask students to write and solve a two-step equation that could be used to model a given situation. For example, give students the following problems.

  - Tyler scores 7 points in each of the first 3 quarters of his basketball game and 4 points in the last quarter. How many total points does he score in the game?

    $$7 \times 3 + 4 = p$$

    The order of operations indicates that multiplication should be done before addition, so start with $7 \times 3$, which is 21. Then, add $21 + 4$ to get 25. The solution is $p = 25$ points.

  - Maureen collects 38 rocks while on vacation. She gives 8 rocks to her friend and then divides the rest of the rocks equally into 6 buckets. How many rocks are in each bucket?

    $$(38 - 8) \div 6 = r$$

    The parentheses must be evaluated first. Since $38 - 8 = 30$, the equation is now $30 \div 6 = r$. This can be rewritten as a related fact using multiplication, giving the equation $30 = 6 \times r$. The number that multiplies by 6 to get 30 is 5. The solution is 5 rocks in each bucket.

- Ask students to match a word problem to an estimation equation that would help them estimate the answer.

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Malia has 30 colored pencils that she divides equally into a pencil holder and a pencil box. She then takes 3 pencils out of the pencil box. How many pencils are left in the pencil box?

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Operations and Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Use operations, patterns, and estimation strategies to solve problems (may include word problems).

## ► Eligible Content

B-O.3.1.1 Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.2 Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.3 Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers.

B-O.3.1.4 Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols).

## ► Instructional Supports

- Ask students to study a two-step word problem and the associated equation and identify what the letter represents in each problem. For example, give students the equation $3 \times 8 \div f = 6$ and this problem: “Julie has 3 bunches of flowers with 8 flowers in each bunch. She puts the flowers into 6 different vases. She puts the same number of flowers in each vase. How many flowers are in each vase?” The variable $f$ represents the number of flowers in each vase.
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- Ask students to evaluate expressions using the order of operations. For example, give students the following expressions.

    - $3 \times 9 + 7$

      34

    - $18 – 3 \times 5$

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    - $48 \div 8 + 6$

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- Ask students to use estimation to determine whether the answer provided for a word problem is most likely correct or incorrect. Then ask students to solve the problem to find the actual correct answer. For example, give students this problem: “It takes Jack 9 minutes to walk to the bus stop. He walked to the bus stop 4 days last week. On the fifth day, Jack walked all the way to school, which took 32 minutes. What is the total number of minutes Jack spent walking to get to school last week?” Give students the provided answer of 68 minutes. A possible student response is shown.

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## ► Identifying and Addressing Common Misconceptions

- Some students think that the letter that is used as a variable makes a difference in an equation. Ask students whether either of the equations $16 \times f + 3 = 35$ or $16 \times g + 3 = 35$ can be used to represent this situation: “Amir has 16 small marbles that all have the same mass and a larger marble that has a mass of 3 grams. All his marbles have a total mass of 35 grams. What is the mass of each small marble?” Some students may say that only the second equation is correct because $g$ represents grams. Others may say that the variable needs to be an $m$ for marble or an $s$ for small marble. Remind students of problems they have solved in which a box, circle, or question mark was used to represent the unknown. Discuss why using a different symbol does not change the validity of the equation and help students become comfortable with using many different symbols to indicate unknowns. Discuss why using $s$, $m$, or $g$ may be more helpful than using $f$ but not any more valid.

- Some students may think that the value of a variable is connected to the order of the letters in the alphabet. Give students the information that the value of $c$ is 5 and ask them what, if anything, can be concluded about $d$. Some students may think that $d = 6$ since $d$ is one letter after $c$ or that $d$ must be greater than $c$ because it is later in the alphabet. Give students the same questions using an empty box and a circle to represent the unknowns and help them conclude that the second unknown could be any number regardless of what is known about the first variable. Help students understand that using letters as unknowns works in the same manner.

## ► Enrichment

- Introduce students to the idea of a variable as an unknown that can represent multiple values rather than just one value. For example, give students a paper ruler with the first three inches cut off and the expression $m - 3$. Help students measure various objects using the shortened ruler and recognize that 3 inches can be subtracted from any measurement shown on the ruler to find the true length of the object. Discuss how the expression fits the situation and how the use of the variable is different from solving equations.
- Ask students to solve simple equations with one or two unknowns. It may be helpful to use shapes instead of letters as the unknowns if students are more accustomed to writing their answer in a box, circle, or some other shape. For example, ask students to find the missing numbers in the following equation.

  $$\Delta + \Delta + \Delta = 21$$

  Emphasize that all the triangles must have the same value.

  For the equation below with two unknowns, ask students to find several pairs of solutions that solve the equation. In this situation, the numbers represented by the triangle and square can change. However, if one number changes, the other number must also change.

  $$(\Delta - \square) + 5 = 8$$

- Ask students to determine upper and lower bounds on non-whole number solutions to two-step problems. For example, Sandra stacks five identical blocks on top of a 1-inch book to a total height of 13 inches. Ask students how tall the blocks are. While students may not have enough experience with fractions and decimals to calculate the exact answer, guide them to recognize that five 2-inch blocks would make a height of 11 inches because 10 + 1 = 11, so the blocks Sandra uses must be greater than 2 inches tall. However, five 3-inch blocks would make a height of 16 inches because 15 + 1 = 16. Therefore, the blocks Sandra uses must be greater than 2 inches and less than 3 inches tall.

## ► Key Terms:

equation, estimation, expression, mental computation, order of operations, reasonableness, rounding, two-step problem, unknown quantity, variable

# B-O.3.1.1, B-O.3.1.2, B-O.3.1.3, & B-O.3.1.4

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Use operations, patterns, and estimation strategies to solve problems (may include word problems).

## ► Eligible Content

B-O.3.1.1 Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.2 Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers.

B-O.3.1.3 Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers.

B-O.3.1.4 Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols).

## ► Instructional Supports

- Ask students to study a two-step word problem and the associated equation and identify what the letter represents in each problem. For example, give students the equation $3 \times 8 \div f = 6$ and this problem: “Julie has 3 bunches of flowers with 8 flowers in each bunch. She puts the flowers into 6 different vases. She puts the same number of flowers in each vase. How many flowers are in each vase?” The variable $f$ represents the number of flowers in each vase.
- Ask students to match an equation with the two-step word problem that it represents. For example, ask students to match the following three word problems and equations.

| | |
|---|---|
| Jody makes a total of 36 charms in 4 days. She makes 8 charms on each of the first 3 days. How many charms does Jody make on the fourth day? | $36 \times 3 - c = 8$ |
| Mick buys 3 packages of cookies that each have 36 cookies. After each student in his grade eats one cookie, there are 8 cookies left. How many cookies did the students eat? | $36 - (8 + c) = 3$ |
| Mr. Bing has 36 dollars to spend at a cheese store. He buys some cheese and a slicer and has 3 dollars left. The slicer costs 8 dollars. How much does the cheese cost? | $8 \times 3 + c = 36$ |

- Ask students to write an equation with a letter as a variable to represent a two-step word problem. For example, give students this problem: "Nick sells candy bars to raise money for his club. He has a total of 72 candy bars to sell. He sells 16 candy bars on Monday and 23 candy bars on Tuesday. How many candy bars does Nick have remaining?" Two possible equations include $72 = 16 + 23 + c$ and $72 - 16 - 23 = c$.

- Ask students to evaluate expressions using the order of operations. For example, give students the following expressions.
    - $3 \times 9 + 7$

      34

    - $18 - 3 \times 5$

      3

    - $48 \div 8 + 6$

      12

- Ask students to write and solve a two-step equation that could be used to model a given situation. For example, give students the following problems.

- Tyler scores 7 points in each of the first 3 quarters of his basketball game and 4 points in the last quarter. How many total points does he score in the game?

  $$7 \times 3 + 4 = p$$

  The order of operations indicates that multiplication should be done before addition, so start with $7 \times 3$, which is 21. Then, add $21 + 4$ to get 25. The solution is $p = 25$ points.

- Maureen collects 38 rocks while on vacation. She gives 8 rocks to her friend and then divides the rest of the rocks equally into 6 buckets. How many rocks are in each bucket?

  $$(38 - 8) \div 6 = r$$

  The parentheses must be evaluated first. Since $38 - 8 = 30$, the equation is now $30 \div 6 = r$. This can be rewritten as a related fact using multiplication, giving the equation $30 = 6 \times r$. The number that multiplies by 6 to get 30 is 5. The solution is 5 rocks in each bucket.

- Ask students to match a word problem to an estimation equation that would help them estimate the answer.

| | |
|---|---|
| Joni gives her dog 3 snacks and 2 meals every day. How many total times does Joni feed her dog in 30 days? | $30 \div 2 - 3 = 12$ |
| Harry has a total of 30 marbles. He gives 2 marbles to his sister and 3 marbles to his brother. How many marbles does Harry have left? | $30 - (2 + 3) = 25$ |
| Malia has 30 colored pencils that she divides equally into a pencil holder and a pencil box. She then takes 3 pencils out of the pencil box. How many pencils are left in the pencil box? | $(3 + 2) \times 30 = 150$ |

- Ask students to use estimation to determine whether the answer provided for a word problem is most likely correct or incorrect. Then ask students to solve the problem to find the actual correct answer. For example, give students this problem: “It takes Jack 9 minutes to walk to the bus stop. He walked to the bus stop 4 days last week. On the fifth day, Jack walked all the way to school, which took 32 minutes. What is the total number of minutes Jack spent walking to get to school last week?” Give students the provided answer of 68 minutes. A possible student response is shown.

To estimate, round 9 minutes to 10 and 32 minutes to 30.

$$10 \times 4 + 30 = 70$$

The answer is most likely correct since 68 is close to 70.

## ► Identifying and Addressing Common Misconceptions

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## ▶ Enrichment

- Introduce students to the idea of a variable as an unknown that can represent multiple values rather than just one value. For example, give students a paper ruler with the first three inches cut off and the expression $m - 3$. Help students measure various objects using the shortened ruler and recognize that 3 inches can be subtracted from any measurement shown on the ruler to find the true length of the object. Discuss how the expression fits the situation and how the use of the variable is different from solving equations.
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  $$\Delta + \Delta + \Delta = 21$$

  Emphasize that all the triangles must have the same value.

  For the equation below with two unknowns, ask students to find several pairs of solutions that solve the equation. In this situation, the numbers represented by the triangle and square can change. However, if one number changes, the other number must also change.

  $$(\Delta - \square) + 5 = 8$$

- Ask students to determine upper and lower bounds on non-whole number solutions to two-step problems. For example, Sandra stacks five identical blocks on top of a 1-inch book to a total height of 13 inches. Ask students how tall the blocks are. While students may not have enough experience with fractions and decimals to calculate the exact answer, guide them to recognize that five 2-inch blocks would make a height of 11 inches because $10 + 1 = 11$, so the blocks Sandra uses must be greater than 2 inches tall. However, five 3-inch blocks would make a height of 16 inches because $15 + 1 = 16$. Therefore, the blocks Sandra uses must be greater than 2 inches and less than 3 inches tall.

## ► Key Terms:

equation, estimation, expression, mental computation, order of operations, reasonableness, rounding, two-step problem, unknown quantity, variable

# B-O.3.1.5

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Operations and Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Use operations, patterns, and estimation strategies to solve problems (may include word problems).

## ▶ Eligible Content

B-O.3.1.5 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and/or explain them using properties of operations.

## ▶ Instructional Supports

**Sequences**

- Remind students that when consecutive terms in a sequence always differ by the same amount, an arithmetic pattern is formed. For example, in the sequence 2, 7, 12, 17, . . ., each term in the sequence differs from the previous term by 5. Therefore, it has an arithmetic pattern.
- Ask students to determine whether a given sequence has an arithmetic pattern and explain how they determined their answer. For example, give students the following sequence.

  2, 4, 7, 10, 13, . . .

  Students should find that the difference between the first two terms of the sequence is 2. They should also notice that the difference between the remaining consecutive terms of the sequence is 3. Students should determine that because the difference between consecutive terms is not a constant sum or multiple, this is not an arithmetic pattern.
- Ask students to find a missing number in a given arithmetic sequence and explain how they determined their answer. For example, give students the following sequence and ask them to find the term that belongs in the blank.

  5, 9, 13, 17, ____

Students should find that each number in the sequence is 4 more than the previous number, and therefore they can determine that the missing number in the sequence is 21.

- Ask students to write the first five numbers in a sequence when given a starting number and a rule for the sequence. For example, give students a starting value of 3 and the rule "multiply by 2." Students should begin with the first number in the sequence, 3. They should then multiply this number by 2 in order to find the second number in the sequence, which is 6. Students can repeat this again, multiplying 6 by 2 to get 12, which is the third number in the sequence. This process can be repeated two more times to find the fourth and fifth values of 24 and 48.
- Ask students to identify the starting number and the rule for a given sequence. For example, give students the sequence 17, 15, 13, 11, 9, . . . . Students should identify the starting value of the sequence as 17. Then, students should find that each number in the sequence is 2 less than the preceding number. Therefore, they should determine that the rule is to subtract 2.

**Patterns of arithmetic**

- Ask students to find patterns in multiples of numbers in a 10 by 10 multiplication table and explain the patterns using properties of operations. For example, ask students to circle all multiples of 5 on a 10 by 10 multiplication table and describe patterns in the circled numbers. One possible pattern students may identify is that all multiples of 5 end in either 0 or 5, while multiples of all the other odd numbers can end in a variety of digits. Some other patterns students may notice for other sets of multiples are shown.
    - The digits in multiples of 3 add up to a multiple of 3.
    - A number multiplied by 1 is always the number itself.
    - The tens digits of multiples of 9 always decrease by 1, while the ones digits increase by 1.
    - In each diagonal going up and to the right, the largest numbers are always in the middle of the diagonal.

- Give students an addition table and ask them to identify any numeric or visual patterns they observe and explain the patterns using properties of operations. Some possible patterns students may notice are shown.
    - Even and odd numbers form a checkerboard pattern.
    - An even number plus an even number is always even.
    - Each diagonal going up and to the right contains only one number.
    - Each diagonal going down and to the right counts by twos.

## ► Identifying and Addressing Common Misconceptions

- Some students may confuse the starting number of a sequence with a rule for adding. Ask students to identify the rule for the following sequence.

  3, 5, 7, 9, 11, 13, . . .

  Some students may think that all sequences start at zero and state that the rule is "add 3." Ask students to identify the first two terms and find the difference between them. If necessary, remind them that sequences do not necessarily have to start with zero.

- Some students may not be certain whether a pattern observed in a small sample is guaranteed to always be true. Ask students how many terms of a pattern must be observed in order to be certain the pattern continues. Students may give a variety of answers. Help students understand that patterns that are created using a given rule are guaranteed to continue. For example, the sequence of numbers created using a starting value of 8 and a rule of "add 5" is certain to always follow the rule because it has been defined that way. However, some patterns that are observed in a sample may not always be true. For example, students may observe on a 10 by 10 multiplication table that the digits of multiples of 9 always sum to 9. However, looking beyond those bounds shows that neither $0 \times 9$ nor $11 \times 9$ have digits that sum to 9.

## ► Enrichment

- Ask students to identify rules for sequences that follow a 2-step rule. For example, give students the following sequence.

  2, 5, 11, 23, 47, . . .

  Students should determine that the sequence has a starting value of 2 and the rule is to multiply by 2 and add 1.
- Ask students to work backwards in a sequence to determine a missing starting number.
- Ask students to construct and find patterns in a 100's table (numbers 1–100 with 10 numbers in each row) and explain the patterns using properties of operations.

## ► Key Terms:

addition table, arithmetic pattern, consecutive, decompose, multiplication table, sequence, starting value, term

# C-G.1.1.1 & C-G.1.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Reason with shapes and their attributes.

Analyze characteristics of polygons.

## ► Eligible Content

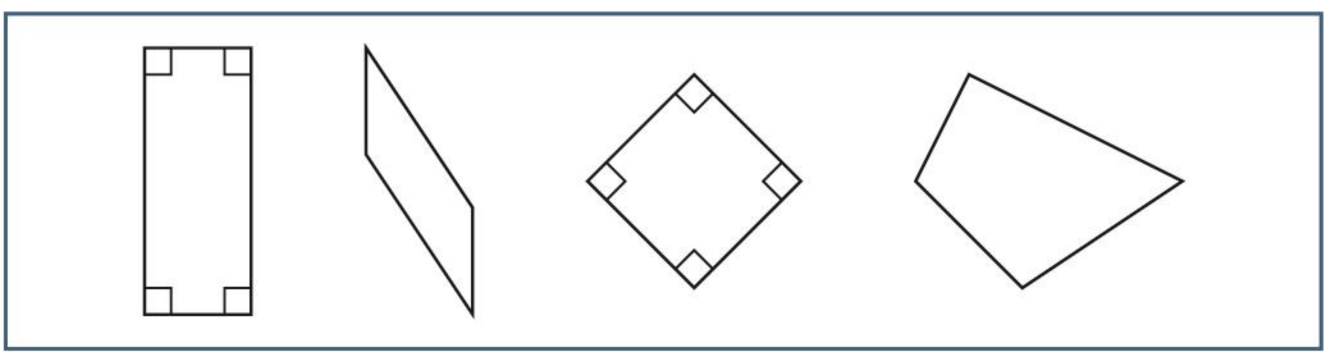
C-G.1.1.1 Explain that shapes in different categories may share attributes, and that the shared attributes can define a larger category.

C-G.1.1.2 Recognize rhombi, rectangles, and squares as examples of quadrilaterals, and/or draw examples of quadrilaterals that do not belong to any of these subcategories.

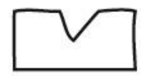
## ► Instructional Supports

**Polygons**

- Ask students to classify a group of polygons based on a common characteristic of those polygons. For example, the polygons shown can all be classified as quadrilaterals since the shapes each have exactly four sides.

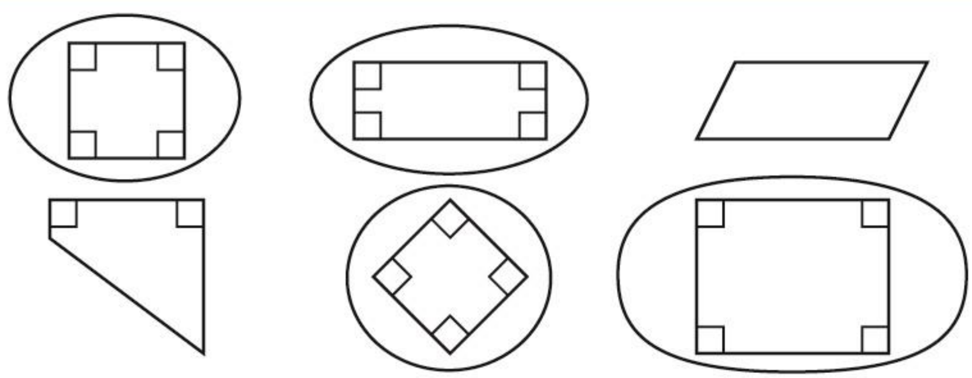


- Ask students to create polygons that include or do not include certain characteristics. For example, give students a table that has a row for polygons that have 4 right angles and a row for polygons that do not and columns for quadrilaterals and not quadrilaterals. Have students create a polygon for each section that fits the description of its row and column. A possible student example is shown.

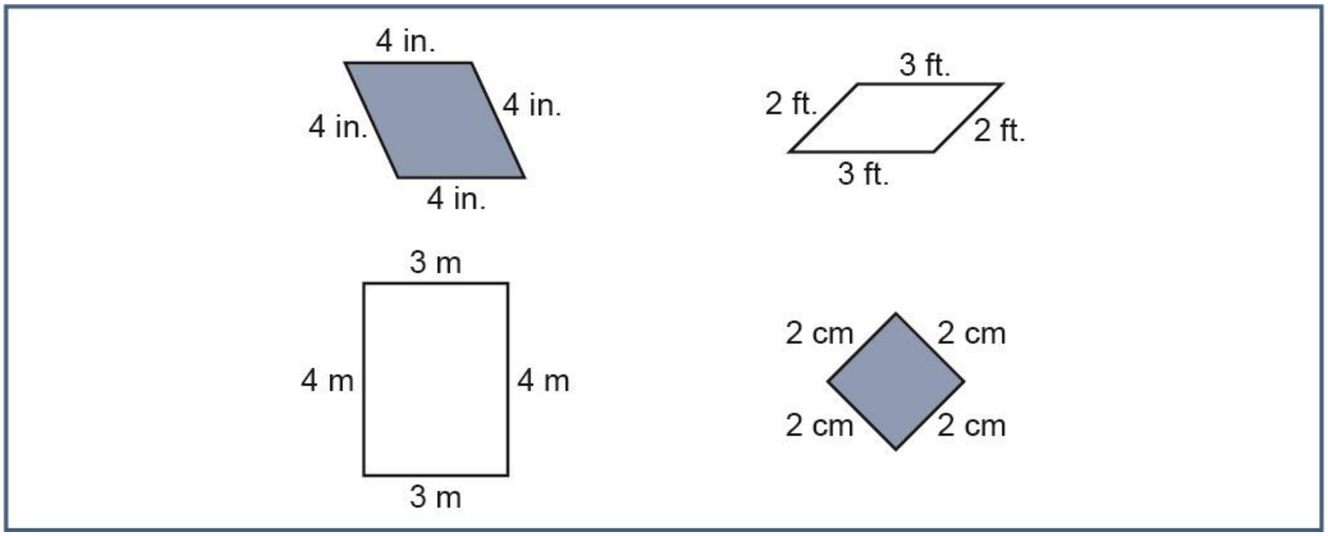
| | Quadrilateral | Not a Quadrilateral |
|---|---|---|
| Has 4 right angles |  |  |
| Does not have 4 right angles |  |  |

**Quadrilaterals**

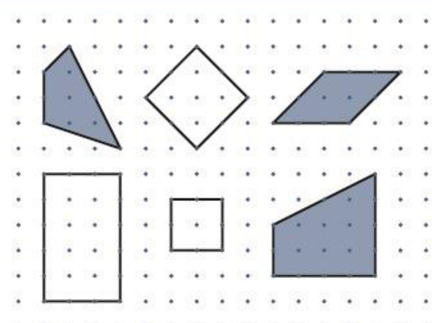
- Ask students to select the rectangles from a set of figures. For example, give students a group of quadrilaterals and ask students to circle or shade the shapes with four right angles. The remaining shapes are quadrilaterals but not rectangles. A possible student example is shown.



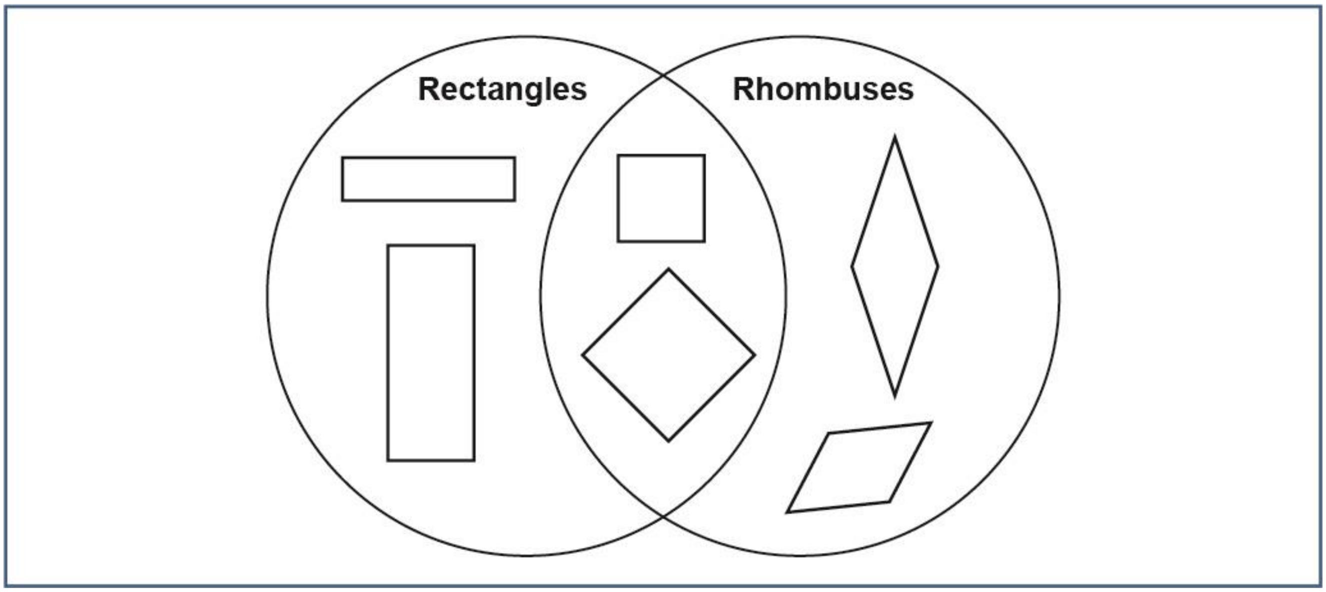
- Ask students to select all rhombuses from a group of quadrilaterals. For example, give students a group of quadrilaterals and ask students to shade or circle the shapes with four equal sides. The remaining shapes are quadrilaterals but not rhombuses. A possible student example is shown.




- Ask students to use grid paper or dot paper to create figures with four sides and recognize the figures that do not fit a certain classification. For example, ask students to draw 6 quadrilaterals and then shade the shapes that are not rectangles as shown.



- Ask students to give the defining criteria of certain quadrilaterals. For example, ask students to name the two characteristics that determine whether a figure is a rectangle. Students respond that the shape must be a quadrilateral (have four sides) and have four right angles. Any figure that fits both of the stated criteria is a rectangle.
- Ask students to recognize that squares fit the classifications of both rhombuses and rectangles. For example, ask students to place quadrilaterals in the following Venn diagram. Help students recognize that squares belong inside both circles.

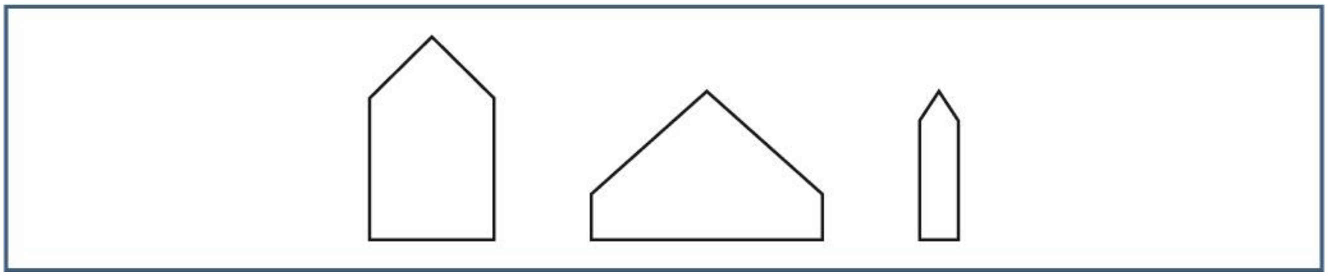



## ► Identifying and Addressing Common Misconceptions

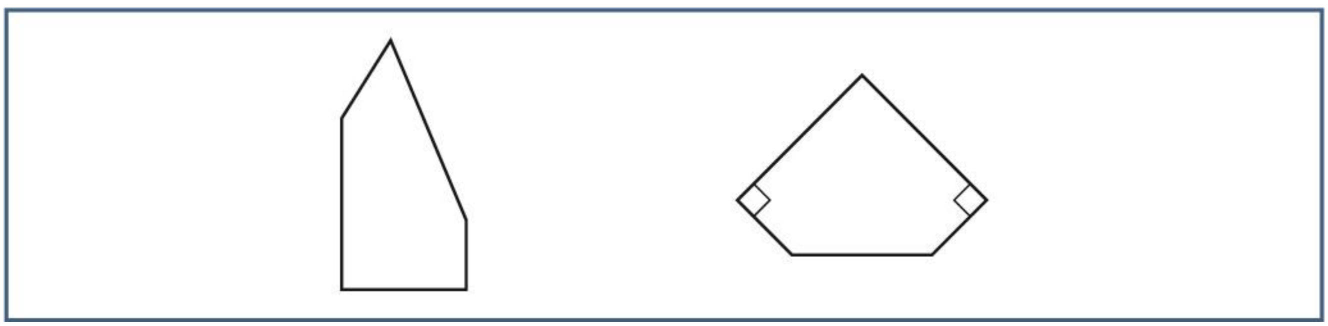
- Students may not think that squares can be classified as rectangles and as rhombuses. Give students several shapes and ask them to circle all the rectangles (or rhombuses). Some students may not circle the squares. Ask students to explain what characteristics allow them to identify a shape as a rectangle (or rhombus). Aside from having four sides, the other defining characteristic is having four right angles (or four equal sides). Then ask students whether the square has those characteristics. Inform students that a square is a special type of rectangle because it has four right angles and that a square is also a special type of rhombus because it has four equal sides.
- Students may think that the only kind of pentagon is a regular pentagon. Give students several shapes and ask them to identify all the pentagons. Some students may not recognize non-regular pentagons to be pentagons. Define a pentagon for students and have them draw different pentagons, including ones with different side lengths and ones that are concave (define concave if necessary).
- Students may categorize all triangles as acute triangles because all triangles have at least one acute angle. Give students several triangles and ask them to classify them by their angle measures. Then ask them to color or highlight all the acute angles. Because every triangle always has at least one acute angle, the presence of an acute angle is not a very useful way of categorizing triangles. In order to be an acute triangle, a triangle must have three acute angles.

## ► Enrichment

- Give students a set of shapes that have common characteristics. Ask students to identify the characteristics they believe were used to create the group and give the group a name. Then ask students to find shapes that may challenge those definitions. For example, students may suggest the defining characteristic of the pentagons shown is that they have two right angles and name their category "houses."



Some pentagons that might challenge that definition are shown.



Help students refine their criteria in order to define their shapes as accurately as possible.

- Ask students to create quadrilaterals with two pairs of equal sides and discuss why it may not be useful to have a single term to describe these quadrilaterals. Point out to students that figures in which the equal sides are adjacent have patterns that are different from figures in which the equal sides are not adjacent. Therefore, it is more useful to have two separate terms for the two groups.

- Give students a set of characteristics and ask them to draw polygons that fit the given characteristics. For example, ask students to draw four hexagons that are concave, one hexagon with exactly 2 right angles, one hexagon with exactly 3 right angles, one hexagon with exactly 4 right angles, and one hexagon with exactly 6 right angles.

- Ask students to characterize the association between rectangles and squares using an analogy or another example. For example, a greyhound is a dog, just like a square is a rectangle, but the definition of a greyhound is more specific than the definition of a dog because a greyhound is a specific type of dog. Similarly, the definition of a square is more specific than the definition of a rectangle because a square is a specific type of rectangle.

## ► Key Terms:

angle, attributes, category, opposite sides, parallelogram, quadrilateral, rectangle, rhombus, shape, square, subcategory, trapezoid

# C-G.1.1.1 & C-G.1.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Reason with shapes and their attributes.

Analyze characteristics of polygons.

## ► Eligible Content

C-G.1.1.1 Explain that shapes in different categories may share attributes, and that the shared attributes can define a larger category.

C-G.1.1.2 Recognize rhombi, rectangles, and squares as examples of quadrilaterals, and/or draw examples of quadrilaterals that do not belong to any of these subcategories.

## ► Instructional Supports

**Polygons**

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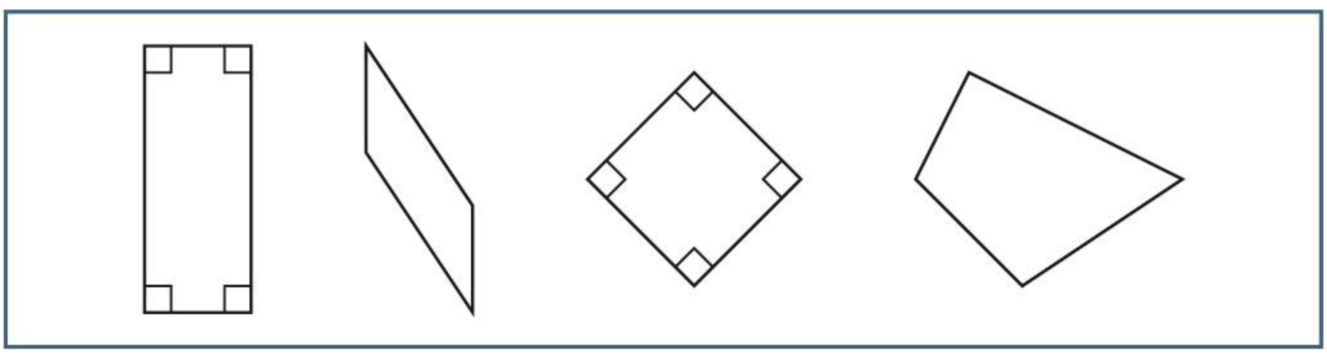

- Ask students to create polygons that include or do not include certain characteristics. For example, give students a table that has a row for polygons that have 4 right angles and a row for polygons that do not and columns for quadrilaterals and not quadrilaterals. Have students create a polygon for each section that fits the description of its row and column. A possible student example is shown.

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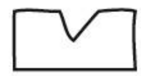
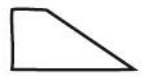
| Has 4 right angles |  |  |
| Does not have 4 right angles |  |  |

**Quadrilaterals**

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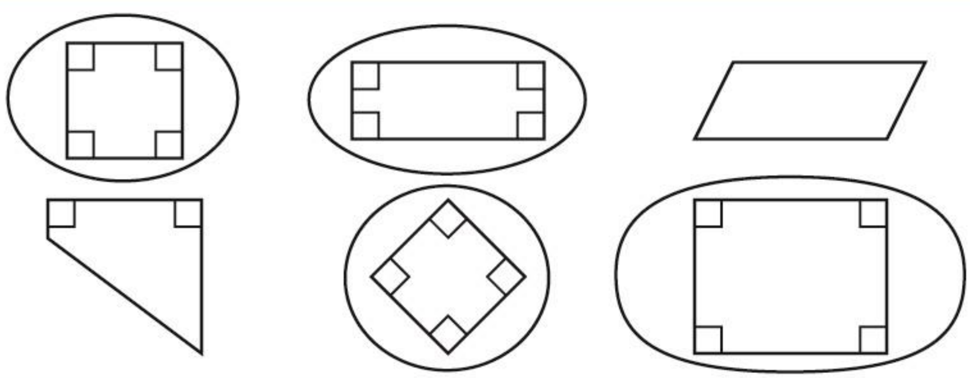

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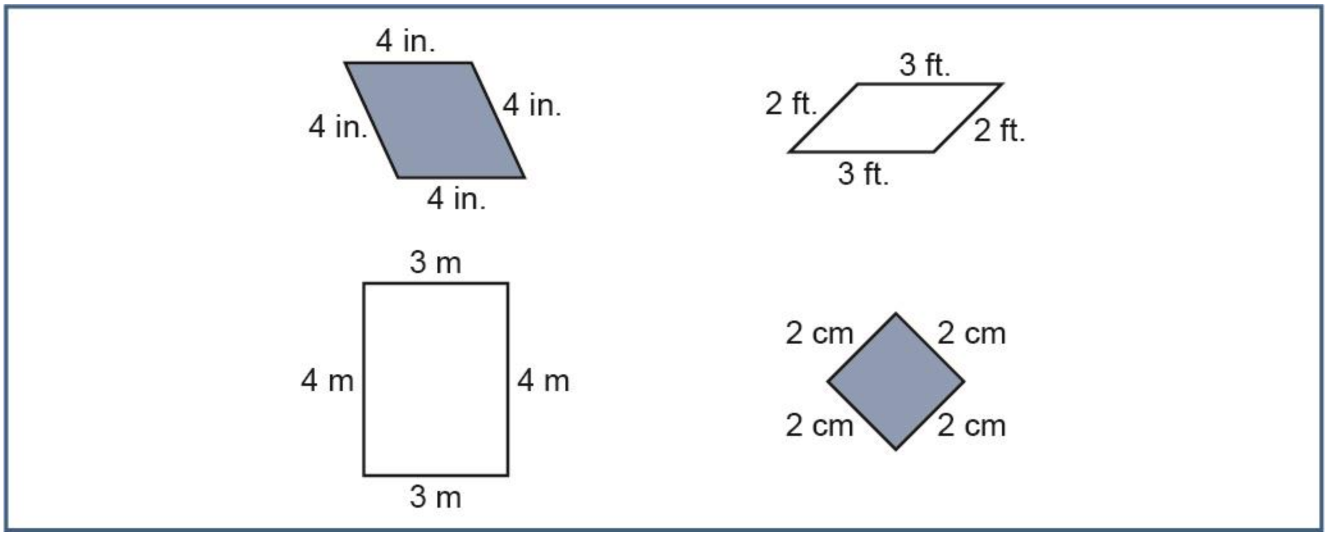


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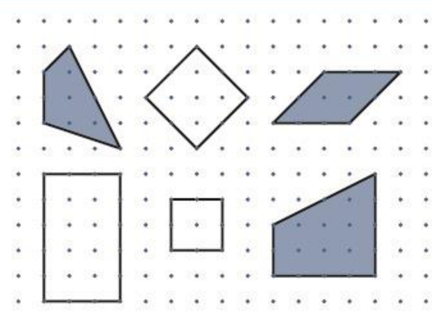

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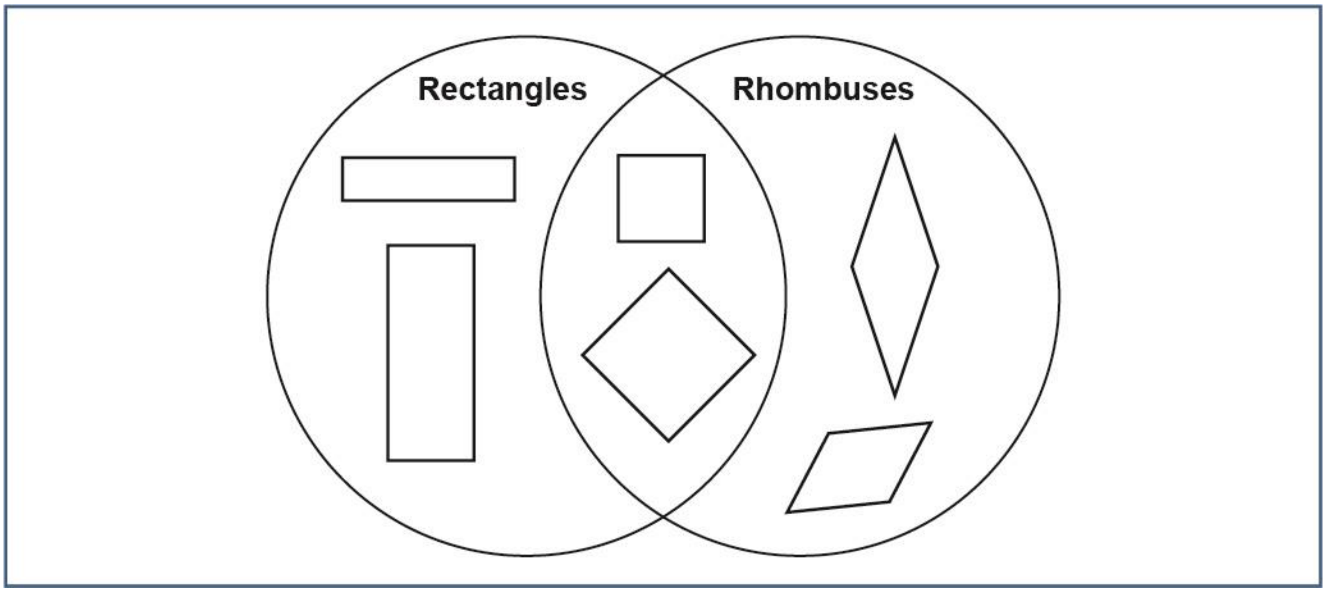
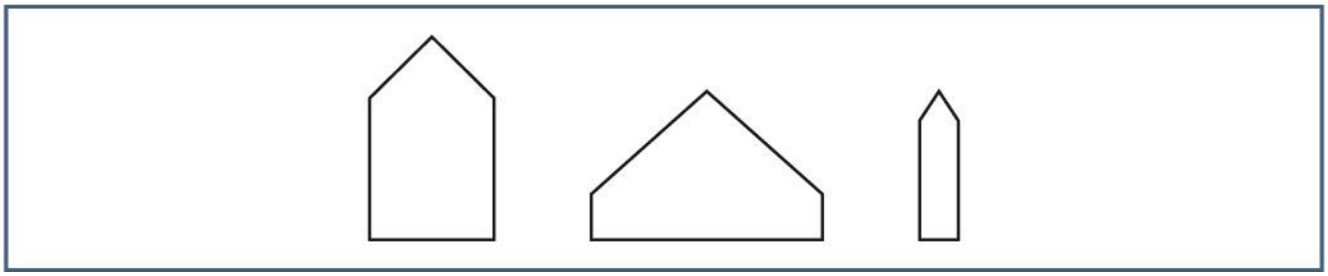



## ► Identifying and Addressing Common Misconceptions

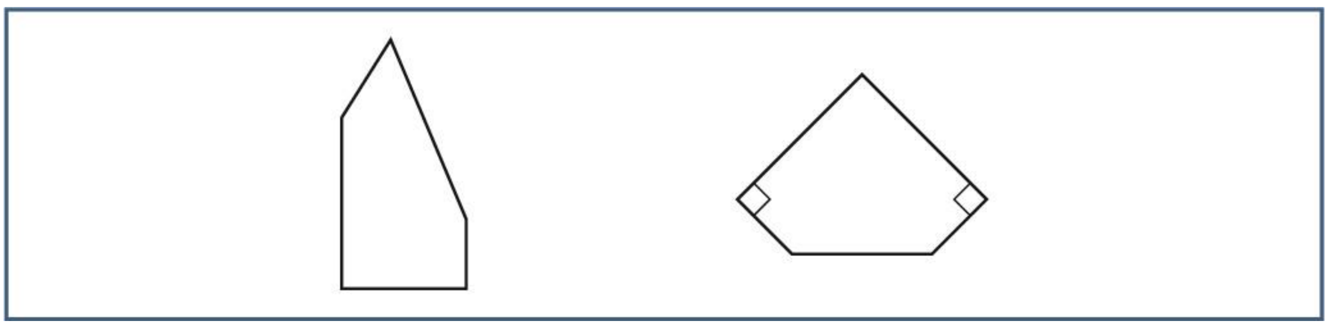
- Students may not think that squares can be classified as rectangles and as rhombuses. Give students several shapes and ask them to circle all the rectangles (or rhombuses). Some students may not circle the squares. Ask students to explain what characteristics allow them to identify a shape as a rectangle (or rhombus). Aside from having four sides, the other defining characteristic is having four right angles (or four equal sides). Then ask students whether the square has those characteristics. Inform students that a square is a special type of rectangle because it has four right angles and that a square is also a special type of rhombus because it has four equal sides.
- Students may think that the only kind of pentagon is a regular pentagon. Give students several shapes and ask them to identify all the pentagons. Some students may not recognize non-regular pentagons to be pentagons. Define a pentagon for students and have them draw different pentagons, including ones with different side lengths and ones that are concave (define concave if necessary).
- Students may categorize all triangles as acute triangles because all triangles have at least one acute angle. Give students several triangles and ask them to classify them by their angle measures. Then ask them to color or highlight all the acute angles. Because every triangle always has at least one acute angle, the presence of an acute angle is not a very useful way of categorizing triangles. In order to be an acute triangle, a triangle must have three acute angles.

## ► Enrichment

- Give students a set of shapes that have common characteristics. Ask students to identify the characteristics they believe were used to create the group and give the group a name. Then ask students to find shapes that may challenge those definitions. For example, students may suggest the defining characteristic of the pentagons shown is that they have two right angles and name their category “houses.”



Some pentagons that might challenge that definition are shown.



Help students refine their criteria in order to define their shapes as accurately as possible.

- Ask students to create quadrilaterals with two pairs of equal sides and discuss why it may not be useful to have a single term to describe these quadrilaterals. Point out to students that figures in which the equal sides are adjacent have patterns that are different from figures in which the equal sides are not adjacent. Therefore, it is more useful to have two separate terms for the two groups.

- Give students a set of characteristics and ask them to draw polygons that fit the given characteristics. For example, ask students to draw four hexagons that are concave, one hexagon with exactly 2 right angles, one hexagon with exactly 3 right angles, one hexagon with exactly 4 right angles, and one hexagon with exactly 6 right angles.

- Ask students to characterize the association between rectangles and squares using an analogy or another example. For example, a greyhound is a dog, just like a square is a rectangle, but the definition of a greyhound is more specific than the definition of a dog because a greyhound is a specific type of dog. Similarly, the definition of a square is more specific than the definition of a rectangle because a square is a specific type of rectangle.

## ► Key Terms:

angle, attributes, category, opposite sides, parallelogram, quadrilateral, rectangle, rhombus, shape, square, subcategory, trapezoid

# C-G.1.1.3

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Geometry

Reason with shapes and their attributes.

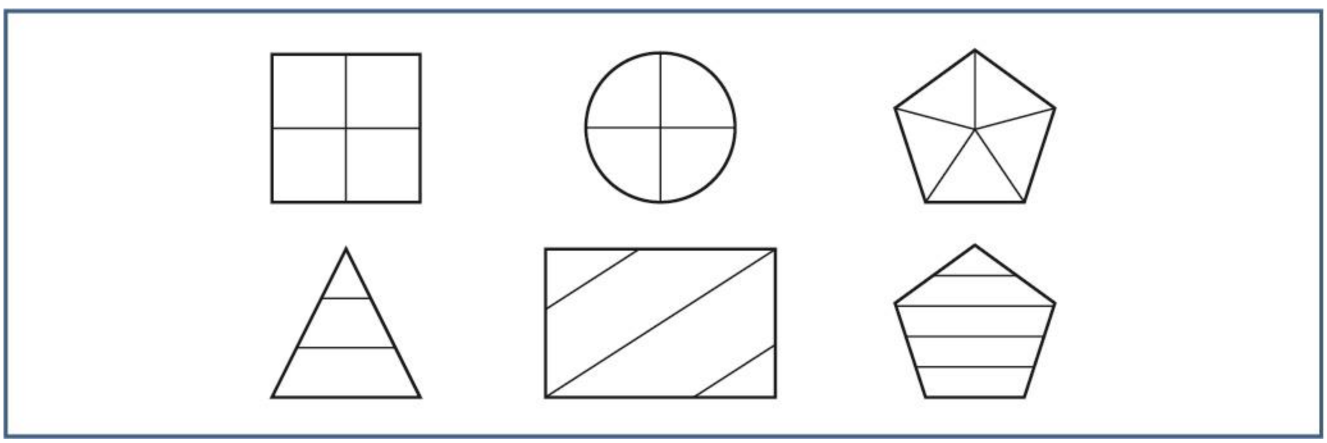
Analyze characteristics of polygons.

## ► Eligible Content

C-G.1.1.3 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

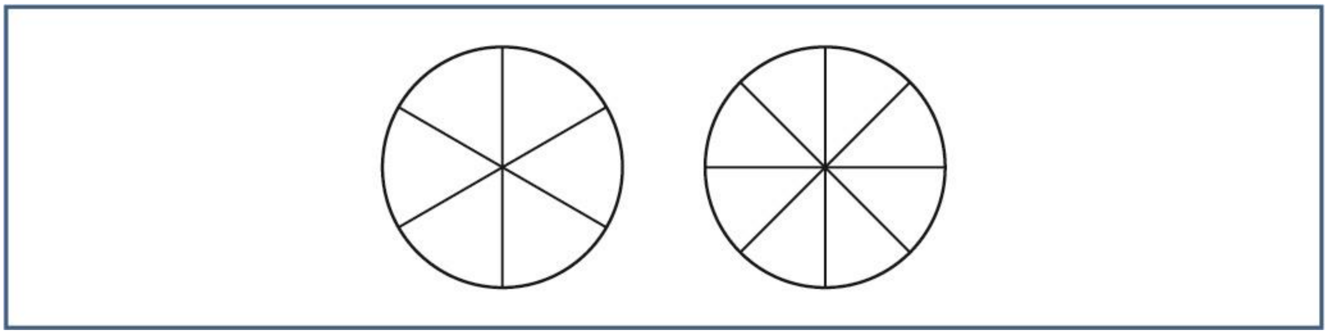
## ► Instructional Supports

- Ask students to determine which shapes have been partitioned into parts with equal areas and which shapes have not. For example, give students the following shapes.

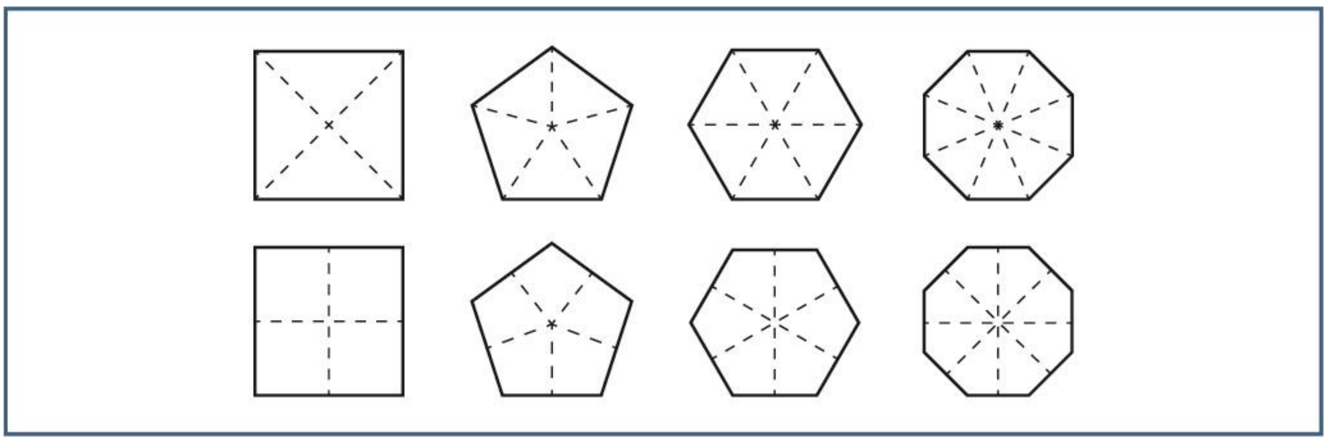


The shapes in the first row have been partitioned into parts with equal areas, while the shapes in the second row have not.

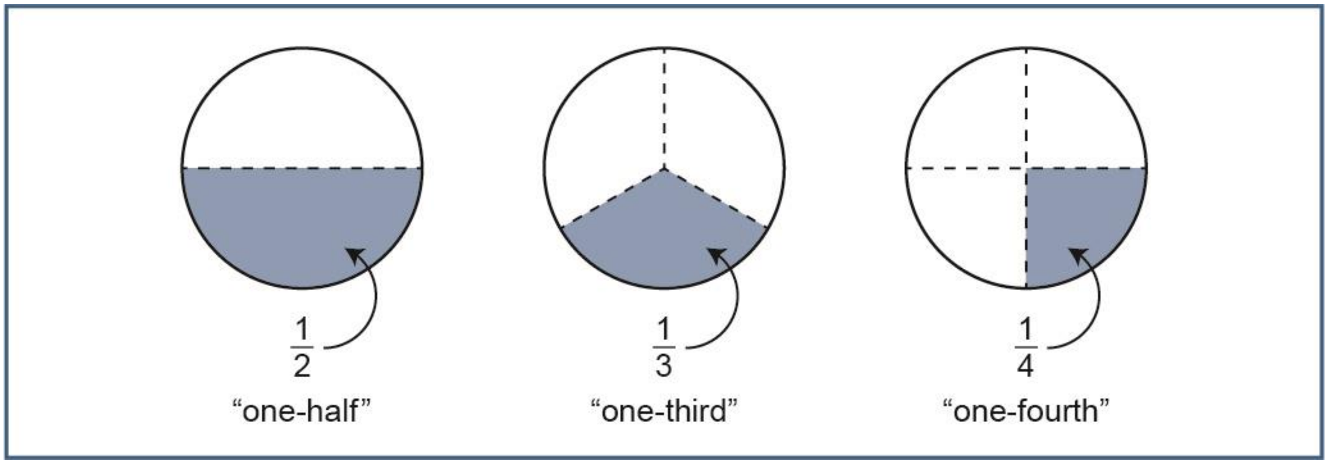
- Ask students to partition a shape into different numbers of parts, each of which has an equal area. For example, ask students to partition circles into 6 or 8 equal parts. Possible responses are shown.



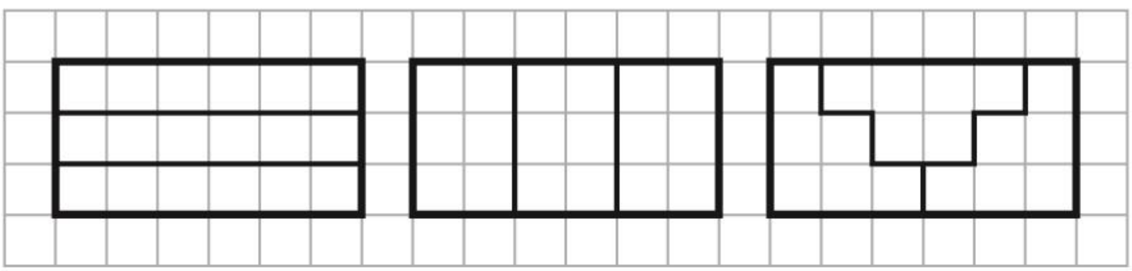
- Ask students to identify two different ways to partition regular polygons into equal parts, both using the center of the polygon as a starting point. Ask students to describe the resulting shapes depending on the method used to partition the polygon. For example, students may note that when connecting the center to the vertices, the result is a number of triangles equal to the number of sides the polygon has. Likewise, students may note that when connecting the center to the midpoint of the sides, the result is a number of quadrilaterals equal to the number of sides the polygon has.



- Give students a shape that has been partitioned into equal parts and ask them to write unit fractions to express the area of a single part in relation to the area of the whole figure. Students should write the unit fractions and also recognize how to read the unit fractions using the proper wording. For example, give students the following figures. Students should recognize that the areas of the single parts are $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of the area of an entire circle.




- Ask students to partition shapes into parts with equal areas in multiple ways and explain why each of the parts represents the same unit fraction despite having a different shape. For example, give students a rectangle on grid paper and ask them to partition it into three equally sized parts. Some possible partitions are shown.

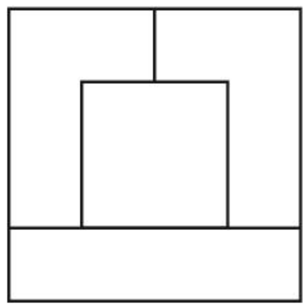


Help students explain that each part represents the same fraction, $\frac{1}{3}$, of the original shape because three copies of the shape can be used to perfectly cover the original shape (possibly with some cutting and rearranging). Confirm that each shape represents the same area by counting the squares on the grid paper.

## ▶ Identifying and Addressing Common Misconceptions

- Some students do not recognize that the partitions of a shape must have equal areas for each partition to represent the same unit fraction. Give students a partition that does not create equal areas and ask them to identify the fraction shown by a single part. Some students may answer as if the parts were equal. Ask students to cut rectangular pieces of paper into 2 different-sized parts and then indicate which of the 2 pieces is larger. Discuss why it does not make sense to give the same numerical value to both pieces when one piece is larger than the other.

- Some students believe that when a shape is partitioned into equal parts, all the parts must be the same shape. Give students the following partition and ask them whether each part has an area that is $\frac{1}{4}$ the area of the entire shape.

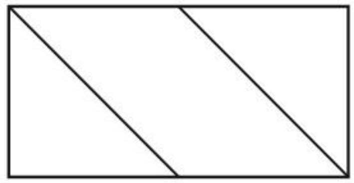


Cut out 4 copies of each shape and show that 4 copies of each shape can be used to fill the entire square. Discuss why this shows that each shape is $\frac{1}{4}$ of the entire figure.

- Some students may describe the area of a single part of an equal partition as a unit fraction using a part-part ratio rather than a part-whole ratio. Give students a figure that is partitioned into 4 equal parts and ask them to identify the fraction that represents a single part. Some students may give the answer of $\frac{1}{3}$ with the 3 representing the other 3 parts of the figure. Remind students that because the fraction describes how each part relates to the whole figure, the denominator must reflect the number of parts in the whole figure.

## ► Enrichment

- Ask students to create equal partitions and use a fraction to describe more than one part of the partition.
- Ask students to divide shapes into unequal areas and describe the area of each shape as greater than or less than the appropriate unit fraction, based on the number of areas into which the shape was divided. For example, give students the following rectangle that has been divided into 3 unequal parts.



Ask students to determine whether each part is greater than or less than $\frac{1}{3}$ of the figure.

- Ask students to combine identical shapes to create new shapes and then describe the area of each individual shape as a fraction of the area of the entire shape. For example, ask students to combine two $1 \times 3$ rectangles into a $2 \times 3$ rectangle in which each small rectangle represents $\frac{1}{2}$ the area of the entire shape. Then ask students to include another $1 \times 3$ rectangle to form a $3 \times 3$ square and note that, because the whole has changed, the partition that used to represent $\frac{1}{2}$ the area now represents $\frac{1}{3}$ the area, even though the size of the partition is unchanged.

## ► Key Terms:

area, decompose, fraction, partition, shape

# D-M.1.1.1 & D-M.1.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Determine or calculate time and elapsed time.

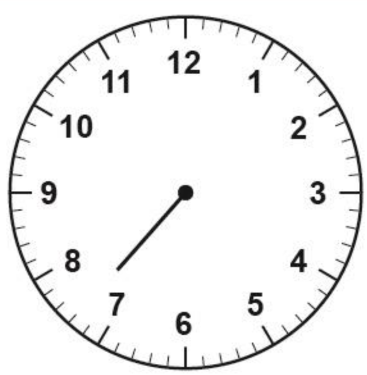
## ► Eligible Content

D-M.1.1.1 Tell, show, and/or write time (analog) to the nearest minute.

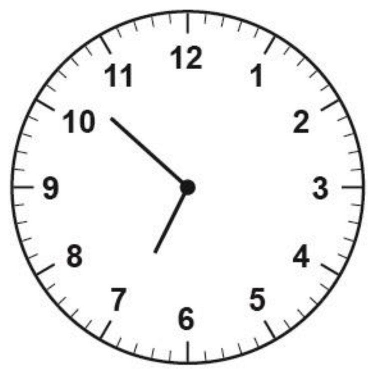
D-M.1.1.2 Calculate elapsed time to the minute in a given situation (total elapsed time limited to 60 minutes or less).

## ► Instructional Supports

- Discuss how to tell time to the nearest minute by drawing on strategies of skip-counting by 5 and adding on any residual minutes that may be left over. Help students recognize that a clock is broken into 5-minute intervals. For example, give students the following situations.
    - Ask students to determine the number of minutes shown by the minute hand of a clock. If the minute hand is pointing 2 tick marks past the 7, that represents 7 groups of 5-minute intervals plus an additional 2 minutes. Skip-counting by 5 seven times gives 35 minutes. Adding on the additional 2 tick marks gives 37 minutes.

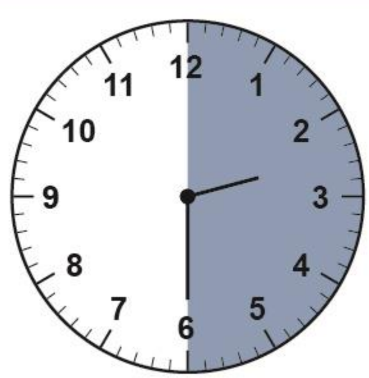


- Ask students to tell time to the nearest minute from a clock face using skip-counting. For example, give students the following picture.

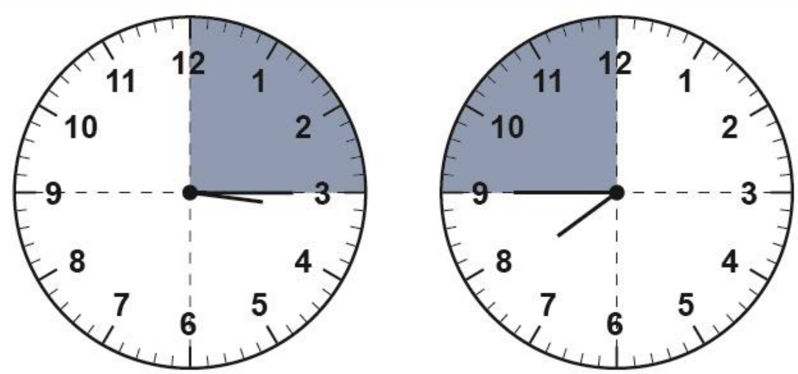



The hour hand is past 6 but not quite to 7, so the hour must be 6. The minute hand is 2 tick marks past 10. Skip-counting by 5 ten times gives 50 minutes. Adding the 2 additional tick marks gives a total of 52 minutes. Therefore, the time shown on the clock is 6:52.

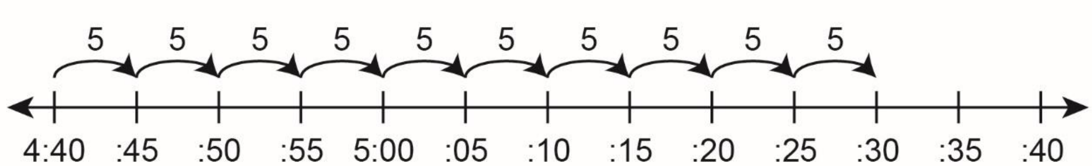
- Ask students to show that a clock can be divided into equal intervals representing $\frac{1}{4}$ or $\frac{1}{2}$ of an hour, just as a circle can be divided into parts equal to $\frac{1}{4}$ or $\frac{1}{2}$ of the circle. These fractional representations of an hour can be used to identify times.
    - In the figure shown, the hour hand is between the 2 and 3, and the minute hand is at the 6, so the time is 2:30. The time can also be read as half past two, because 30 minutes is half of the whole hour.



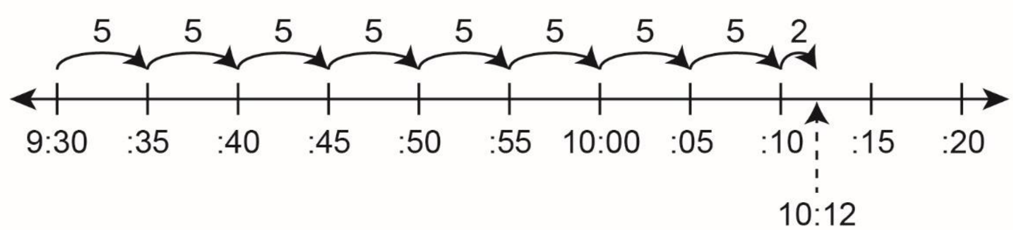
- In the following diagram, both clocks show an hour divided into 4 equal parts, which each correspond to fourths or quarters of the hour. In the clock on the left, the hour hand is between the 3 and 4, so the hour is 3. Because the minute hand started at 12 and moved one quarter of the way around the circle, the time can be read as "one quarter past 3." For the clock on the right, the hour hand is between the 7 and 8, which indicates that the hour is still 7 and not quite 8. The minute hand moved three-quarters of the way toward completing the 7 o'clock hour and reaching 8 o'clock, leaving one quarter of an hour left to go. Therefore, the time can also be read as "one quarter to eight."




- Discuss with students how to use their previous understanding of number line diagrams or tape diagrams to determine the time an event will be completed.
  - Ask students to create modified diagrams where the whole numbers represent hours and each tick mark between the hours represents a 5-minute interval. These diagrams can be used to represent transitions from one hour to the next. For example, if a class begins at 4:40 and ends at 5:30, how many minutes long is the class? If a student creates a number line that starts at 4:40 and skip-counts until they reach 5:30, they could make the diagram shown and conclude that the class would last 50 minutes.

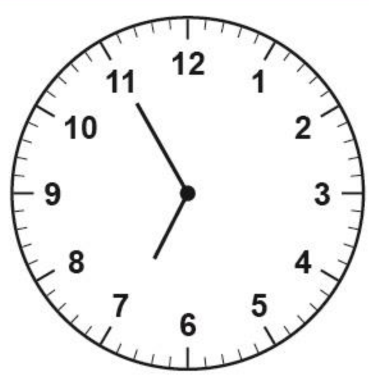


- Ask students to count forward on a number line to determine the ending time of an event. For example, Shane is in a 42-minute class that started at 9:30. The ending time of the class can be determined by skip-counting forward on a number line by increments of 5 minutes from 9:30 until 10:10 and then adding on the remaining 2 minutes to get an end time of 10:12.




## ▶ Identifying and Addressing Common Misconceptions

- Students sometimes incorrectly round the hour to the next higher hour when the hour hand is almost at the next number. Ask students what hour is represented on the clock shown.




Some students will round the hour up to 7 because the hour hand is so close to 7. Ask students whether the minute hand has completed a full hour or has completed only part of an hour. Because the minute hand has not finished a full hour, the hour hand has not yet reached the 7. Therefore, the time is 6:55, not 7:55.

- Some students may interpret $\frac{1}{2}$ hour as 50 minutes or $\frac{1}{4}$ hour as 25 minutes. Ask students how many minutes are in one full hour. Then ask how many minutes must be in a half hour or in a quarter hour. Remind students that each number on the clock is separated by 5 minutes, and, if necessary, confirm this by skip-counting by 5 twelve times. Then show students a circle divided into 2 or 4 equal parts. Show that the 3 on the clock is associated with $\frac{1}{4}$ of the circle and that skip-counting by 5 three times is 15 minutes. Similarly show that the 6 on the clock is associated with $\frac{1}{2}$ of the circle and that skip-counting by 5 six times is 30 minutes.

## ► Enrichment

- Ask students to determine the elapsed time of an event where the student must pay attention to whether the beginning and ending times are AM or PM. Use times that cross noon and times that cross midnight.
- Ask students to determine the time to the next full hour by reading 9:47 as “13 minutes to 10” because 47 is 13 less than 60.
- Give students the time on a digital clock and have them approximate the locations of the hands on an analog clock, noting that for a time such as 8:23, the minute hand is just more than halfway between the 4 and 5 and the hour hand is slightly less than halfway between the 8 and 9.

## ► Key Terms:

analog clock, half past an hour, interval, number line, quarter past the hour, quarter to an hour, tick mark

# D-M.1.1.1 & D-M.1.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Determine or calculate time and elapsed time.

## ► Eligible Content

D-M.1.1.1 Tell, show, and/or write time (analog) to the nearest minute.

D-M.1.1.2 Calculate elapsed time to the minute in a given situation (total elapsed time limited to 60 minutes or less).

## ► Instructional Supports

- Discuss how to tell time to the nearest minute by drawing on strategies of skip-counting by 5 and adding on any residual minutes that may be left over. Help students recognize that a clock is broken into 5-minute intervals. For example, give students the following situations.
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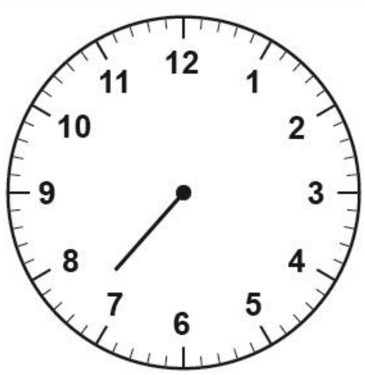

- Ask students to tell time to the nearest minute from a clock face using skip-counting. For example, give students the following picture.

The hour hand is past 6 but not quite to 7, so the hour must be 6. The minute hand is 2 tick marks past 10. Skip-counting by 5 ten times gives 50 minutes. Adding the 2 additional tick marks gives a total of 52 minutes. Therefore, the time shown on the clock is 6:52.

- Ask students to show that a clock can be divided into equal intervals representing $\frac{1}{4}$ or $\frac{1}{2}$ of an hour, just as a circle can be divided into parts equal to $\frac{1}{4}$ or $\frac{1}{2}$ of the circle. These fractional representations of an hour can be used to identify times.
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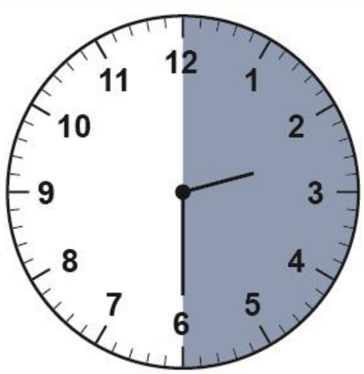
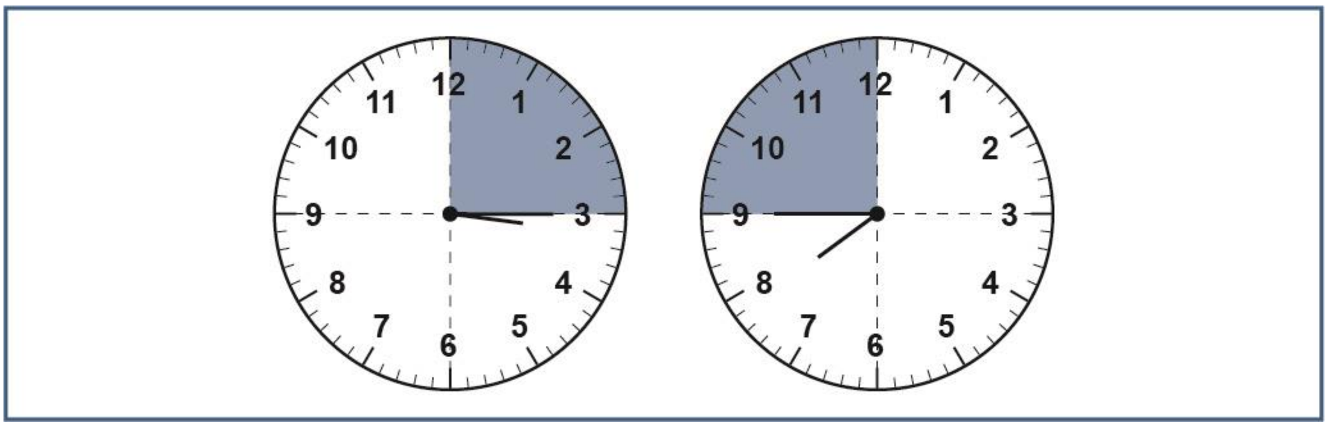
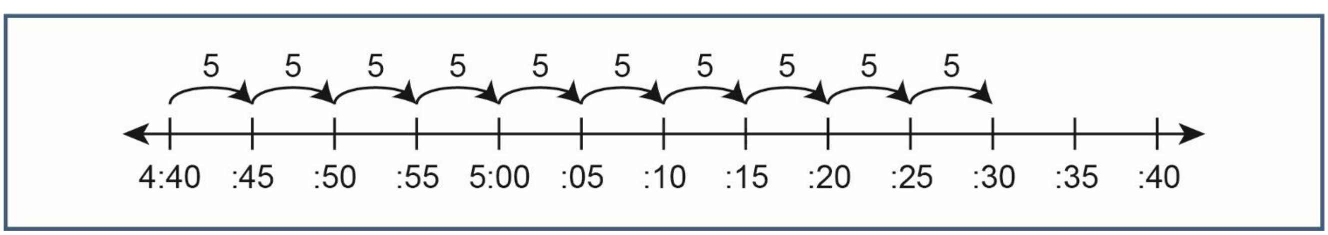


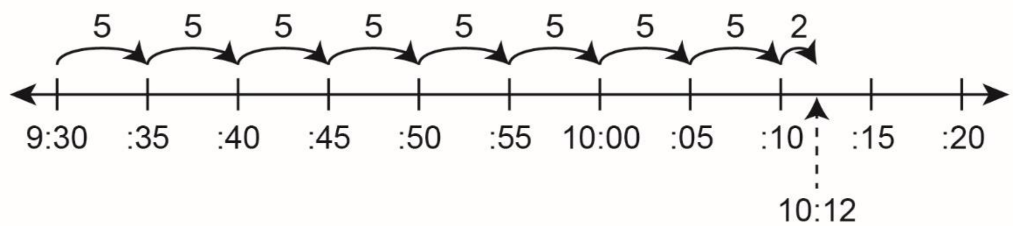
- In the following diagram, both clocks show an hour divided into 4 equal parts, which each correspond to fourths or quarters of the hour. In the clock on the left, the hour hand is between the 3 and 4, so the hour is 3. Because the minute hand started at 12 and moved one quarter of the way around the circle, the time can be read as "one quarter past 3." For the clock on the right, the hour hand is between the 7 and 8, which indicates that the hour is still 7 and not quite 8. The minute hand moved three-quarters of the way toward completing the 7 o'clock hour and reaching 8 o'clock, leaving one quarter of an hour left to go. Therefore, the time can also be read as "one quarter to eight."




- Discuss with students how to use their previous understanding of number line diagrams or tape diagrams to determine the time an event will be completed.
  - Ask students to create modified diagrams where the whole numbers represent hours and each tick mark between the hours represents a 5-minute interval. These diagrams can be used to represent transitions from one hour to the next. For example, if a class begins at 4:40 and ends at 5:30, how many minutes long is the class? If a student creates a number line that starts at 4:40 and skip-counts until they reach 5:30, they could make the diagram shown and conclude that the class would last 50 minutes.



- Ask students to count forward on a number line to determine the ending time of an event. For example, Shane is in a 42-minute class that started at 9:30. The ending time of the class can be determined by skip-counting forward on a number line by increments of 5 minutes from 9:30 until 10:10 and then adding on the remaining 2 minutes to get an end time of 10:12.




## ► Identifying and Addressing Common Misconceptions

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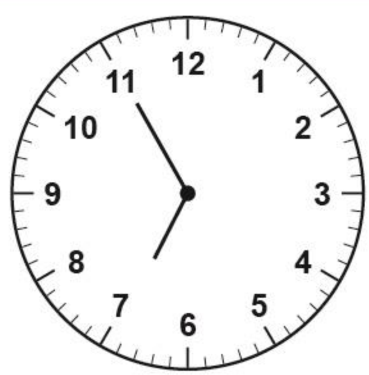



Some students will round the hour up to 7 because the hour hand is so close to 7. Ask students whether the minute hand has completed a full hour or has completed only part of an hour. Because the minute hand has not finished a full hour, the hour hand has not yet reached the 7. Therefore, the time is 6:55, not 7:55.

- Some students may interpret $\frac{1}{2}$ hour as 50 minutes or $\frac{1}{4}$ hour as 25 minutes. Ask students how many minutes are in one full hour. Then ask how many minutes must be in a half hour or in a quarter hour. Remind students that each number on the clock is separated by 5 minutes, and, if necessary, confirm this by skip-counting by 5 twelve times. Then show students a circle divided into 2 or 4 equal parts. Show that the 3 on the clock is associated with $\frac{1}{4}$ of the circle and that skip-counting by 5 three times is 15 minutes. Similarly show that the 6 on the clock is associated with $\frac{1}{2}$ of the circle and that skip-counting by 5 six times is 30 minutes.

## ► Enrichment

- Ask students to determine the elapsed time of an event where the student must pay attention to whether the beginning and ending times are AM or PM. Use times that cross noon and times that cross midnight.
- Ask students to determine the time to the next full hour by reading 9:47 as "13 minutes to 10" because 47 is 13 less than 60.
- Give students the time on a digital clock and have them approximate the locations of the hands on an analog clock, noting that for a time such as 8:23, the minute hand is just more than halfway between the 4 and 5 and the hour hand is slightly less than halfway between the 8 and 9.

## ► Key Terms:

analog clock, half past an hour, interval, number line, quarter past the hour, quarter to an hour, tick mark

# D-M.1.2.1 & D-M.1.2.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Use the attributes of liquid volume (capacity), mass, and length of objects.

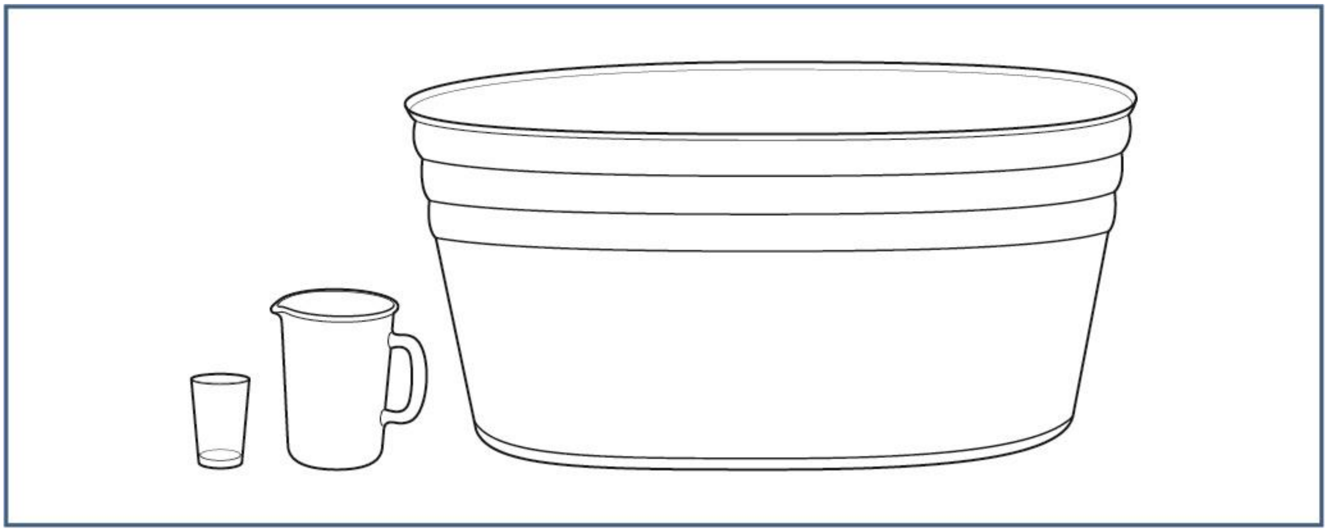
D-M.1.2.1 Measure and estimate liquid volumes and masses of objects using standard units (cups (c), pints (pt), quarts (qt), gallons (gal), ounces (oz.), and pounds (lb)) and metric units (liters (l), grams (g), and kilograms (kg)).

D-M.1.2.2 Add, subtract, multiply, and/or divide to solve one-step word problems involving masses or liquid volumes that are given in the same units.

## ► Instructional Supports

**What are mass and volume?**

- Discuss the relationship between the size of a container and the liquid volume of the container. The larger a container, the more capacity it has and therefore the more liquid it can hold. The capacity can be measured in terms of a fixed unit. For example, a container that can hold 1 liter of water is used 3 times to completely fill up a cooking pot. Therefore, the capacity of the cooking pot is 3 liters. Ask students to compare the relative capacities of different containers. For example, given a drinking glass, a pitcher, and a water trough, students order the items from least capacity to greatest capacity.

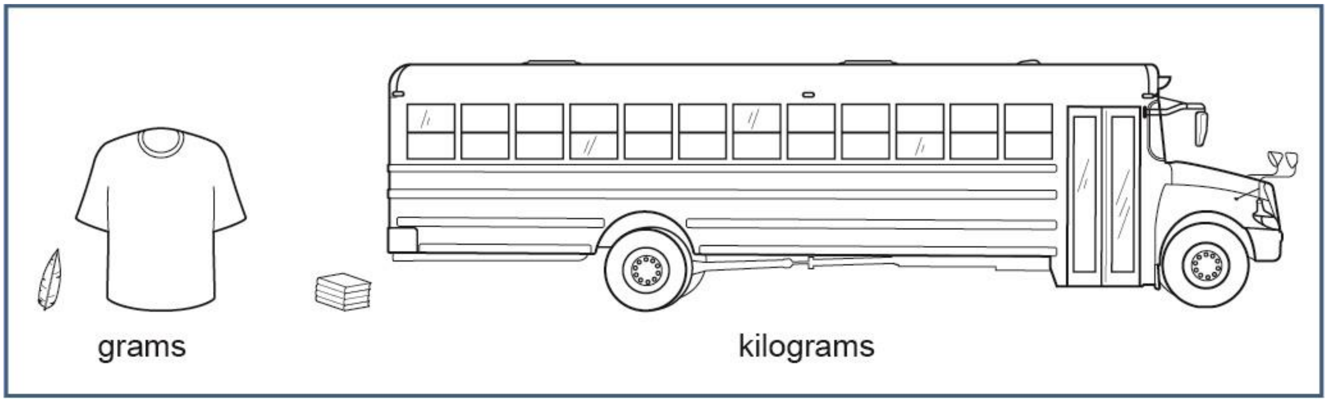


- Discuss with students the concept that mass and capacity are not related. For example, ask students to compare a folded paper grocery bag and an open paper grocery bag. The two bags have the same mass because they are made from the same amount of material, however, the opened paper bag has a greater capacity (i.e., more space inside) so that it can be filled with groceries.

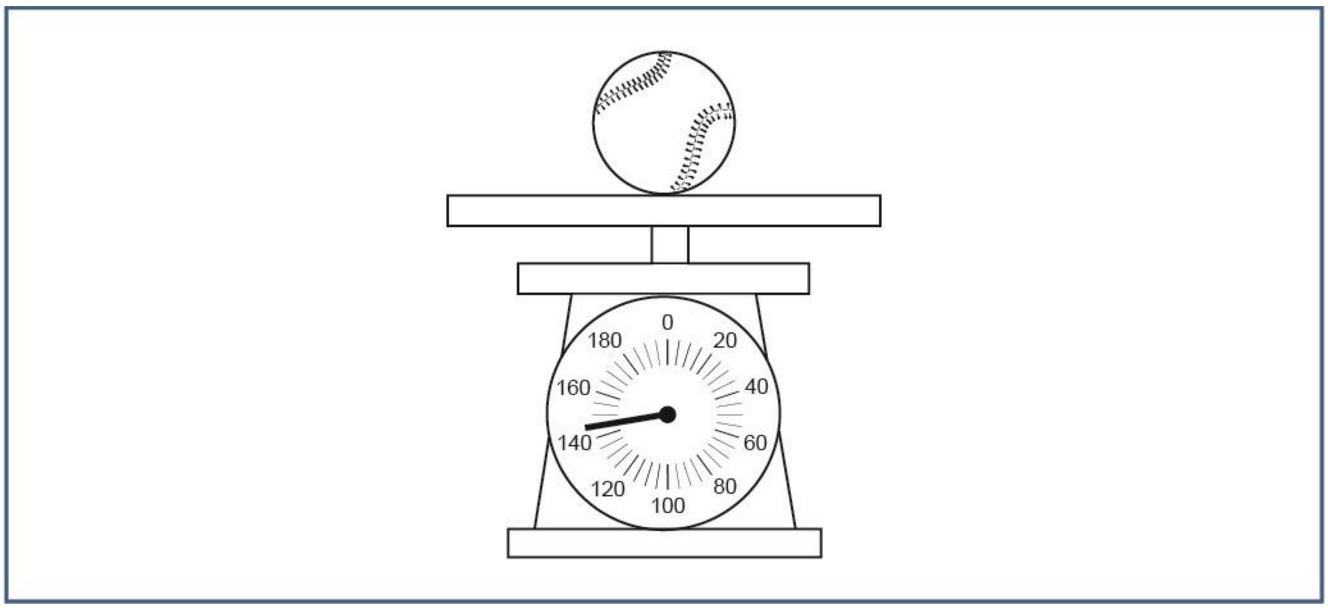
**Measuring mass and volume**

- Discuss the relative sizes of different units of measurement for mass. A kilogram is a better unit of measurement for larger objects such as a person or a car whereas a gram is a better unit of measurement for smaller objects such as a pencil or a number cube. One way to describe this is by asking students to think about whether they could easily pick up an object with one hand. If they could, the object is probably best measured in grams. If they could not easily pick it up, the object is probably best measured in kilograms.

- Ask students to determine which unit of measurement of mass would be best for a variety of objects. Given a list of several objects or pictures of objects, students determine which objects are best measured in grams and which are best measured in kilograms. For example, a feather and a shirt should both be measured in grams, whereas a stack of books and a school bus should both be measured in kilograms.



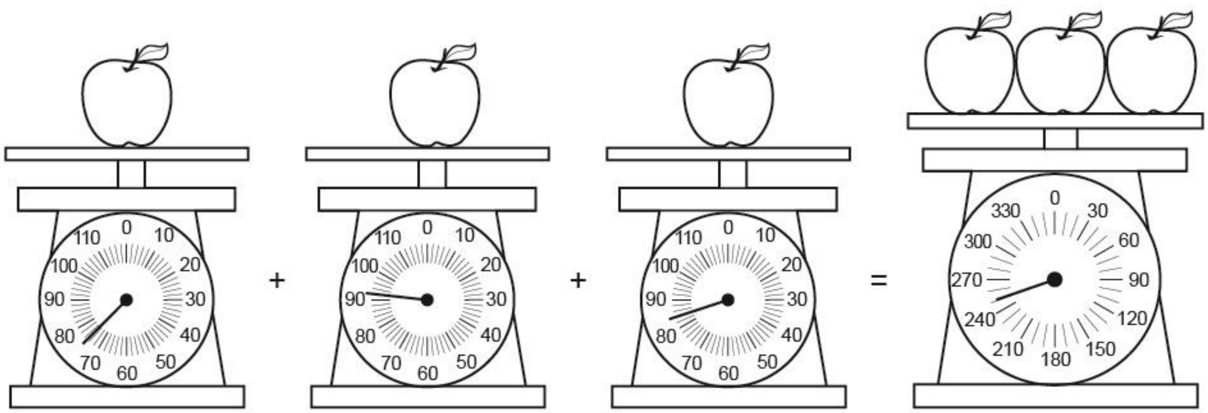

- Discuss how to measure the mass of objects using a scale. Perform multiple in-class demonstrations determining the mass or capacity of a variety of objects and have the students report and compare their results. For example, the scale shows a baseball with a mass of 145 grams.




- Ask students to measure the volume of liquid in a container. For example, given a 2-liter pitcher marked out by $\frac{1}{4}$-liter increments that is $\frac{3}{4}$ full, the students can determine that the amount of water in the container is $1\frac{1}{2}$ liters.


**Computations with mass and volume**

- Ask students to determine the total mass of multiple objects in a group. For example, if the masses of three apples are 75 grams, 92 grams, and 84 grams, the total mass of the apples is 251 grams because 75 + 92 + 84 = 251.




- Ask students to determine volume resulting from multiplication. For example, a bottle holds 2 liters of water. It takes 30 bottles to fill a fish tank, so the volume of the fish tank is 60 liters because 2 × 30 = 60.

## ► Identifying and Addressing Common Misconceptions

- Some students will incorrectly connect the concepts of size and mass, thinking that an object that is larger must have a greater mass. Ask students which would weigh more, a large box of paper towels or a 12-pack of soda. Students that say the large box of paper towels weighs more may have this misconception and need to have hands-on experiences working with large, light objects such as a bath sponge and small, heavy objects such as a brick.

- Some students will add or subtract measurements without considering the units or dimensions the unit is representing. Ask students to determine the combined mass of objects when the masses are given in different units such as a 10-kilogram bag of sugar and a 2-gram packet of sugar.
- Some students will not be able to recognize that containers of different shapes can have the same capacity. They may assume that if one container is taller than another, then it must hold more liquid. Ask students to estimate the capacities of two containers with different shapes but equal volumes. For example, find the volumes of a tall, narrow graduated cylinder and a short, wide beaker. Demonstrate that the two containers have the same volume by pouring the contents of one into the other.

## ► Enrichment

- Discuss other units of measurement such as milliliters and milligrams and their relative sizes for measuring the capacity and mass of extremely small objects, such as a small syringe of water or a grain of rice.
- Discuss other measuring tools for mass, such as a balance scale or a triple beam balance.
- Introduce students to the way the distance an object is from the fulcrum affects how a system balances and how a 1-gram object placed 5 units from the fulcrum can balance a 5-gram object placed 1 unit from the fulcrum.

## ► Key Terms:

capacity, gram, kilogram, liquid volume, liter, mass, matter

# D-M.1.2.1 & D-M.1.2.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Use the attributes of liquid volume (capacity), mass, and length of objects.

## ► Eligible Content

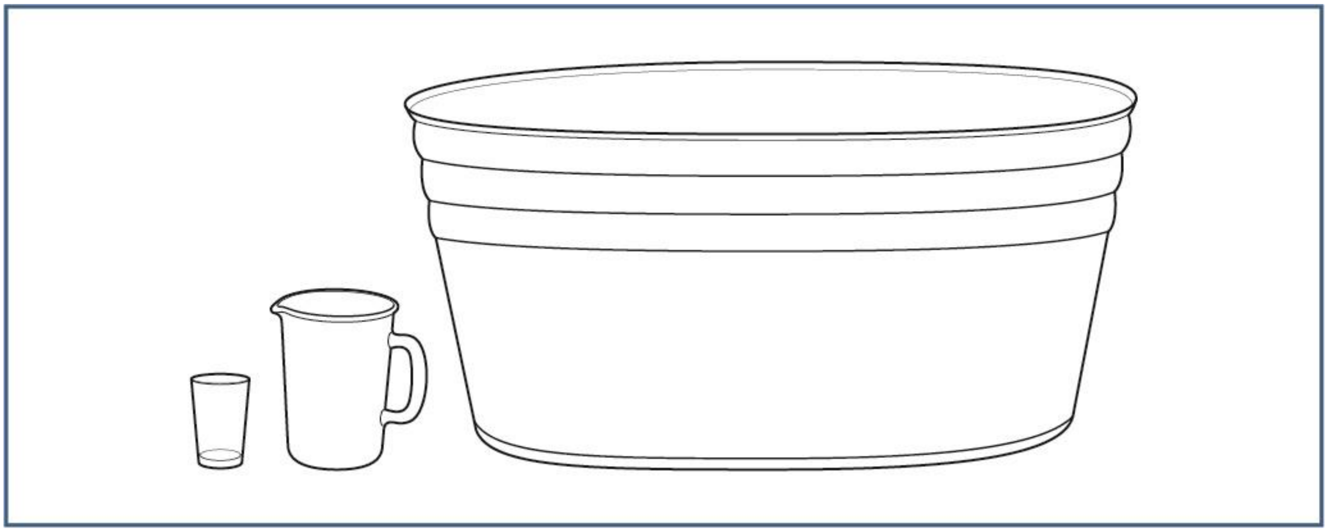
D-M.1.2.1 Measure and estimate liquid volumes and masses of objects using standard units (cups (c), pints (pt), quarts (qt), gallons (gal), ounces (oz.), and pounds (lb)) and metric units (liters (l), grams (g), and kilograms (kg)).

D-M.1.2.2 Add, subtract, multiply, and/or divide to solve one-step word problems involving masses or liquid volumes that are given in the same units.

## ► Instructional Supports

**What are mass and volume?**

- Discuss the relationship between the size of a container and the liquid volume of the container. The larger a container, the more capacity it has and therefore the more liquid it can hold. The capacity can be measured in terms of a fixed unit. For example, a container that can hold 1 liter of water is used 3 times to completely fill up a cooking pot. Therefore, the capacity of the cooking pot is 3 liters. Ask students to compare the relative capacities of different containers. For example, given a drinking glass, a pitcher, and a water trough, students order the items from least capacity to greatest capacity.

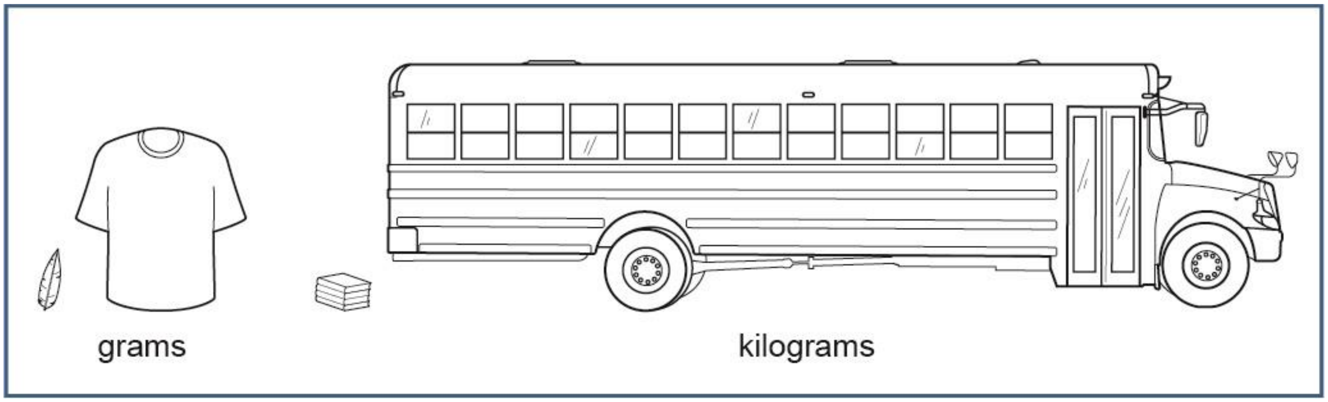


- Discuss with students the concept that mass and capacity are not related. For example, ask students to compare a folded paper grocery bag and an open paper grocery bag. The two bags have the same mass because they are made from the same amount of material, however, the opened paper bag has a greater capacity (i.e., more space inside) so that it can be filled with groceries.

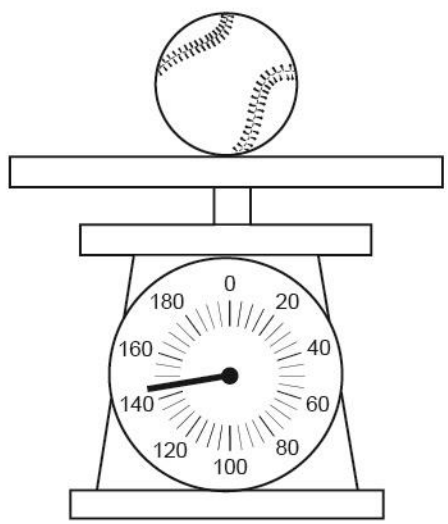
**Measuring mass and volume**

- Discuss the relative sizes of different units of measurement for mass. A kilogram is a better unit of measurement for larger objects such as a person or a car whereas a gram is a better unit of measurement for smaller objects such as a pencil or a number cube. One way to describe this is by asking students to think about whether they could easily pick up an object with one hand. If they could, the object is probably best measured in grams. If they could not easily pick it up, the object is probably best measured in kilograms.

- Ask students to determine which unit of measurement of mass would be best for a variety of objects. Given a list of several objects or pictures of objects, students determine which objects are best measured in grams and which are best measured in kilograms. For example, a feather and a shirt should both be measured in grams, whereas a stack of books and a school bus should both be measured in kilograms.



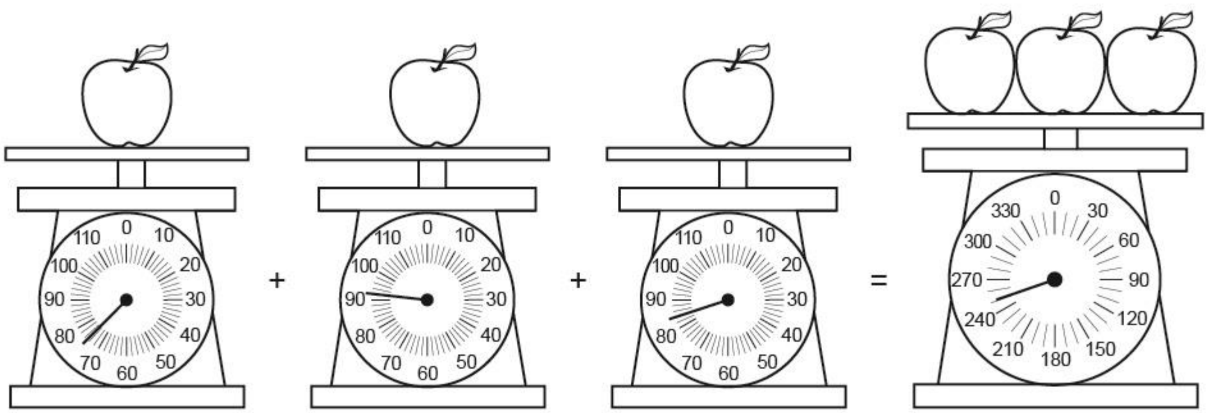

- Discuss how to measure the mass of objects using a scale. Perform multiple in-class demonstrations determining the mass or capacity of a variety of objects and have the students report and compare their results. For example, the scale shows a baseball with a mass of 145 grams.




- Ask students to measure the volume of liquid in a container. For example, given a 2-liter pitcher marked out by $\frac{1}{4}$-liter increments that is $\frac{3}{4}$ full, the students can determine that the amount of water in the container is $1\frac{1}{2}$ liters.


**Computations with mass and volume**

- Ask students to determine the total mass of multiple objects in a group. For example, if the masses of three apples are 75 grams, 92 grams, and 84 grams, the total mass of the apples is 251 grams because 75 + 92 + 84 = 251.



- Ask students to determine volume resulting from multiplication. For example, a bottle holds 2 liters of water. It takes 30 bottles to fill a fish tank, so the volume of the fish tank is 60 liters because 2 × 30 = 60.

## ► Identifying and Addressing Common Misconceptions

- Some students will incorrectly connect the concepts of size and mass, thinking that an object that is larger must have a greater mass. Ask students which would weigh more, a large box of paper towels or a 12-pack of soda. Students that say the large box of paper towels weighs more may have this misconception and need to have hands-on experiences working with large, light objects such as a bath sponge and small, heavy objects such as a brick.

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## ► Enrichment

- Discuss other units of measurement such as milliliters and milligrams and their relative sizes for measuring the capacity and mass of extremely small objects, such as a small syringe of water or a grain of rice.
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- Introduce students to the way the distance an object is from the fulcrum affects how a system balances and how a 1-gram object placed 5 units from the fulcrum can balance a 5-gram object placed 1 unit from the fulcrum.

## ► Key Terms:

capacity, gram, kilogram, liquid volume, liter, mass, matter

# D-M.1.2.3 & D-M.2.1.3

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Use the attributes of liquid volume (capacity), mass, and length of objects.

## ▶ Eligible Content

D-M.1.2.3 Use a ruler to measure lengths to the nearest quarter inch or centimeter.

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Represent and interpret data.

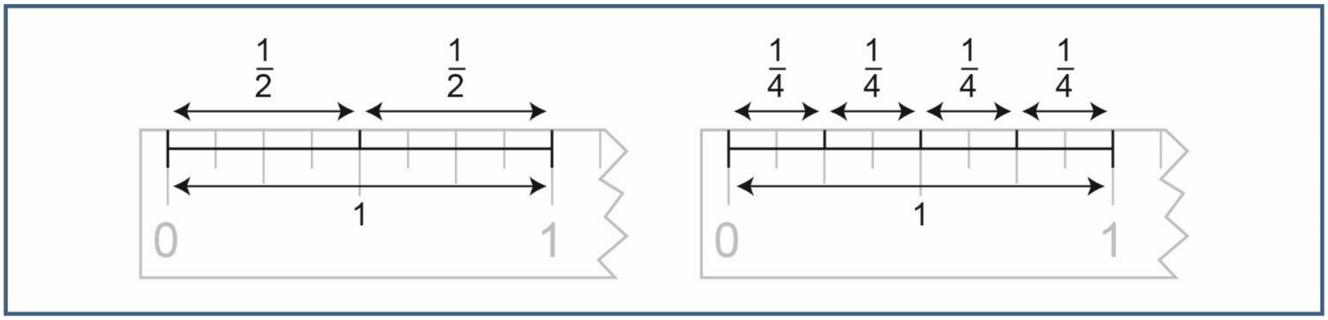
Organize, display, and answer questions based on data.

## ▶ Eligible Content

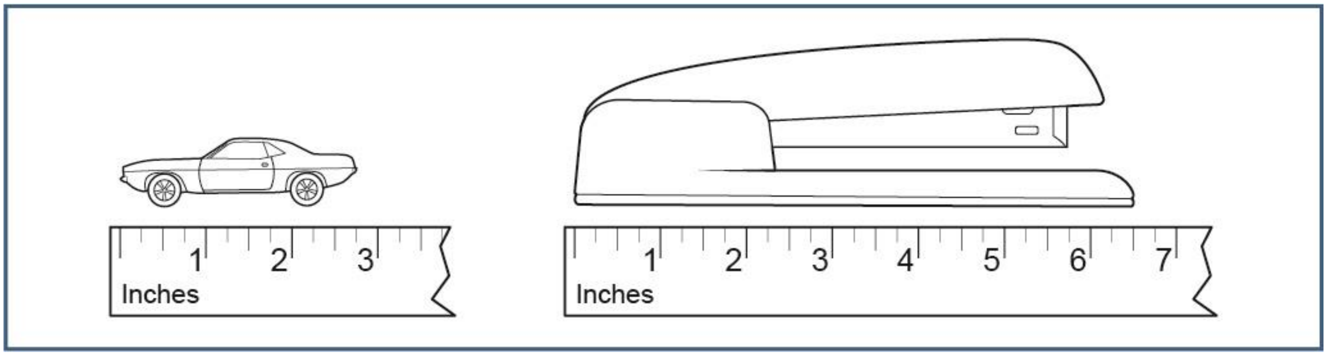
D-M.2.1.3 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Display the data by making a line plot, where the horizontal scale is marked in appropriate units—whole numbers, halves, or quarters.

## ▶ Instructional Supports

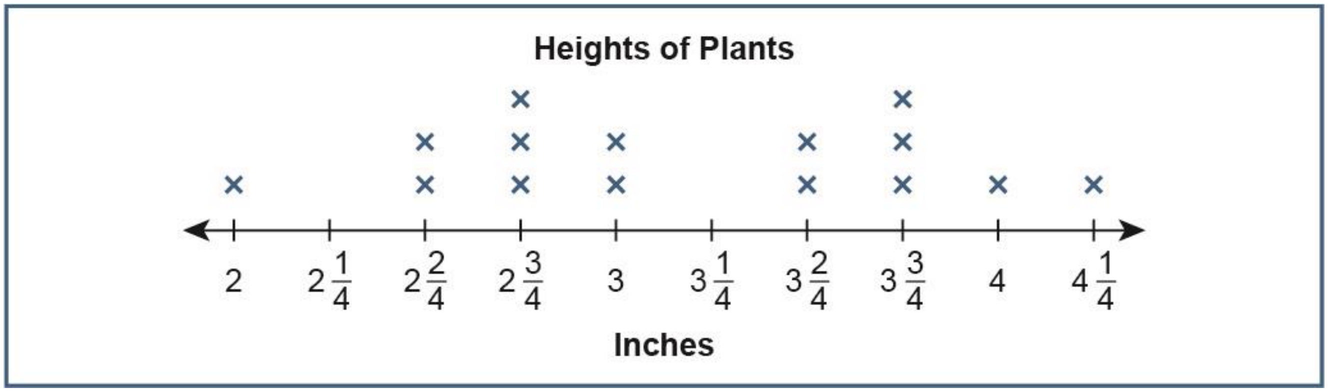
- Ask students to read a ruler to fractional values by recognizing that each tick mark between whole number units divides the whole into equally sized parts. For example, a ruler with one tick mark between whole units has each whole divided into 2 equal parts, or halves, and each part can be represented by the fraction $\frac{1}{2}$. Three tick marks divide whole units into 4 equal parts so the size of each part can be represented with the fraction $\frac{1}{4}$ or multiples of $\frac{1}{4}$ such as $\frac{2}{4}$ or $\frac{3}{4}$.




- Ask students to measure objects to the nearest quarter-inch or half-inch when an object is measured from the beginning of the ruler or measuring tape. For example, the measure of the toy car shown is $2\frac{3}{4}$ inches since the tick marks divide each inch into 4 equal parts, and the car starts at the 0 and ends at the third tick mark past the 2. The length of the stapler shown is $6\frac{1}{2}$ inches since the large tick marks divide each inch into 2 equal parts, and the stapler starts at 0 and ends at the large tick mark between the 6 and 7.



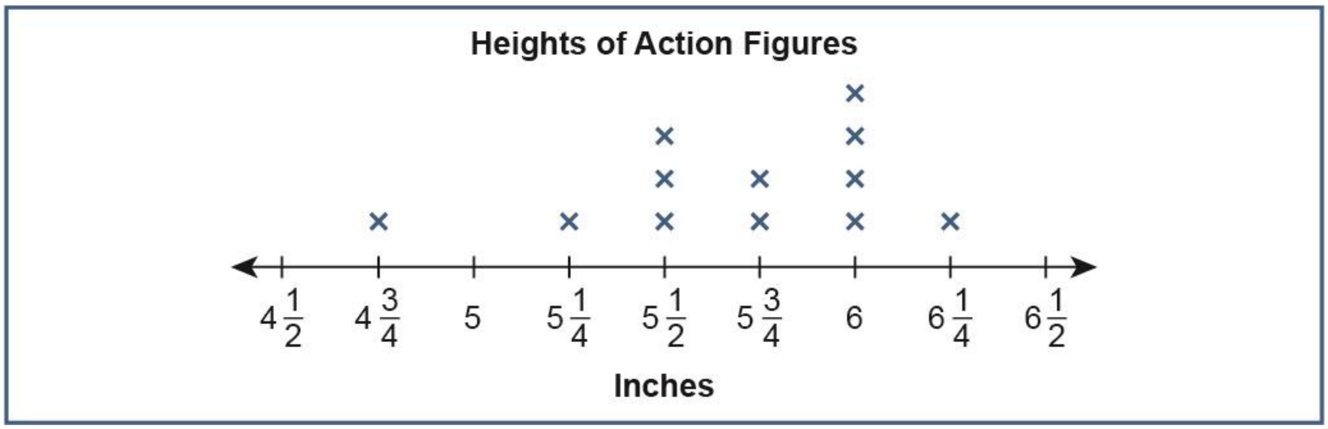

- Ask students to read information from a line plot. For example, give students the following line plot representing the heights of plants. Students can see that there are 5 plants with heights greater than $3\frac{2}{4}$ inches because there are 3 plants with heights of $3\frac{3}{4}$ inches, 1 plant with a height of 4 inches, and 1 plant with a height of $4\frac{1}{4}$ inches.




- Ask students to create a line plot from a set of measurements. For example, the following list shows the heights, in inches, of action figures.

  $$4\tfrac{3}{4}, 6, 5\tfrac{1}{2}, 5\tfrac{1}{4}, 6, 6\tfrac{1}{4}, 5\tfrac{1}{2}, 6, 5\tfrac{3}{4}, 5\tfrac{1}{2}, 5\tfrac{3}{4}, 6$$

  This information can be represented by the line plot shown.




## ► Identifying and Addressing Common Misconceptions

- Some students do not recognize the importance of starting their measurements from 0 on their rulers. Ask students to identify the length of an object on a broken ruler. Some students may read the length on the ruler without considering that the ruler does not start at 0. Ask students to measure the length by counting the tick marks on the ruler and ignoring the numbers shown. Discuss why it is easiest to align one end of the item with 0 on the ruler.
- When working with rulers marked in both quarters and halves, some students will count a single tick mark past the whole number as $\frac{1}{2}$ when asked to measure something to the nearest $\frac{1}{2}$. Ask the

students to measure an object, such as a piece of uncooked spaghetti, that is between 2 and $2\frac{1}{4}$ inches in length. Some students may read the measurement as $2\frac{1}{2}$ inches. Have these students fold strips of paper in halves and quarters to be able to better visualize where half is located between the end points. Then ask students to identify the measurement that each tick mark between 2 and 3 inches represents.

- Some students may have difficulty placing numbers on a number line divided into quarters when the label includes $\frac{1}{2}$ and not $\frac{2}{4}$. Ask students to locate $1\frac{1}{2}$ and $1\frac{2}{4}$ on a number line marked by fourths. Some students may locate $1\frac{1}{2}$ at the first tick mark past 1. Have students fold a strip of paper in half and draw a line on their fold. Then have the students fold the strip of paper in half again and draw a line on the folds with a different color. Show the students that the fold for $\frac{1}{2}$ and $\frac{2}{4}$ are at the same location.

## ► Enrichment

- Ask students to determine the combined length of 2 or more objects when placed end to end on a ruler or measuring tape.
- Ask students to determine the length of objects being measured with a ruler or measuring tape that does not start at 0.

## ► Key Terms:

data, distribution of data, fourths, halves, horizontal scale, line plot, quarters, ruler

# D-M.2.1.1 & D-M.2.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Represent and interpret data.

Organize, display, and answer questions based on data.

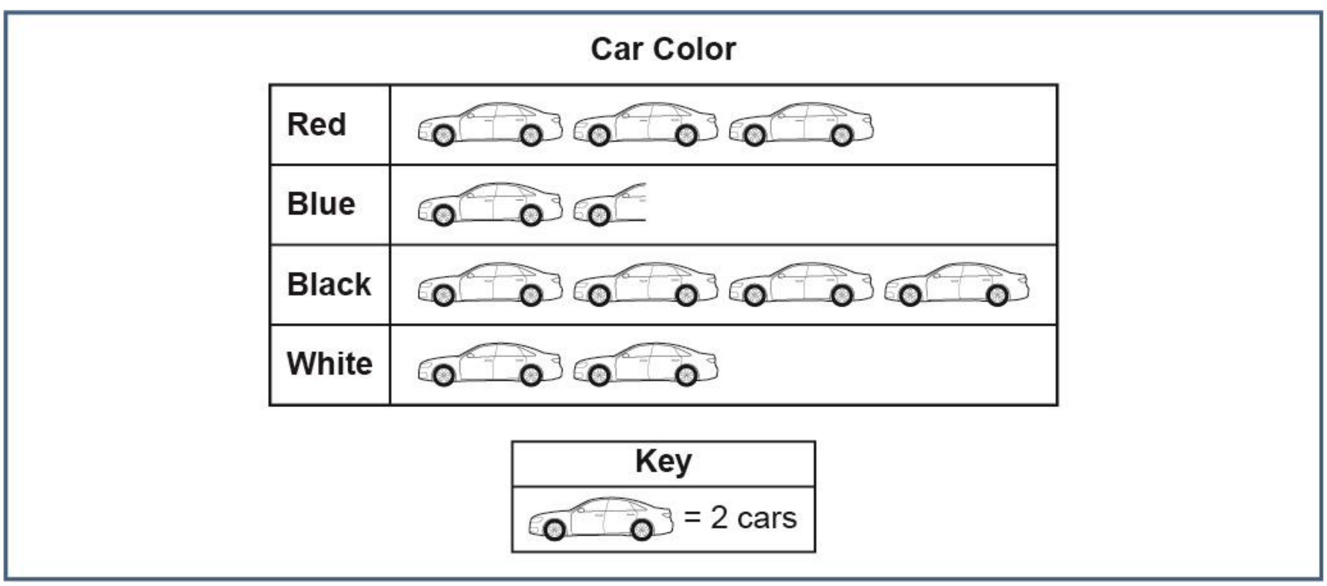
## ► Eligible Content

D-M.2.1.1 Complete a scaled pictograph and/or a scaled bar graph to represent a data set with several categories (scales limited to 1, 2, 5, and 10).

D-M.2.1.2 Solve one- and/or two-step problems using information to interpret data presented in scaled pictographs and/or scaled bar graphs (scales limited to 1, 2, 5, and 10).

## ► Instructional Supports

- Ask students to read information from a given picture graph and discuss the importance of looking at the key before interpreting any of the information in the picture graph. For example, give students the following picture graph representing the colors of cars in a parking lot.

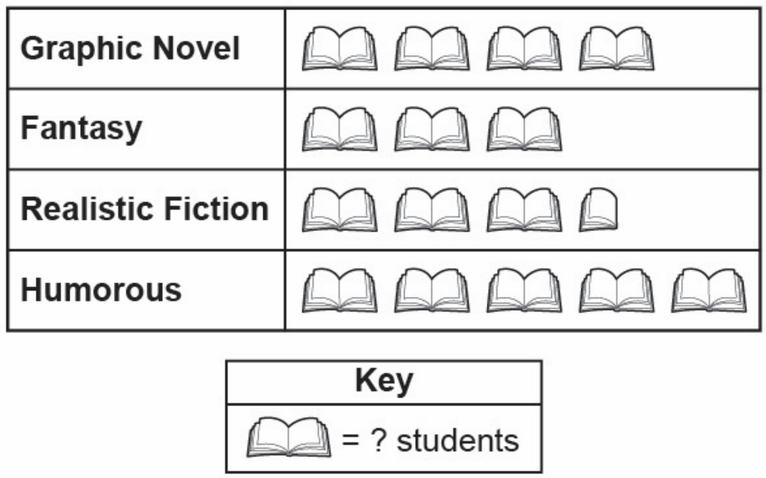


Students identify that the parking lot contains 6 red cars, 3 blue cars, 8 black cars, and 4 white cars. Discuss with students the importance of the key within the context and why the correct response is 6, 3, 8, 4 rather than 3, 1.5, 4, 2.

- Ask students to determine the scale of a picture graph that corresponds to information found in a table of values. For example, give students the following table of Favorite Book Genres and the picture graph containing the same information.

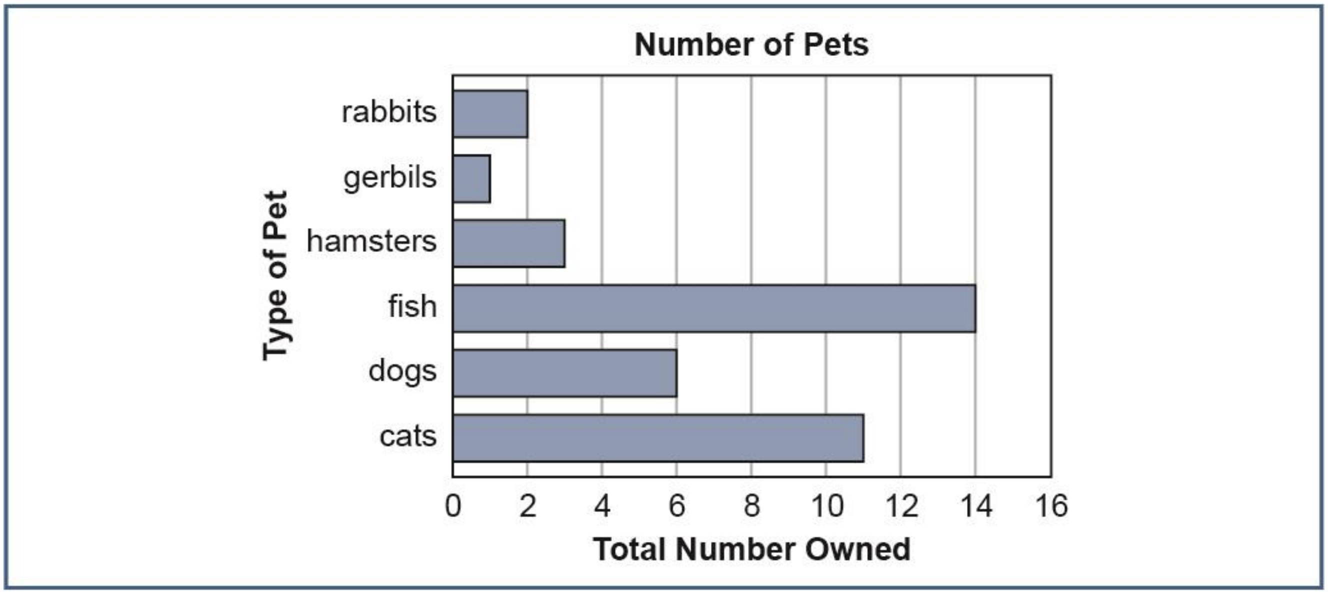
**Favorite Book Genres**

| | |
|---|---|
| Graphic Novel | 8 |
| Fantasy | 6 |
| Realistic Fiction | 7 |
| Humorous | 10 |



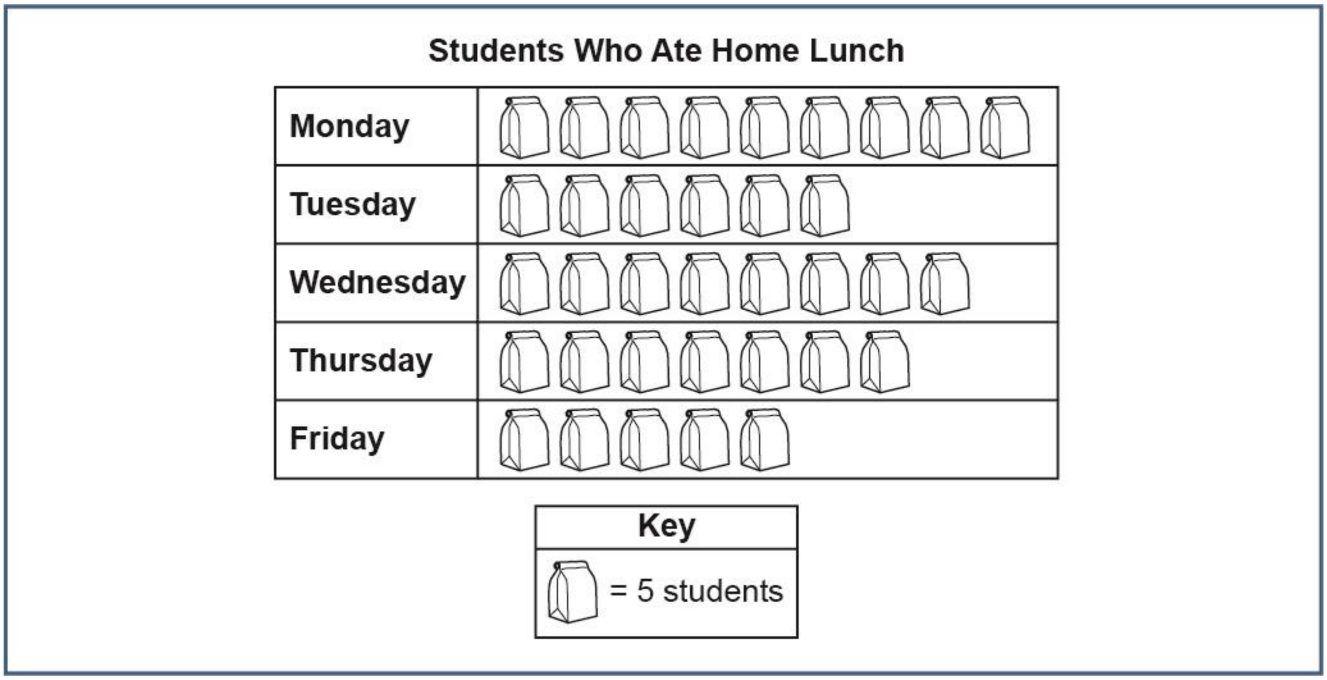

Using number sense to relate the table to the picture graph, students determine the scale of the picture graph is that 1 book represents 2 students.

- Ask students to compare information from two or more bars of a bar graph. For example, give students the following bar graph showing the number of pets owned by students in third grade. Ask them to determine how many more pet cats there are than pet dogs.




Students determine that there are 5 more pet cats than there are pet dogs based on the information in the bar graph.

- Ask students to compare information given in a picture graph. For example, give students the following picture graph showing the number of students who ate a lunch brought from home each day and ask them to determine how many fewer students ate a lunch from home on Friday than on Monday.




The key states that each paper bag represents 5 students, and there are 4 fewer paper bags shown on Friday than there are on Monday. The 4 fewer paper bags represent a total of 20 students. Therefore, there were 20 fewer students who brought lunch from home on Friday than on Monday.

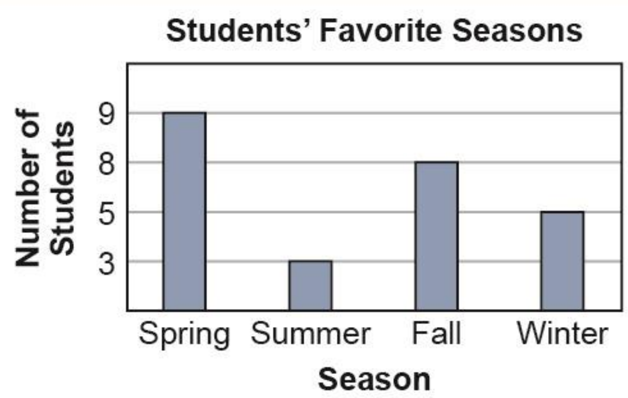
## ► Identifying and Addressing Common Misconceptions

- Some students may not pay attention to the scale of a picture graph. Give students a picture graph and ask what each image in the picture graph represents. Ask students to think of situations where one unit represents multiple objects, like a 6-pack of juice or a carton of 12 eggs. Help students draw parallels between those situations and pictures in a picture graph.
- Some students may create a bar graph that does not use correct scaling. Ask students to create a bar graph using the information in the following table.

**Students' Favorite Seasons**

| Season | Number of Students |
|---|---|
| Spring | 9 |
| Summer | 3 |
| Fall | 8 |
| Winter | 5 |

Some students may create the following bar graph.




Ask students to draw bars of heights 1, 2, 3, 4, 5, 6, 7, 8, and 9 on the graph. Discuss reasons why having bars with inconsistent height increments is problematic (e.g., a bar of height 4 might not be twice as tall as a bar of height 2). Help students create a new bar graph with consistent height increments.

## ► Enrichment

- Discuss the relationships between representing data in a picture graph and representing the same data in a bar graph. Ask students to explain why one option might sometimes be better than the other.
- Ask students to create a bar graph that displays the same data shown on a picture graph or to create a picture graph that displays the same data show on a bar graph.

## ► Key Terms:

bar graph, data, horizontal axis, pictorial key, picture graph, scale, vertical axis

# D-M.2.1.1 & D-M.2.1.2

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Represent and interpret data.

Organize, display, and answer questions based on data.

## ► Eligible Content

D-M.2.1.1 Complete a scaled pictograph and/or a scaled bar graph to represent a data set with several categories (scales limited to 1, 2, 5, and 10).

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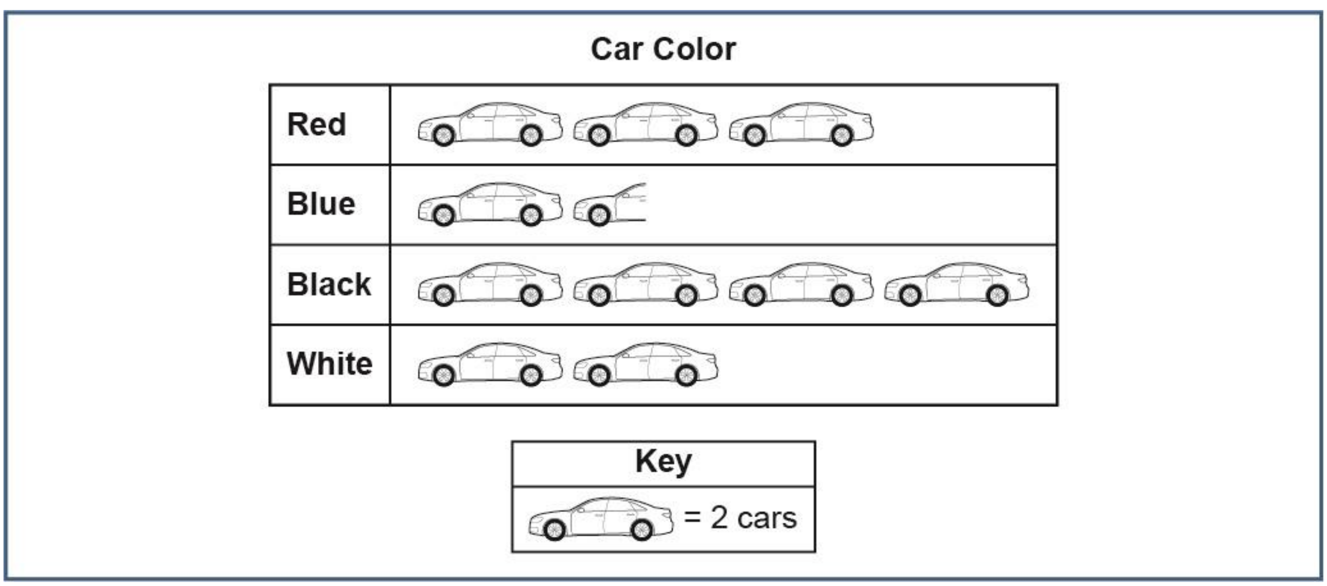

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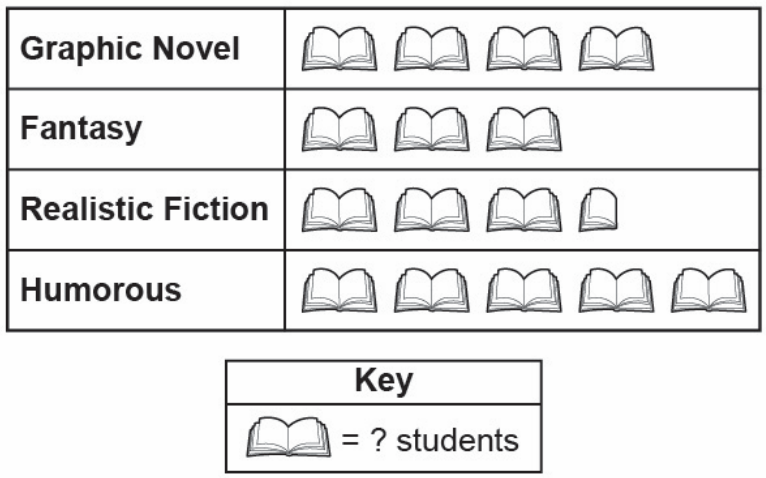


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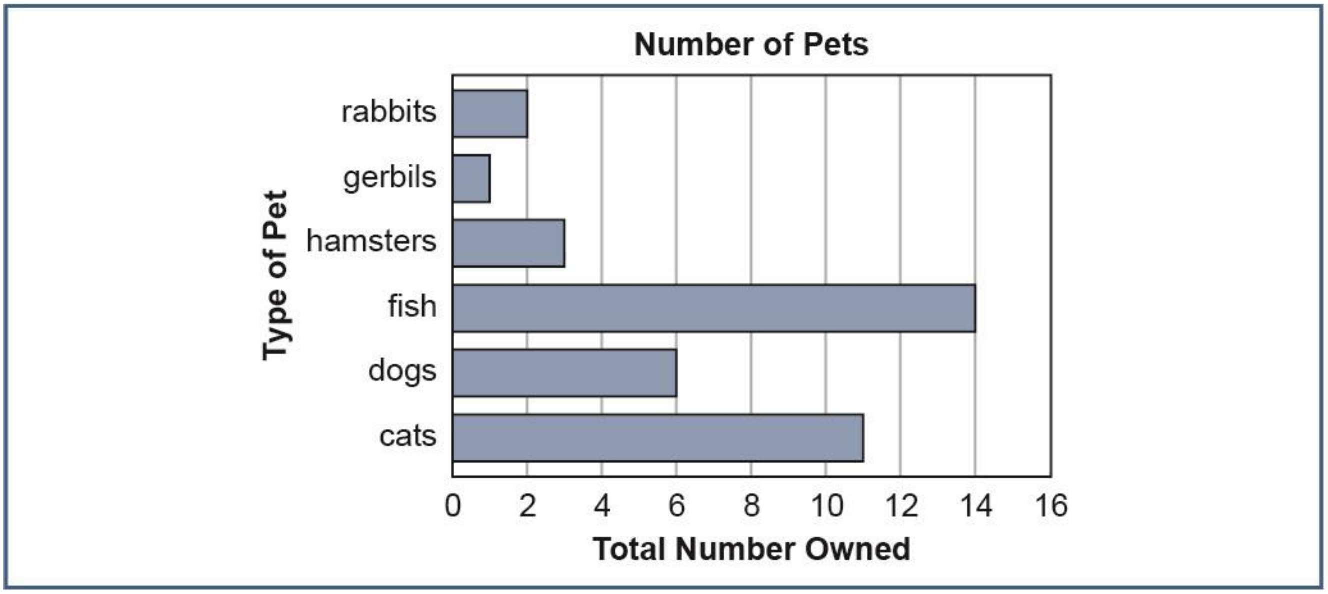


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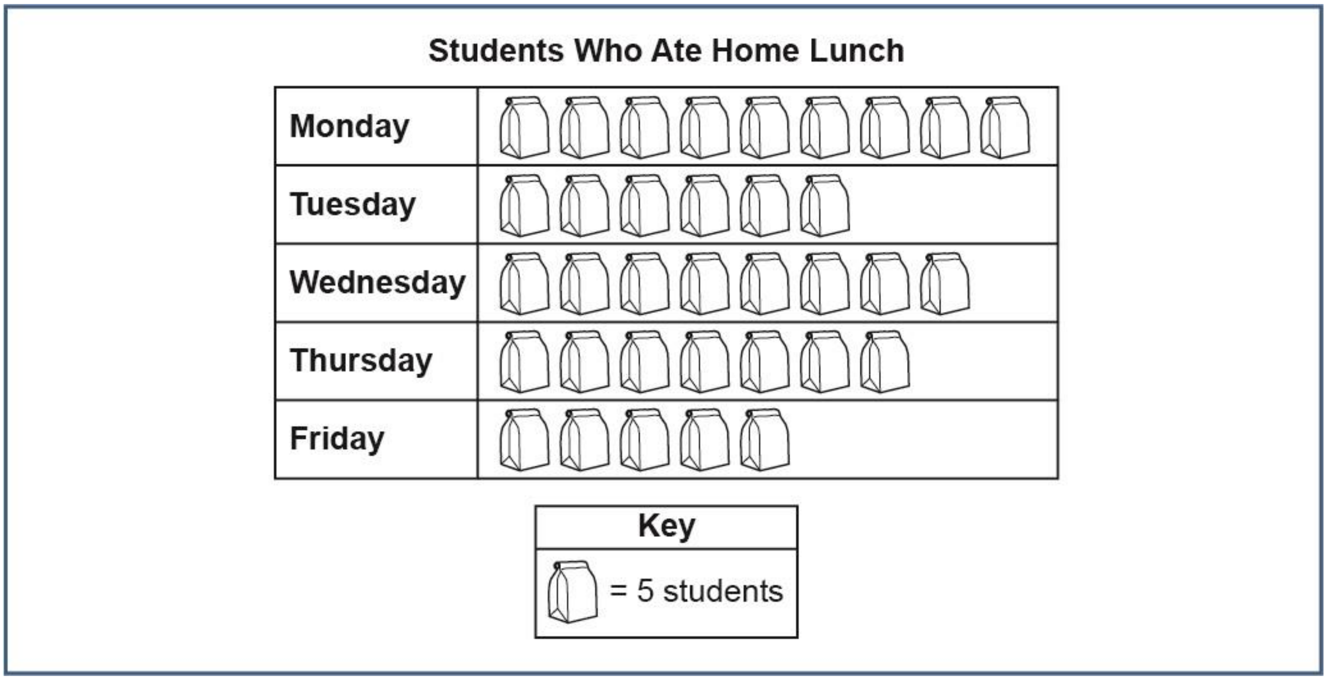


The key states that each paper bag represents 5 students, and there are 4 fewer paper bags shown on Friday than there are on Monday. The 4 fewer paper bags represent a total of 20 students. Therefore, there were 20 fewer students who brought lunch from home on Friday than on Monday.

## ► Identifying and Addressing Common Misconceptions

- Some students may not pay attention to the scale of a picture graph. Give students a picture graph and ask what each image in the picture graph represents. Ask students to think of situations where one unit represents multiple objects, like a 6-pack of juice or a carton of 12 eggs. Help students draw parallels between those situations and pictures in a picture graph.
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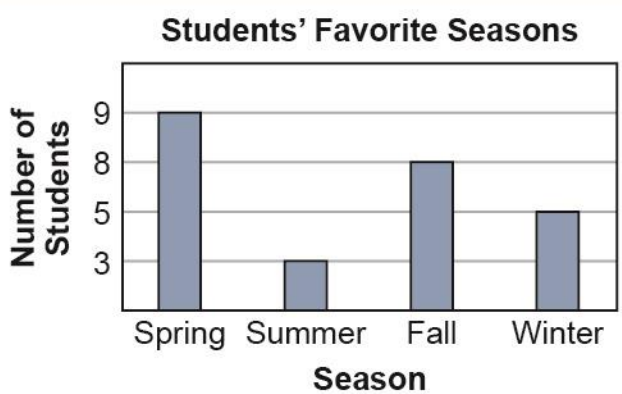



Ask students to draw bars of heights 1, 2, 3, 4, 5, 6, 7, 8, and 9 on the graph. Discuss reasons why having bars with inconsistent height increments is problematic (e.g., a bar of height 4 might not be twice as tall as a bar of height 2). Help students create a new bar graph with consistent height increments.

## ► Enrichment

- Discuss the relationships between representing data in a picture graph and representing the same data in a bar graph. Ask students to explain why one option might sometimes be better than the other.
- Ask students to create a bar graph that displays the same data shown on a picture graph or to create a picture graph that displays the same data show on a bar graph.

## ► Key Terms:

bar graph, data, horizontal axis, pictorial key, picture graph, scale, vertical axis

# D-M.1.2.3 & D-M.2.1.3

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Use the attributes of liquid volume (capacity), mass, and length of objects.

## ► Eligible Content

D-M.1.2.3 Use a ruler to measure lengths to the nearest quarter inch or centimeter.

## ► Reporting Category, Assessment Anchor, Anchor Descriptor

Represent and interpret data.

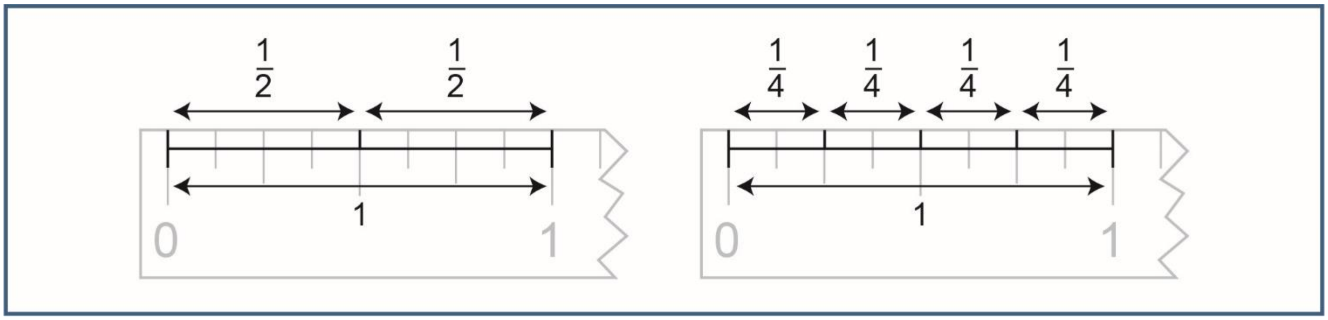
Organize, display, and answer questions based on data.

## ► Eligible Content

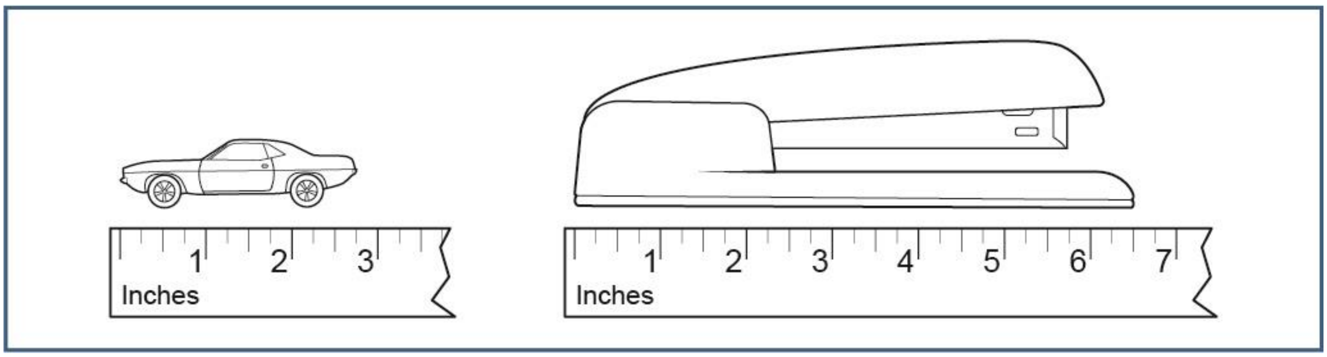
D-M.2.1.3 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Display the data by making a line plot, where the horizontal scale is marked in appropriate units—whole numbers, halves, or quarters.

## ► Instructional Supports

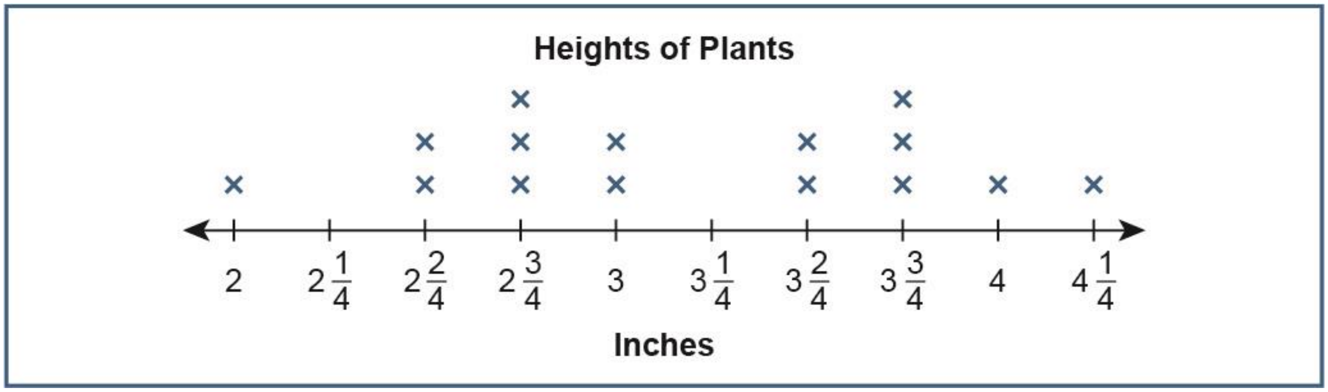
- Ask students to read a ruler to fractional values by recognizing that each tick mark between whole number units divides the whole into equally sized parts. For example, a ruler with one tick mark between whole units has each whole divided into 2 equal parts, or halves, and each part can be represented by the fraction $\frac{1}{2}$. Three tick marks divide whole units into 4 equal parts so the size of each part can be represented with the fraction $\frac{1}{4}$ or multiples of $\frac{1}{4}$ such as $\frac{2}{4}$ or $\frac{3}{4}$.




- Ask students to measure objects to the nearest quarter-inch or half-inch when an object is measured from the beginning of the ruler or measuring tape. For example, the measure of the toy car shown is $2\frac{3}{4}$ inches since the tick marks divide each inch into 4 equal parts, and the car starts at the 0 and ends at the third tick mark past the 2. The length of the stapler shown is $6\frac{1}{2}$ inches since the large tick marks divide each inch into 2 equal parts, and the stapler starts at 0 and ends at the large tick mark between the 6 and 7.



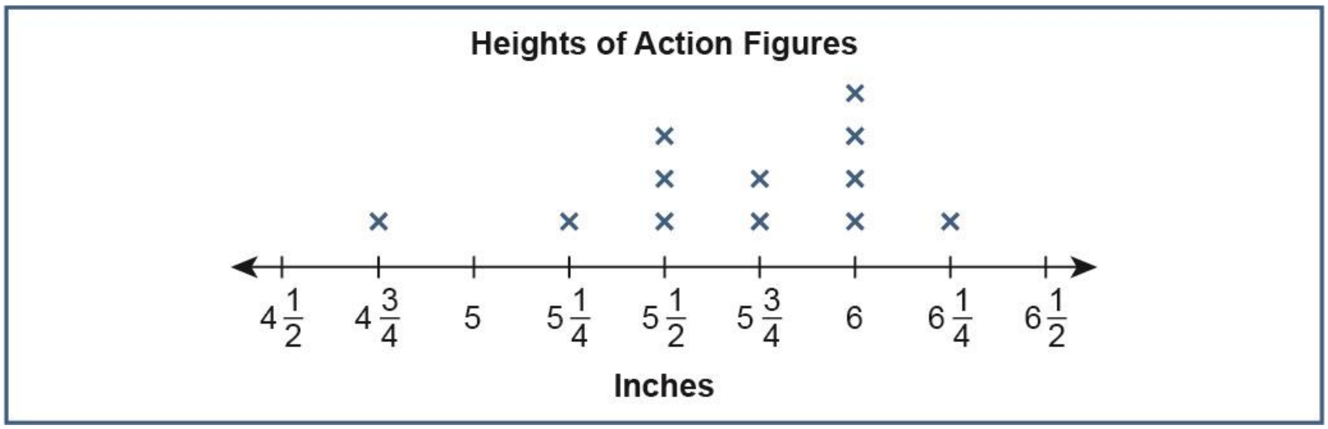

- Ask students to read information from a line plot. For example, give students the following line plot representing the heights of plants. Students can see that there are 5 plants with heights greater than $3\frac{2}{4}$ inches because there are 3 plants with heights of $3\frac{3}{4}$ inches, 1 plant with a height of 4 inches, and 1 plant with a height of $4\frac{1}{4}$ inches.




- Ask students to create a line plot from a set of measurements. For example, the following list shows the heights, in inches, of action figures.

$$4\frac{3}{4}, 6, 5\frac{1}{2}, 5\frac{1}{4}, 6, 6\frac{1}{4}, 5\frac{1}{2}, 6, 5\frac{3}{4}, 5\frac{1}{2}, 5\frac{3}{4}, 6$$

This information can be represented by the line plot shown.




## ► Identifying and Addressing Common Misconceptions

- Some students do not recognize the importance of starting their measurements from 0 on their rulers. Ask students to identify the length of an object on a broken ruler. Some students may read the length on the ruler without considering that the ruler does not start at 0. Ask students to measure the length by counting the tick marks on the ruler and ignoring the numbers shown. Discuss why it is easiest to align one end of the item with 0 on the ruler.

- When working with rulers marked in both quarters and halves, some students will count a single tick mark past the whole number as $\frac{1}{2}$ when asked to measure something to the nearest $\frac{1}{2}$. Ask the students to measure an object, such as a piece of uncooked spaghetti, that is between 2 and $2\frac{1}{4}$ inches in length. Some students may read the measurement as $2\frac{1}{2}$ inches. Have these students fold strips of paper in halves and quarters to be able to better visualize where half is located between the end points. Then ask students to identify the measurement that each tick mark between 2 and 3 inches represents.

- Some students may have difficulty placing numbers on a number line divided into quarters when the label includes $\frac{1}{2}$ and not $\frac{2}{4}$. Ask students to locate $1\frac{1}{2}$ and $1\frac{2}{4}$ on a number line marked by fourths. Some students may locate $1\frac{1}{2}$ at the first tick mark past 1. Have students fold a strip of paper in half and draw a line on their fold. Then have the students fold the strip of paper in half again and draw a line on the folds with a different color. Show the students that the fold for $\frac{1}{2}$ and $\frac{2}{4}$ are at the same location.

## ► Enrichment

- Ask students to determine the combined length of 2 or more objects when placed end to end on a ruler or measuring tape.
- Ask students to determine the length of objects being measured with a ruler or measuring tape that does not start at 0.

## ► Key Terms:

data, distribution of data, fourths, halves, horizontal scale, line plot, quarters, ruler

# D-M.3.1.1

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

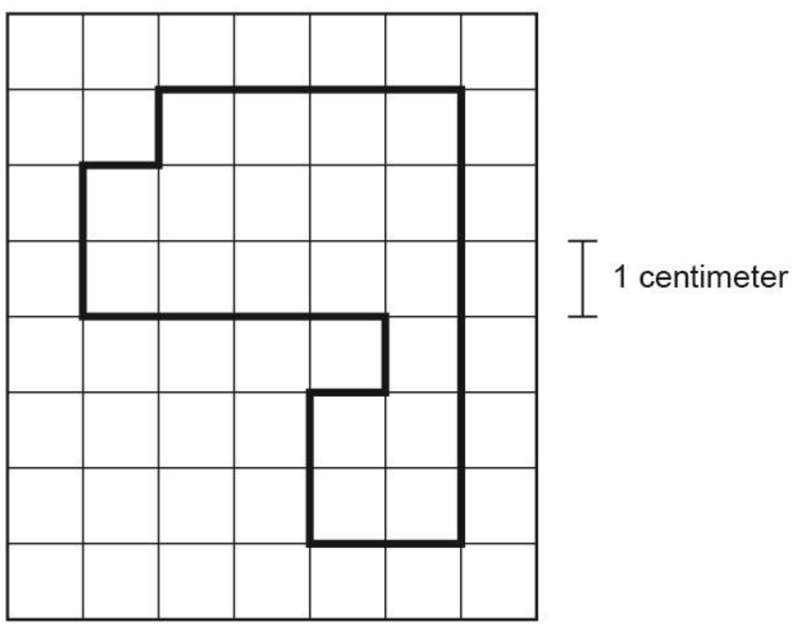
Find the areas of plane figures.

## ▶ Eligible Content

D-M.3.1.1 Measure areas by counting unit squares (square cm, square m, square in., square ft, and non-standard square units).

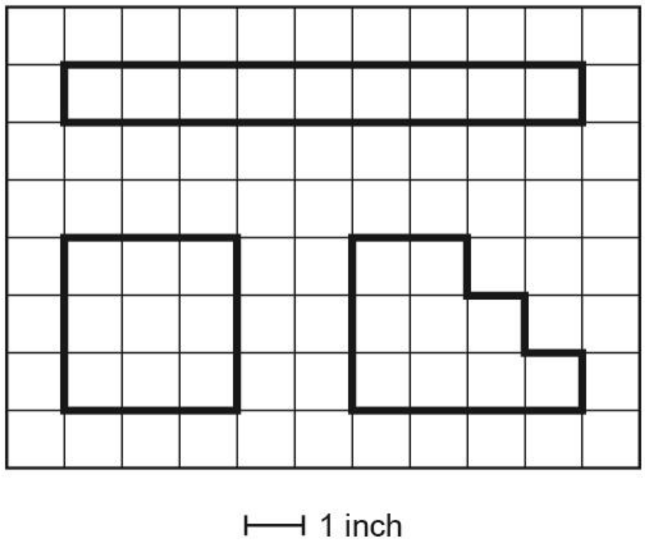
## ▶ Instructional Supports

- Ask students to measure the area of a shape drawn on graph paper by counting the number of unit squares in the figure. For example, give students the figure shown on a 1-centimeter grid.

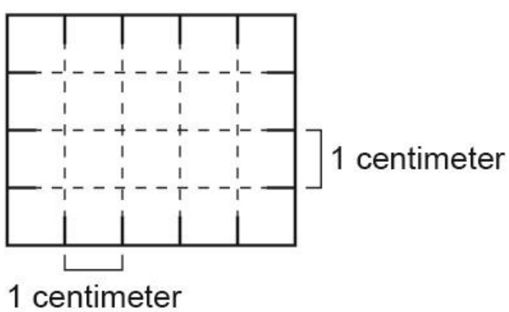



Because each segment of the grid is 1 centimeter, each square is 1 square centimeter. There are 19 squares in the figure, so the figure has an area of 19 square centimeters.

- Ask students to draw figures on graph paper with a given area. For example, ask students to draw multiple shapes with areas of 9 square inches. Some possible figures are shown.

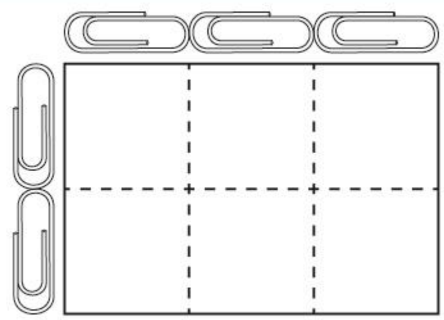



- Ask students to use a ruler to draw a 1-centimeter grid to measure the area of a given figure. For example, give students a rectangle that is 4 centimeters by 5 centimeters, without any measurement indicators shown. Ask them to use a ruler to mark each centimeter along the edges of the figure and then connect the lines to create the grid.



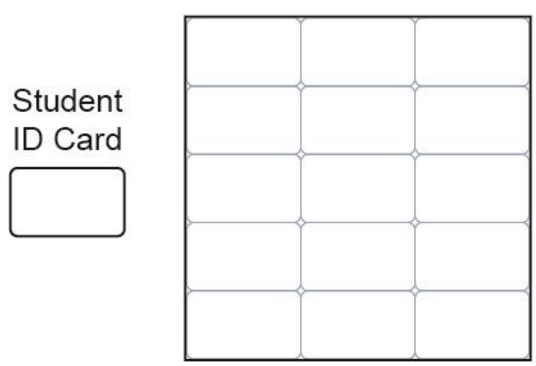

Students can count the squares to conclude that the rectangle has an area of 20 square centimeters.

- Ask students to measure the area of a figure using an improvised length unit. For example, give students a rectangle with a length that is equal to the length of 3 paper clips and a width that is equal to the length of 2 paper clips (but do not tell them the lengths). Ask students to use a paper clip to mark paper clip lengths along the edges of the figure and then connect the lines to create a grid.



Students can count the squares to conclude that the rectangle has an area of 6 square "paper clip lengths."

- Ask students to measure the area of a figure using an improvised rectangular area unit. For example, ask students to use the face of a student ID card to measure the area of a rectangle. A possible example is shown.




Students can describe the area of the rectangle as "15 student ID cards." Note that while most measures of area are square units where the unit in question is a linear unit, not all measures of area have this form. The "acre" is one such example.

## ► Identifying and Addressing Common Misconceptions

- Some students may incorrectly assume that each small square on a piece of grid paper has an area of 1 square inch or 1 square centimeter. Give students a piece of half-inch graph paper and ask them to shade in a square with an area of exactly 1 square inch. Some students may shade in the smallest possible square on the graph paper. Measure the length of each small square and then show that 2 side lengths are needed to make a whole inch. Because a square inch is a square with side lengths of 1 inch, the student should shade 4 of the small squares to make a larger 2 by 2 square with an area of 1 square inch.

- Some students may confuse units of area and length. Ask students to measure the sum of the side lengths of a square with sides that each measure 1 inch. Some students may give an answer of 4 square inches. Emphasize that the word “square” before a given unit means the unit is measuring area and shade the inside of a square to illustrate the concept. Emphasize that when a basic unit of length is given, such as inches or centimeters, then the unit is measuring length, and highlight the sides of the square to illustrate the concept.
- Some students may not consistently use the same side of an improvised object when measuring a shape. Give students a rectangular eraser and ask them to measure the area of a rectangle with a width equal to the length of 3 erasers and a length equal to the length of 4 erasers. Some students may use the short edge of the eraser to measure one side of the rectangle and the long edge of the eraser to measure the other side of the rectangle. Help students draw the grid and notice that the grid does not create squares. Therefore, the area units being used are not square units. Use this as an opportunity to point out the importance of being clear about units when measuring.

## ► Enrichment

- Ask students to explore the relationship between the size of a small square on graph paper and the number of squares needed to make a square with an area of 1 square unit. For example, with quarter-inch graph paper, each small square has side lengths that measure a quarter-inch, so a unit square will be a 4 by 4 square that contains 16 small squares. Likewise, with half-inch graph paper, each small square has side lengths that measure a half-inch, so a unit square will be a 2 by 2 square that contains 4 small squares.
- Ask students to explore the relationship between common units of area such as square inches and square feet. Ask students to decompose a square with a side length of 1 foot into squares that each have an area of 1 inch and then determine how many 1-inch squares are needed to make a 1-foot square.

- Ask students to estimate areas of shapes that are not rectilinear by visualizing how partial squares may combine into complete squares.

## ▶ Key Terms:

area, plane figure, square centimeter, square foot, square inch, square meter, unit square

# D-M.3.1.2

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

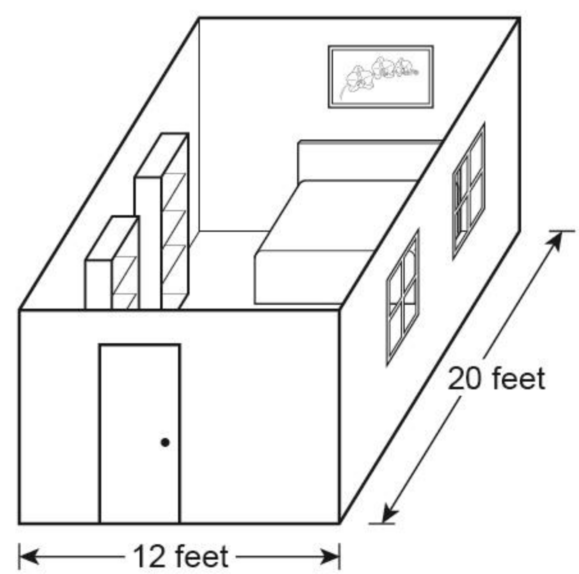
Find the areas of plane figures.

## ▶ Eligible Content

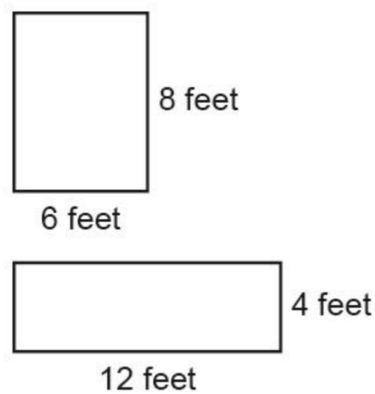
D-M.3.1.2 Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and/or mathematical problems, and /or represent whole-number products as rectangular areas in mathematical reasoning.

## ▶ Instructional Supports

- Ask students to determine the amount of material needed to cover a rectangular area given the side lengths. For example, Alberto wants to carpet a rectangular room with a length of 20 feet and a width of 12 feet. Students determine that he needs 240 square feet of carpet because $20 \times 12 = 240$.



- Ask students to determine possible whole-number side lengths of rectangles that have a given area. For example, Kayla has enough grass seed to cover a rectangle in her yard with an area of 48 square feet. What are the side lengths of a possible rectangular area she could cover? One possible rectangle she could cover has a length of 8 feet and a width of 6 feet. Another possible rectangle she could cover has a length of 12 feet and a width of 4 feet.




## ► Identifying and Addressing Common Misconceptions

- Some students may only consider proper factors when finding rectangles that have a given area. For example, ask students to find all the rectangles with side lengths that are whole numbers that have an area of 35 square units. Some students may respond that the only possible rectangle is one with a width of 5 units and a length of 7 units. Remind students that the product $1 \times 35 = 35$ and ask students to use this product to describe another rectangle.
- Some students may think that there are at most 2 unique rectangles that can be constructed with a given area. Ask students how many unique rectangles can be constructed with an area of 30 square units. Some students may only find 2 rectangles. Ask students to list factors of 30 and point out that each of those numbers can be a side length in a rectangle with that area.

## ► Enrichment

- Ask students to explore the relationship between the side lengths of rectangles that have the same area. For example, give students a rectangle that is 6 inches by 8 inches and a second rectangle that is 2 inches by 24 inches. Both rectangles have an area of 48 square inches. Have students compare 6 with 2 and compare 8 with 24. Students should note that one dimension of the first rectangle is three times as large as a dimension of the second rectangle and the other dimension of the first rectangle is three times as small as the other dimension of the second rectangle. That is, $6 \div 3 = 2$ and $8 \times 3 = 24$.

- Ask students to construct rectangles with areas that are whole numbers of square units but with side lengths that are fractions (or even mixed numbers) by cutting up paper unit squares.
- Ask students to determine an area measurement for which there exists only one unique rectangle with whole-number side lengths.
- Have students explore the number of rectangles that can be made with a given area and whole-number side lengths. Are there areas that can be represented by an even number of different rectangles? Are there areas that can be represented by an odd number of different rectangles? Students should pay attention to areas that are prime numbers or are perfect squares.

## ► Key Terms:

area, multiplication, product, rectangle, whole number

# D-M.4.1.1

## ▶ Reporting Category, Assessment Anchor, Anchor Descriptor

Measurement and Data

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Find and use the perimeters of plane figures.

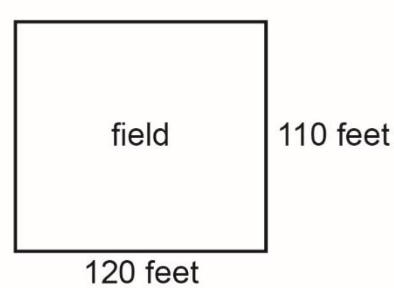
## ▶ Eligible Content

D-M.4.1.1 Solve real world and/or mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles. Rectangles may have the same perimeter and different areas or the same area and different perimeters. Use the same units throughout the problem.

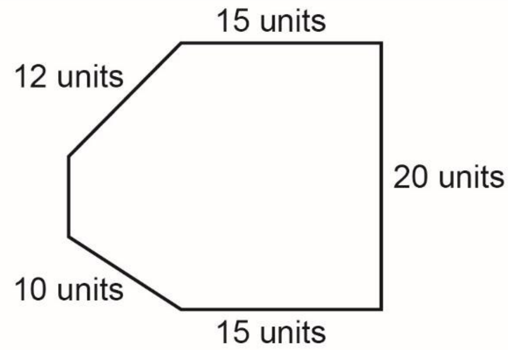
## ▶ Instructional Supports

**Perimeter**

- Ask students to determine the perimeter of a figure by adding the lengths of all the sides of the figure. For example, if a fence goes around a rectangular field with side lengths of 120 feet and 110 feet, the length of the fence is 460 feet (120 + 110 + 120 + 110 = 460).

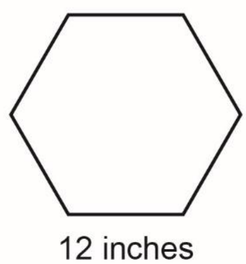



- Ask students to use given side lengths and the perimeter to determine unknown side lengths. For example, if a hexagon has a perimeter of 78 units and given side lengths of 12 units, 15 units, 20 units, 15 units, and 10 units as shown, ask students to find the missing side length.

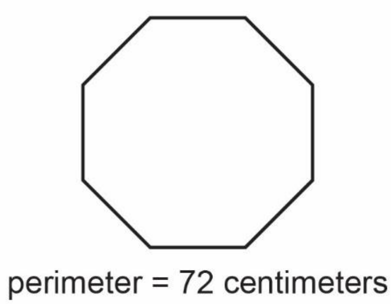



The length of the sixth side can be found by subtracting 78 – 12 – 15 – 20 – 15 – 10 = 6, so the length of the sixth side is 6 feet.

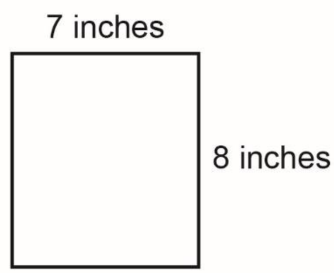
- Ask students to determine the perimeter of a regular polygon by multiplying the number of sides by the length of any of the sides. For example, a regular hexagon has side lengths of 12 inches. The perimeter of the hexagon can be calculated as 6 × 12 = 72 inches.




- Ask students to determine the side length of a regular polygon when given the perimeter of the polygon. For example, the perimeter of a regular octagon is 72 centimeters. The length of each side must be 9 centimeters because 72 ÷ 8 = 9.



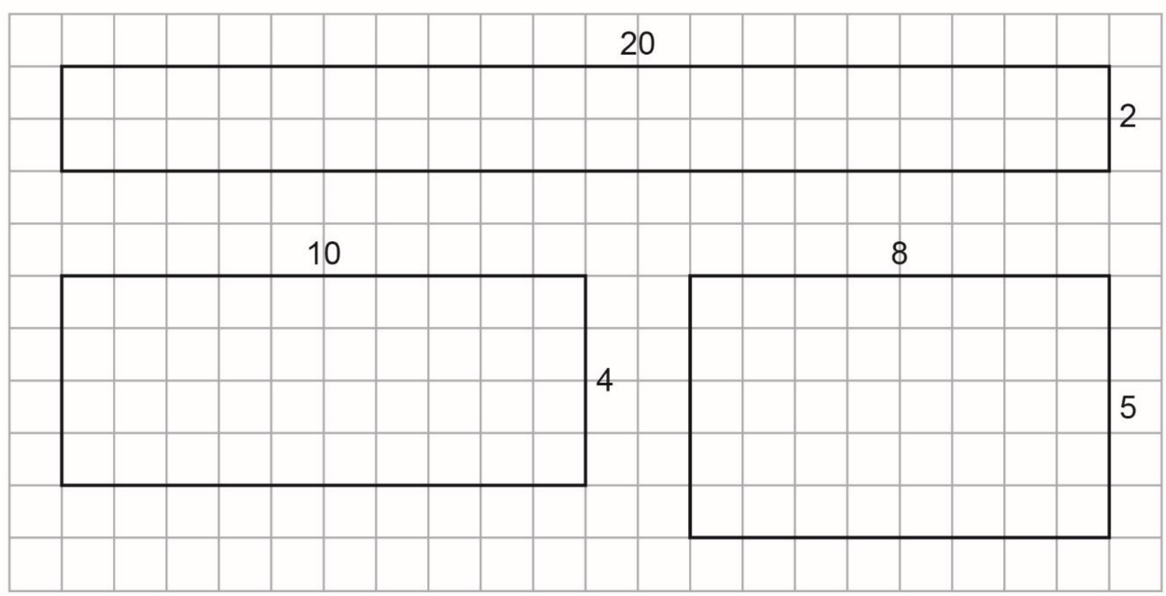
- Ask students to write different expressions to represent the perimeter of a rectangle that is not a square and have them use those expressions to calculate the perimeter. For example, give students the rectangle shown.




To calculate the perimeter of the given rectangle, students can find the sum of $(7 \times 2) + (8 \times 2)$, or students can find the sum of two adjacent sides and double it, expressed as $2 \times (8 + 7)$. Calculating the value of either expression gives a perimeter of 30 inches.

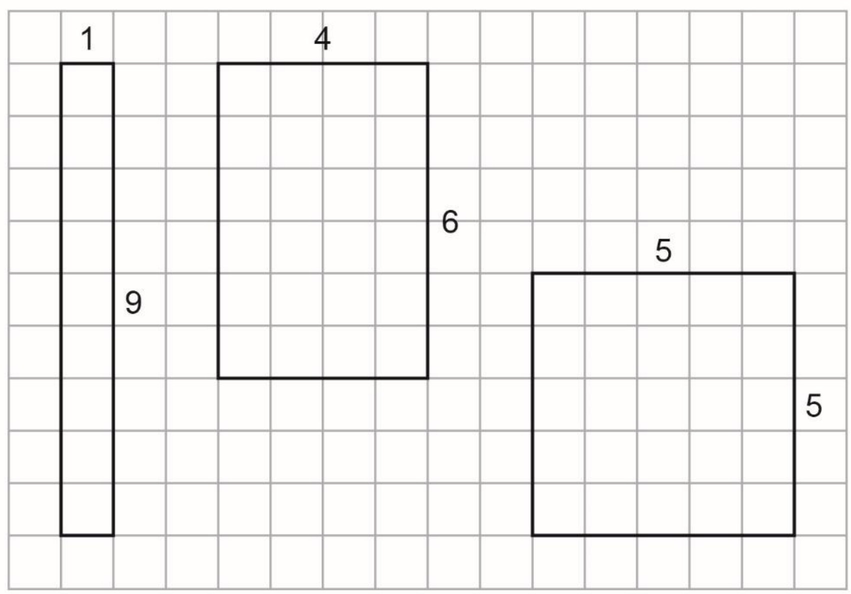
**Area and perimeter**

- Ask students to determine the possible perimeters of a rectangle with a given area. For example, ask students to find the possible perimeters of a rectangle that has an area of 40 square units. Some possible rectangles are shown.



The rectangles shown have perimeters of 44, 28, and 26 units. Other rectangles with different perimeters are also possible.

- Ask students to determine the possible areas of a rectangle with a given perimeter. For example, ask students to find the possible areas of a rectangle that has a perimeter of 20 units. Some possible areas are shown.




The rectangles shown have areas of 9, 24, and 25 square units. Other rectangles with different areas are also possible.

- Ask students to create a list of possible lengths and widths for rectangles that all have a given area and then predict which of the rectangles has the smallest perimeter. For example, using manipulatives, create sets of rectangles that have areas of 4, 9, and 64 square units, and then determine the perimeter of each rectangle. Ask students to make observations about the dimensions of the rectangle in each set that has the smallest perimeter. Students should observe that the rectangle with the smallest perimeter in each set is also a square, and that as the length and width become closer to equal, the perimeter of the rectangle decreases.

**Rectangles with Areas of 4 Square Units**

| Length | Width | Perimeter |
|---|---|---|
| 1 | 4 | 10 |
| 2 | 2 | 8 |
| 4 | 1 | 10 |

**Rectangles with Areas of 9 Square Units**

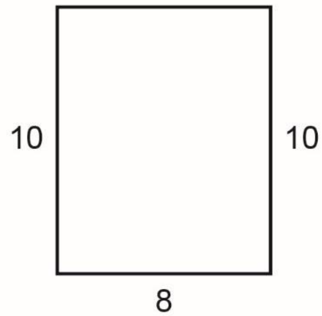
| Length | Width | Perimeter |
|---|---|---|
| 1 | 9 | 20 |
| 3 | 3 | 12 |
| 9 | 1 | 20 |

**Rectangles with Areas of 64 Square Units**

| Length | Width | Perimeter |
|---|---|---|
| 1 | 64 | 130 |
| 2 | 32 | 68 |
| 4 | 16 | 40 |
| 8 | 8 | 32 |
| 16 | 4 | 40 |
| 32 | 2 | 68 |
| 64 | 1 | 130 |

## ▶ Identifying and Addressing Common Misconceptions

- Some students may attempt to find the perimeter by adding only the given side lengths. Ask students to find the perimeter of the rectangle shown.

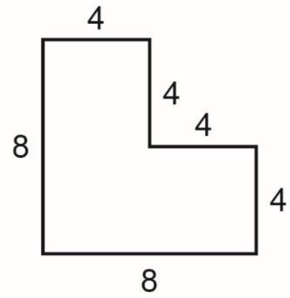



Some students may respond that the perimeter is 28 units because 10 + 10 + 8 = 28. Ask students the length of the missing side. Reinforce that the perimeter is the sum of all the sides and that if any sides are unlabeled, the lengths of those sides need to be determined in order to calculate the perimeter.

- Some students may attempt to find the side lengths of a polygon given the perimeter by dividing the perimeter by the number of sides the polygon has, even if the polygon is not a regular polygon. Ask students if they are able to calculate the side lengths of a hexagon with a perimeter of 60 inches. Some students may respond that each side length is 10 inches. Explain the distinction between a polygon and a regular polygon and that, for the example problem, more information is needed to find the perimeter of each side—for example, the lengths of five of the sides, or the fact that the hexagon is regular.
- Some students may reverse the calculations for area and perimeter, particularly when creating lists of possible rectangles that have a given perimeter and then calculating the area of each perimeter. Ask students to list the dimensions of three unique rectangles that each have a perimeter of 24 units and then to list the area of each rectangle. Some students may give dimensions such as 4 by 6 combined with areas of 20 square units, reversing area and perimeter. Reinforce the distinct concepts of area and perimeter, pointing out that both depend on the side lengths of the rectangle but are calculated in different ways.

## ► Enrichment

- Give students graph paper, a geoboard, or another manipulative and ask them to construct a figure that has a perimeter with side lengths that are shown in a given expression. For example, give students the expression $(4 \times 8) + (4 \times 4)$. Students can construct the following shape.




- Ask students to determine all the different regular polygons that could have a given perimeter, given that the side lengths are equal and must be whole numbers. For example, given that a regular polygon has a perimeter of 90 inches, it could have 3, 5, 6, 9, 10, 15, 18, 30, 45, or 90 sides with whole-number lengths.
- Ask students to estimate the dimensions of a rectangle with a given area and the smallest possible perimeter when the given area is not a perfect square. For example, ask students to estimate which rectangle has the smallest possible perimeter while having an area of 24 square units. Students may note that a 4 by 6 rectangle has a perimeter of 20, so a square with a side length somewhere between 4 and 6 should have a perimeter of less than 20 units while still having an area of 24 square units. Furthermore, students can observe that if the area were 25 square units, the side length needed to minimize the perimeter would be 5, so the square with an area of 24 square units should have a side length just less than 5.

## ► Key Terms:

area, equivalent, perimeter, polygon